Evgeny Epelbaum, RUB

SFB 1245 Topical Lecture Week, IKP, TU Darmstadt
18.-20. August 2025

Effective theory of nuclear interactions

Concepts

Effective field theory, chiral perturbation theory, renormalization, predictive power, KSW vs Weinberg, power counting…

Methods

Effective Lagrangian, heavy-baryon expansion, perturbative calculation of the amplitude, methods to derive nuclear forces (and currents), …

erc European Research Council
Ministerium für Kultur und Wissenschaft des Landes Nordrhein-Westfalen
ERATO Exploratory Research for Advanced Technology

Day 1

Part I: General introduction to EFT

EFT philosophy, renormalization, power counting, construction principles…

Part II: Chiral perturbation theory

Chiral symmetry, effective Lagrangian, chiral expansion, loops, inclusion of nucleons, …

Day 2

Part III: Pionless EFT

Resummation of the amplitude, fine tuning, renormalization conditions, RG analysis,…

Part IV: Inclusion of pions

How to include pions non-perturbatively? Long-range physics and low-energy theorems…

Day 3

Part V: From $\mathscr{L}_{\rm eff}$ to nuclear forces

How to derive nuclear forces from the effective Lagrangian?

Part VI: Gradient flow method

How to introduce regularization in the way consistent with the symmetries?

V: From $\mathscr{L}_{\rm eff}$ to nuclear forces

How to derive nuclear forces from the effective Lagrangian?
What is the current state-of-the-art?

Outline

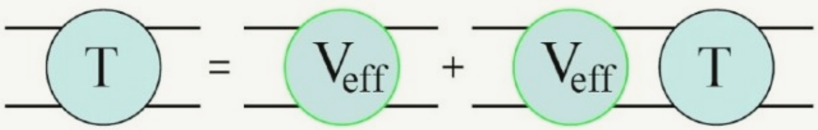
- Methods: S-matrix matching, TOPT, MUT, a path integral approach
- Example: chiral expansion of the 2π-exchange 3N force
- State-of-the-art for nuclear forces

Further reading

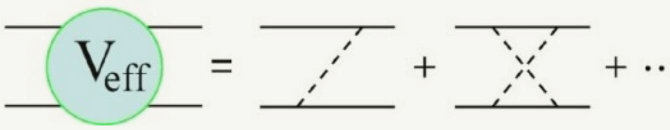
EE, PPNP 57 (2006) 654
EE, Hammer, Meißner, RMP 81 (2009) 1773
Entem, Machleidt, Phys. Rept. 503 (2011) 1
EE, Krebs, Reinert, Front. In Phys. 8 (2020) 98
Krebs, EE, PRC 110 (2024) 044003

Chiral EFT for nuclear systems

Ladder graphs are responsible for the failure of perturbation theory and must be re-summed:



Lippmann-Schwinger equation



derived in ChPT

Nuclear forces and currents = irreducible parts of the amplitude (scheme-dependent)

They can be derived using a variety of methods including [In all cases, utilize a perturba-tive expansion within ChPT]:

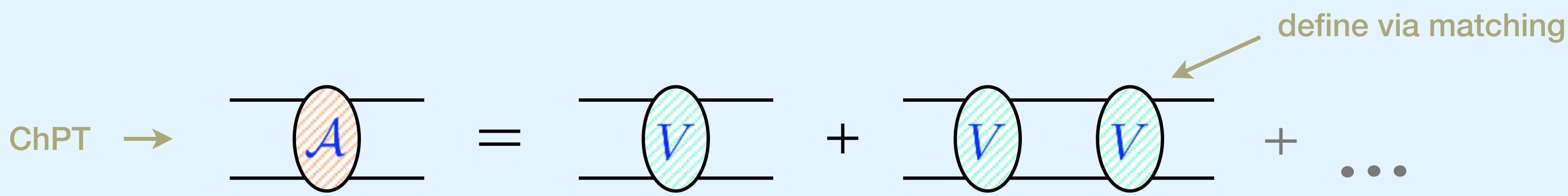
- — S-matrix matching Kaiser et al.
- — time-ordered perturbation theory Pastore, Baroni, Schiavilla et al.
- — method of unitary transformations (UTs) EE, Glöckle, Meißner, Krebs, Kölling
- — path integral approach Krebs, EE

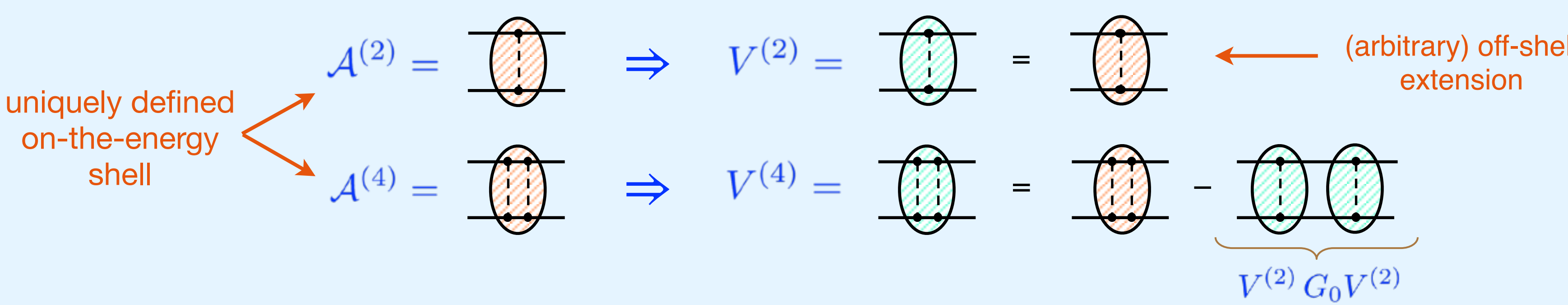
More demanding than just calculating Feynman diagrams:

- — need to subtract reducible pieces in order to avoid double counting
- — have to deal with non-uniqueness of nuclear potentials
- — maintaining renormalizability non-trivial…

S-matrix matching

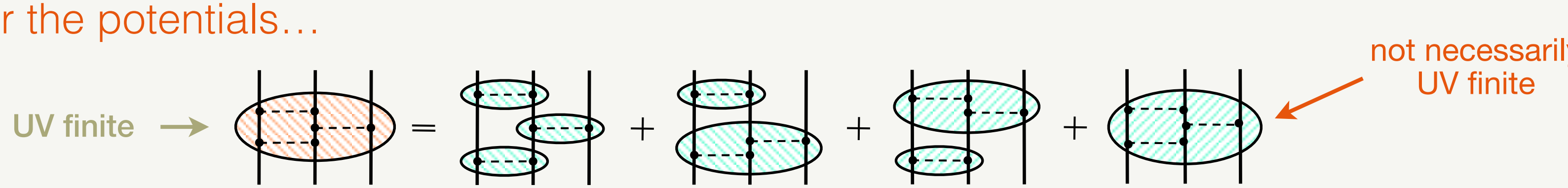
Matching to the amplitude Kaiser et al.





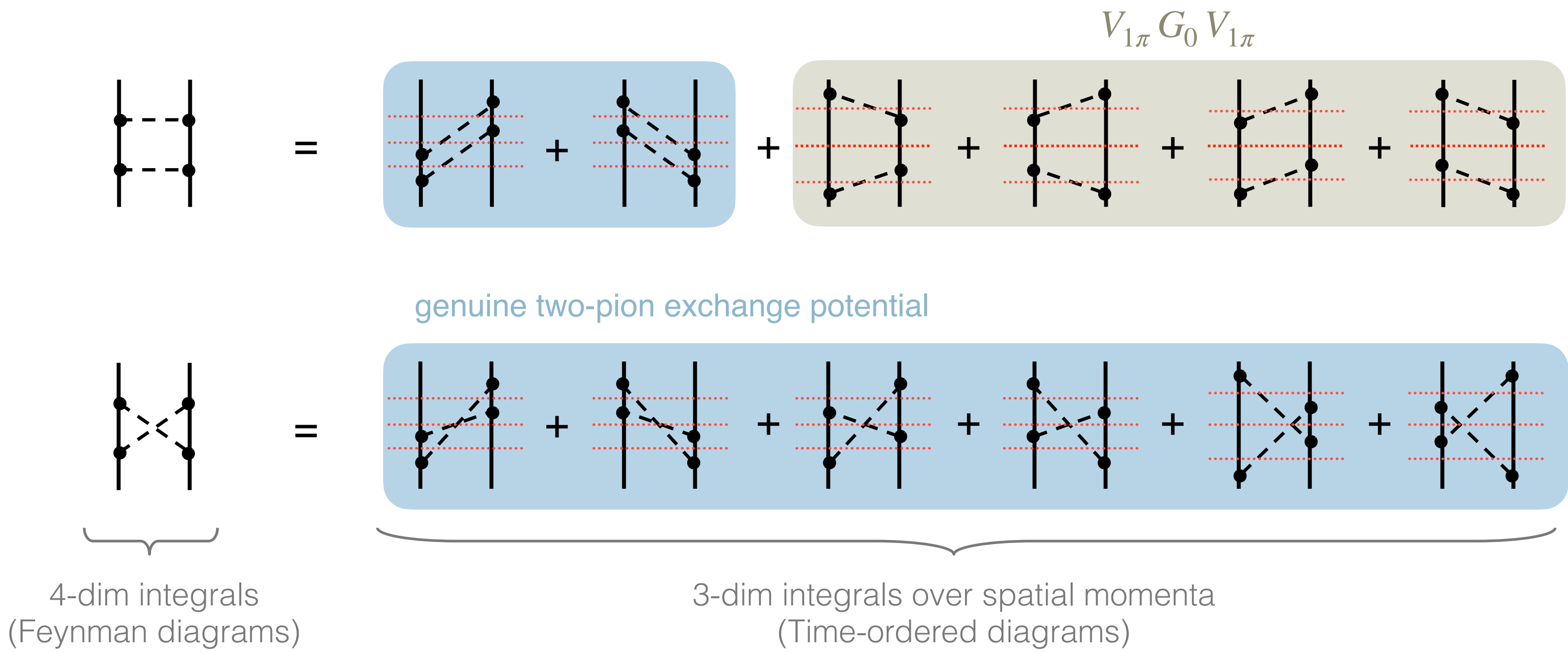
$\Rightarrow$ higher-order terms in the Hamiltonian „know“ about the choice made for the off-shell extension (consistency...)

S-matrix in ChPT is renormalizable (in the EFT sense). But this should not be taken for granted for the potentials…



Renormalization can be enforced by systematically exploiting unitary ambiguities…

Time-ordered perturbation theory



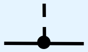
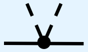
— has been used by Weinberg in his original publications

— leads to energy-dependent potentials which are inconvenient for many-body calculations (the energy dependence can be eliminated)

— changes the normalization of few-nucleon states

Method of Unitary Transformation

Taketani, Mashida, Ohnuma'52; Okubo '54; EE, Glöckle, Meißner, Krebs, Kölling, ...

- Canonical transformation and quantization: $\mathcal{L}_{\pi \mathrm{N}} \longrightarrow \mathcal{H}_{\pi \mathrm{N}} =$  +  + ...

nucleonic states $|N\rangle,\ |NN\rangle,\ \ldots$

EOM: $$\begin{pmatrix} \eta H \eta & \eta H \lambda \\ \lambda H \eta & \lambda H \lambda \end{pmatrix} \begin{pmatrix} |\phi\rangle \\ |\psi\rangle \end{pmatrix} = E \begin{pmatrix} |\phi\rangle \\ |\psi\rangle \end{pmatrix}$$

can not solve (infinite-dimensional eq.)

projectors

states with mesons $|N\pi\rangle,\ |N\pi\pi\rangle,\ \ldots$

- Decouple pions via a suitable UT: $\tilde{H} \equiv U^\dagger \begin{pmatrix} \eta H \eta & \eta H \lambda \\ \lambda H \eta & \lambda H \lambda \end{pmatrix} U = \begin{pmatrix} \eta \tilde{H} \eta & 0 \\ 0 & \lambda \tilde{H} \lambda \end{pmatrix}$

Minimal parametrization of the UT: $U = \begin{pmatrix} \eta(1 + A^\dagger A)^{-1/2} & -A^\dagger(1 + AA^\dagger)^{-1/2} \\ A(1 + A^\dagger A)^{-1/2} & \lambda(1 + AA^\dagger)^{-1/2} \end{pmatrix}, \quad A = \lambda A \eta$

Okubo '54

Require: $\eta \tilde{H} \lambda = \lambda \tilde{H} \eta = 0 \quad \Rightarrow \quad \boxed{\lambda \left(H - [A,\ H] - AHA \right) \eta = 0}$

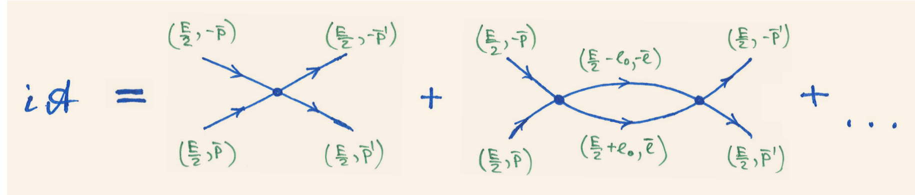
The solution of the nonlinear decoupling equation, calculation of the UT and of the nuclear potentials is carried out in perturbation theory (chiral expansion) EE, EPJA 34 (2007) 197

Notice: Similar methods are widely used in nuclear and many-body physics (Lee-Suzuki)

Path-integral approach

Krebs, EE, PRC 110 (2024) 044003

Pion-less EFT:



$$\mathcal{L} = N^\dagger \left[i\partial_0 + \frac{\vec{\nabla}^2}{2m_N} \right] N - \frac{C_S}{2} (N^\dagger N)^2 + \ldots$$

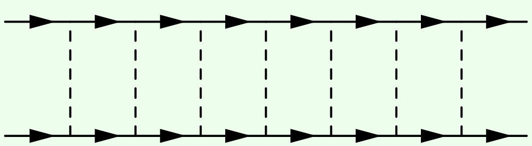
$$\Rightarrow \quad \mathcal{A}_{\rm tree} = \left[C_0 + C_2 (\vec{p}^{\,2} + \vec{p}^{\,\prime 2}) + \ldots \right]$$

Scattering amplitude to 1 loop:

$$-i\mathcal{A}_{1-\rm loop} = \int \frac{d^4 l}{(2\pi)^4} \left[C_0 + C_2 (\vec{p}^{\,2} + \vec{l}^{\,2}) + \ldots \right] \frac{1}{\left(\frac{E}{2} + l_0 - \frac{\vec{l}^{\,2}}{2m_N} + i\epsilon\right)\left(\frac{E}{2} - l_0 - \frac{\vec{l}^{\,2}}{2m_N} + i\epsilon\right)} \left[C_0 + \ldots \right]$$

$$= -i \int \frac{d^3 l}{(2\pi)^3} \left[C_0 + C_2 (\vec{p}^{\,2} + \vec{l}^{\,2}) + \ldots \right] \frac{1}{E - \frac{\vec{l}^{\,2}}{m_N} + i\epsilon} \left[C_0 + (\vec{l}^{\,2} + \vec{p}^{\,\prime 2}) \ldots \right]$$

All l_0-integrals factorize $\Rightarrow$ Lippmann-Schwinger eq. $\mathcal{A} = \mathcal{V} + \mathcal{V}\, G_0\, \mathcal{A}$ with $\mathcal{V} = -\mathcal{L}_{\rm int}$



But l_0-integrals do not factorize for pions due to l_0-dependence of π-propagators…

Idea: $Z[\eta^\dagger, \eta] = A \int \mathcal{D}N^\dagger \, \mathcal{D}N \, \mathcal{D}\pi \, \exp\left(i S^\Lambda_{\rm eff} + i \int d^4x [\eta^\dagger N + N^\dagger \eta] \right)$

Hermann Krebs, EE, 2311.10893

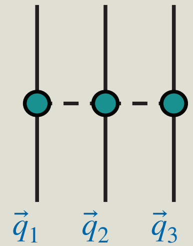
nonlocal redefinitions of $N, N^\dagger$

loops from functional determinant

$$\longrightarrow \quad A \int \mathcal{D}\tilde{N}^\dagger \, \mathcal{D}\tilde{N} \exp\left(i S^\Lambda_{\rm eff,\,N} + i \int d^4x [\eta^\dagger \tilde{N} + \tilde{N}^\dagger \eta] \right)$$

instantaneous

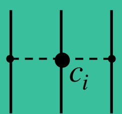
Example: 2π-exchange 3NF



$$V_{3\mathrm{N}} = \frac{\vec\sigma_1\cdot\vec q_1\,\vec\sigma_3\cdot\vec q_3}{(q_1^2+M_\pi^2)\,(q_3^2+M_\pi^2)}\Big[\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_3\,\mathcal{A}(q_2)+\boldsymbol{\tau}_1\times\boldsymbol{\tau}_3\cdot\boldsymbol{\tau}_2\,\vec q_1\times\vec q_3\cdot\vec\sigma_2\,\mathcal{B}(q_2)\Big]$$

$$+\quad \text{short-range terms} \;+\; \text{permutations}$$

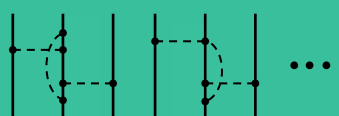
N²LO (Q³)



$$\mathcal{A}^{(3)} = \frac{g_A^2}{8F_\pi^4}\Big[(2c_3-4c_1)M_\pi^2+c_3q_2^2\Big], \qquad \mathcal{B}^{(3)} = \frac{g_A^2c_4}{8F_\pi^4}$$

N³LO (Q⁴)

Bernard, EE, Krebs, Meißner '08



$$\mathcal{A}^{(4)} = \frac{g_A^4}{256\pi F_\pi^6}\Big[\left(4g_A^2+1\right)M_\pi^3+2\left(g_A^2+1\right)M_\pi q_2^2+A(q_2)\left(2M_\pi^4+5M_\pi^2q_2^2+2q_2^4\right)\Big]$$

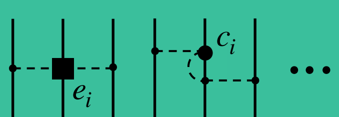
$A(q_2) = \frac{1}{2q_2}\arctan\frac{q_2}{2M_\pi}$

$$\mathcal{B}^{(4)} = -\frac{g_A^4}{256\pi F_\pi^6}\Big[A(q_2)\left(4M_\pi^2+q_2^2\right)+(2g_A^2+1)M_\pi\Big]$$

calculated using DimReg

N⁴LO (Q⁵)

Krebs, Gasparyan, EE '12



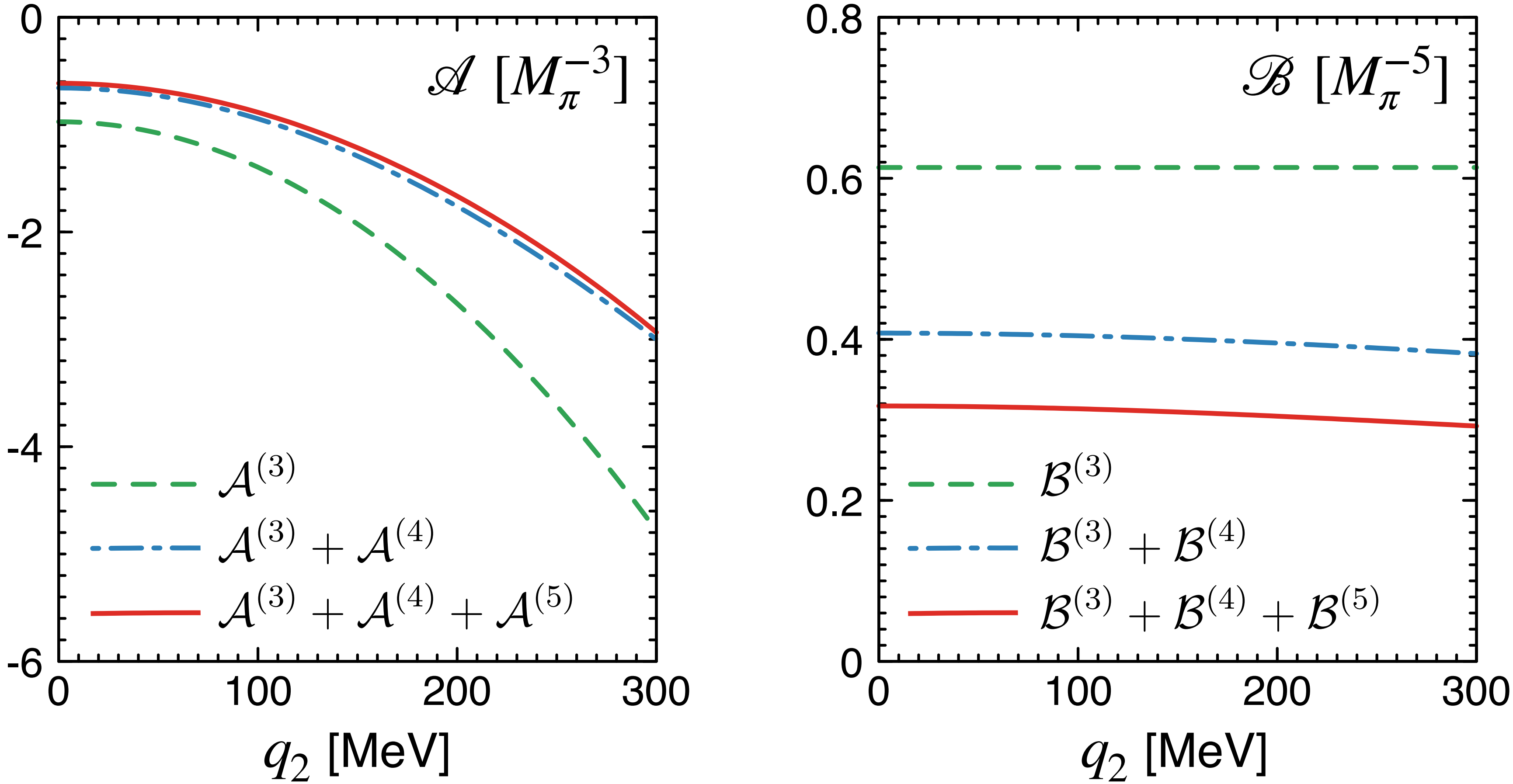
$$\mathcal{A}^{(5)} = \frac{g_A^2\left(M_\pi^2+2q_2^2\right)}{4608\pi^2F_\pi^6}\Big\{\big[6c_1-2c_2-3c_3-2(6c_1-c_2-3c_3)L(q_2)\big]12M_\pi^2 - q_2^2\big[5c_2+18c_3-6L(q_2)(c_2+6c_3)\big]\Big\} + \frac{g_A^2\bar e_{14}}{2F_\pi^4}\left(2M_\pi^2+q_2^2\right)^2$$

$L(q_2) = \frac{\sqrt{q_2^2+4M_\pi^2}}{q_2}\log\frac{\sqrt{q_2^2+4M_\pi^2}+q_2}{2M_\pi}$

$$\mathcal{B}^{(5)} = \frac{g_A^2\bar e_{17}}{2F_\pi^4}\left(2M_\pi^2+q_2^2\right) - \frac{g_A^2c_4}{2304\pi^2F_\pi^6}\Big\{q_2^2\big[5-6L(q_2)\big]+12M_\pi^2\big[2+9g_A^2-2L(q_2)\big]\Big\}$$

calculated using DimReg

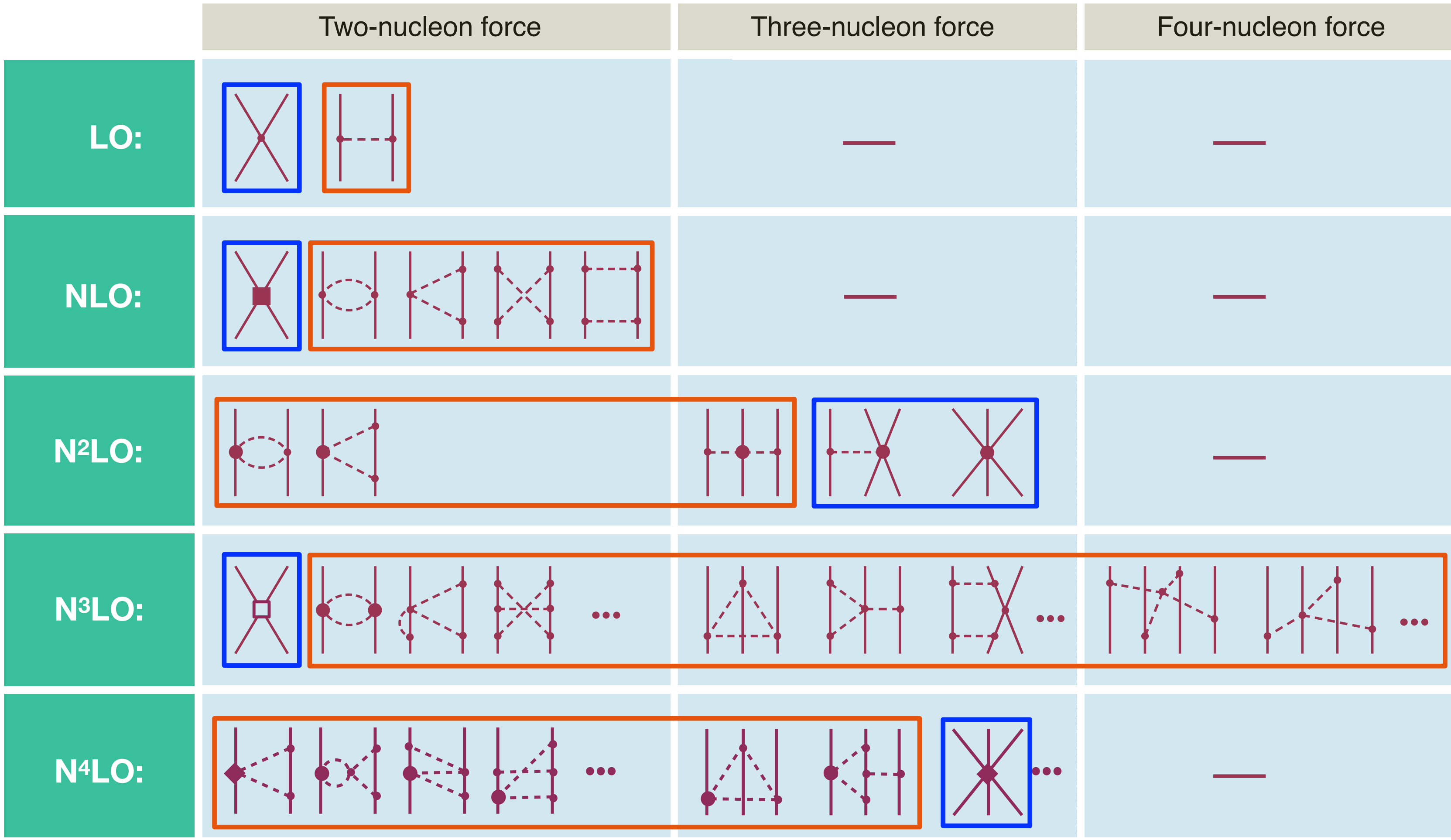
Example: 2π-exchange 3NF

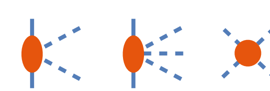


— all LECs c_i and $\bar{e}_i$ are known from the Roy-Steiner-equation analysis of the πN system

— the results are only meaningful (converged) at small momenta $\Rightarrow$ cutoff needed

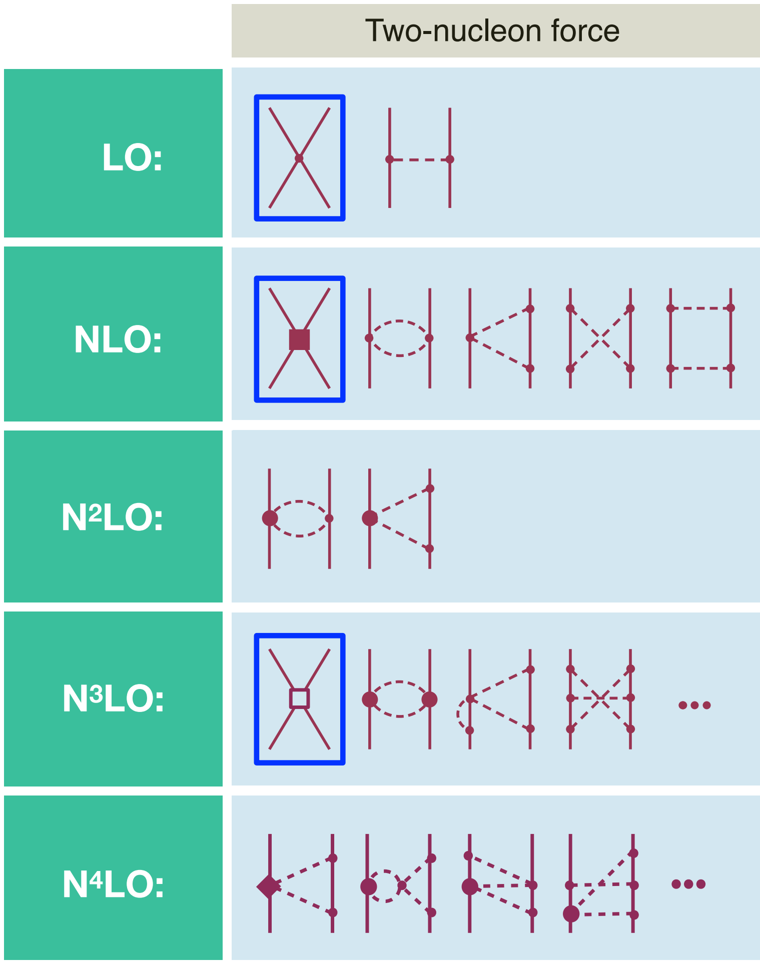
Chiral expansion of nuclear forces



Chiral dynamics: Long-range interactions are predicted in terms of on-shell amplitudes 

Short-range few-N interactions are tuned to experimental data

Chiral expansion of nuclear forces



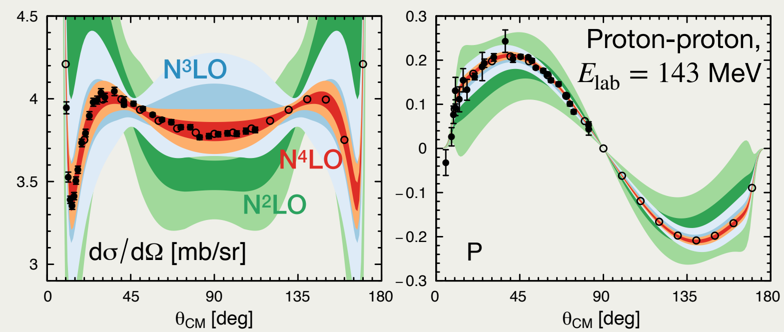
χEFT as a precision tool in the 2N sector

— N⁴LO⁺: currently most accurate and precise NN interactions on the market

— clear evidence of the TPEP from NN data

— almost no residual cutoff dependence

— Bayesian truncation-error estimation



— Precision calculations for 2 nucleons:

$g_{\pi NN} = 13.24 \pm 0.04$ Reinert, Krebs, EE '20

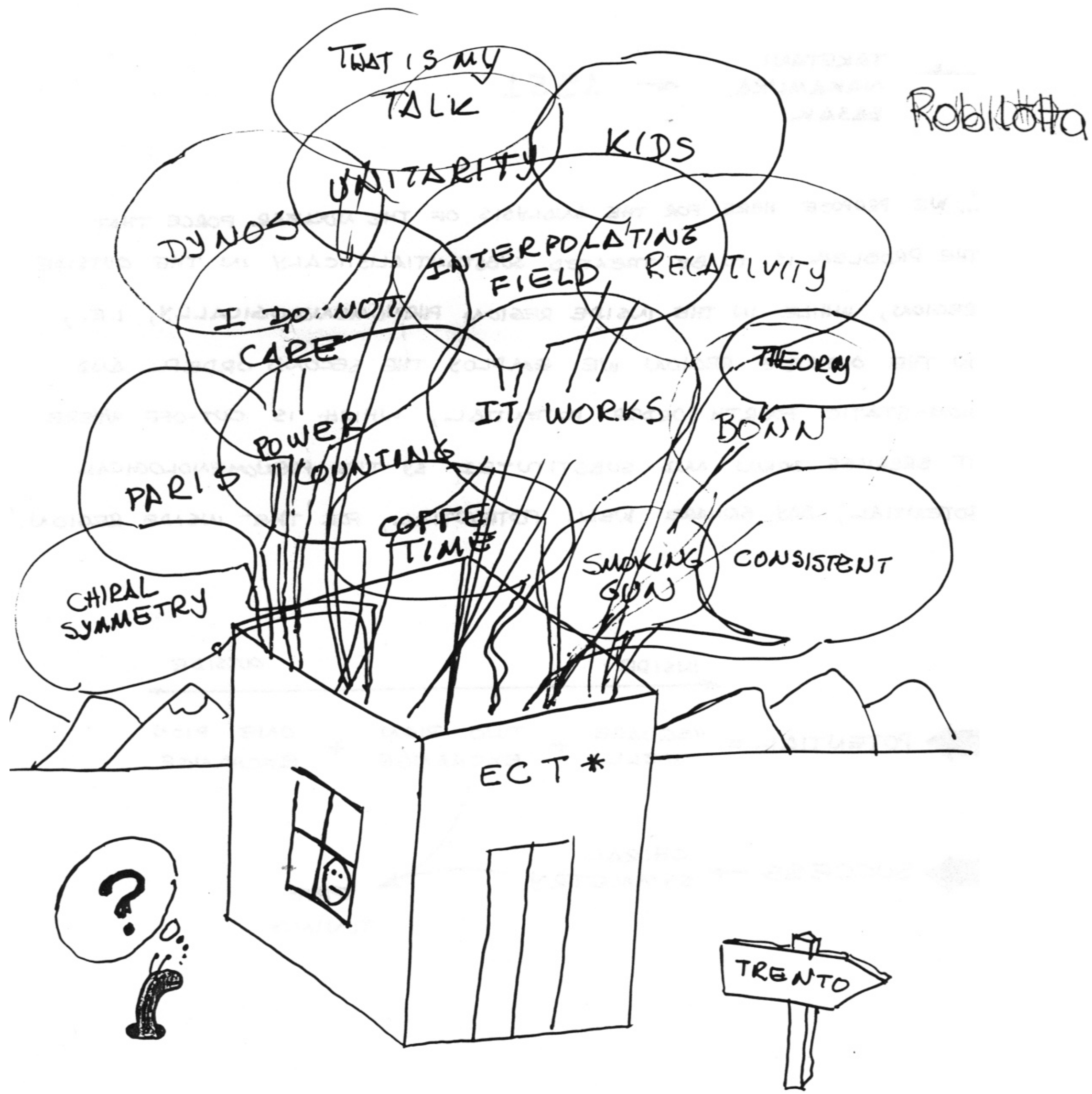
$r_{\rm str}^{^2\rm H} = 1.9729^{+0.0015}_{-0.0012}$ fm Filin et al., '21

Semi-local regularization in momentum space

Reinert, Krebs, EE, EPJA 54 (2018) 86; PRL 126 (2021) 092501

$$V_{1\pi}(q) = \frac{\alpha}{\vec{q}^{\,2} + M_\pi^2}\, e^{-\frac{\vec{q}^{\,2}+M_\pi^2}{\Lambda^2}} + \text{subtraction,} \qquad V_{2\pi}(q) = \frac{2}{\pi}\int_{2M_\pi}^{\infty} d\mu\,\mu\,\frac{\rho(\mu)}{\vec{q}^{\,2}+\mu^2}\, e^{-\frac{\vec{q}^{\,2}+\mu^2}{2\Lambda^2}} + \text{subtractions}$$

+ nonlocal (Gaussian) cutoff for contacts



CHIRAL SYMMETRY ⟵ NN ARCHITECTURE ...

THE GAME IS NOT OVER

25 years of NN chiral architecture

— 2π, 3π NN force worked out to Q^5 in HBChPT Kaiser '00-'02, Entem et al. '15 + Roy-Steiner eq. analysis Ruiz de Elvira et al.'15 to extract LECs from πN data $\Rightarrow$ parameter-free long-range NN force

— clear evidence of the TPEP from NN data Friar, van Kolck, Rentmeester, Timmermans, Birse, McGovern, EE, Krebs, Meißner

— at N⁴LO⁺ as precise as CD Bonn & co. but less param.: 27 vs 40-50 (TPEP!) Reinert et al., Rup et al.

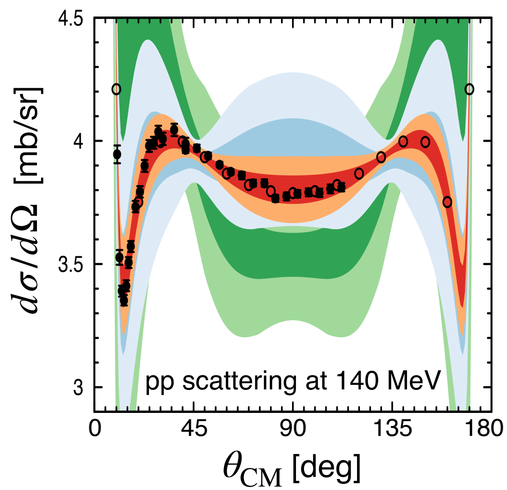
— full-fledged NN PWA using χEFT (data selection, statistical tests, error analysis) Reinert et al.'20

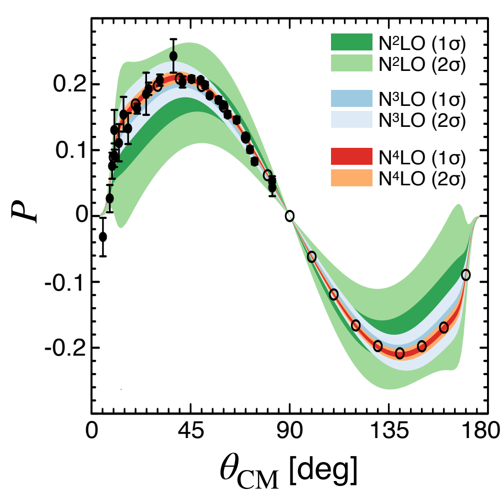
— numerical consistency checks: natural LECs, weak residual Λ-dependence, …

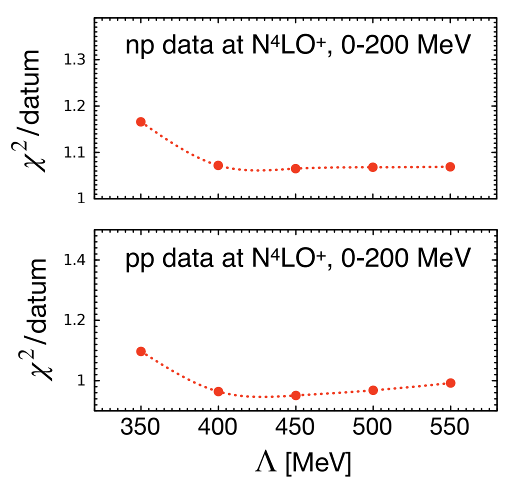
— explicit renormalizability proof of finite-cutoff EFT for 2N at NLO, see talk by Ashot RGI???

— (Bayesian) uncertainty quantification BUQEYE, EKM, Ekström, …

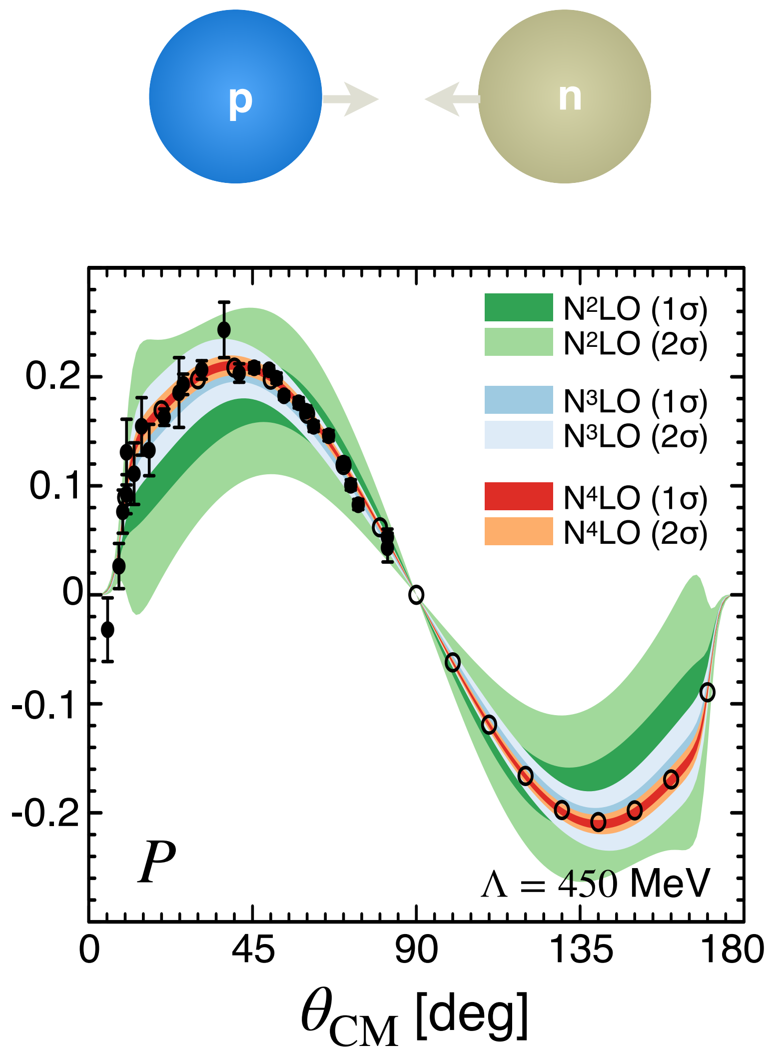
$\Rightarrow$ precision 2N physics, e.g.: $g_{\pi\mathrm{NN}} = 13.24 \pm 0.04$ Reinert, Krebs, EE '20, $r_{\mathrm{str}}^{\,^2\mathrm{H}} = 1.9729^{+0.0015}_{-0.0012}$ fm Filin et al., '21

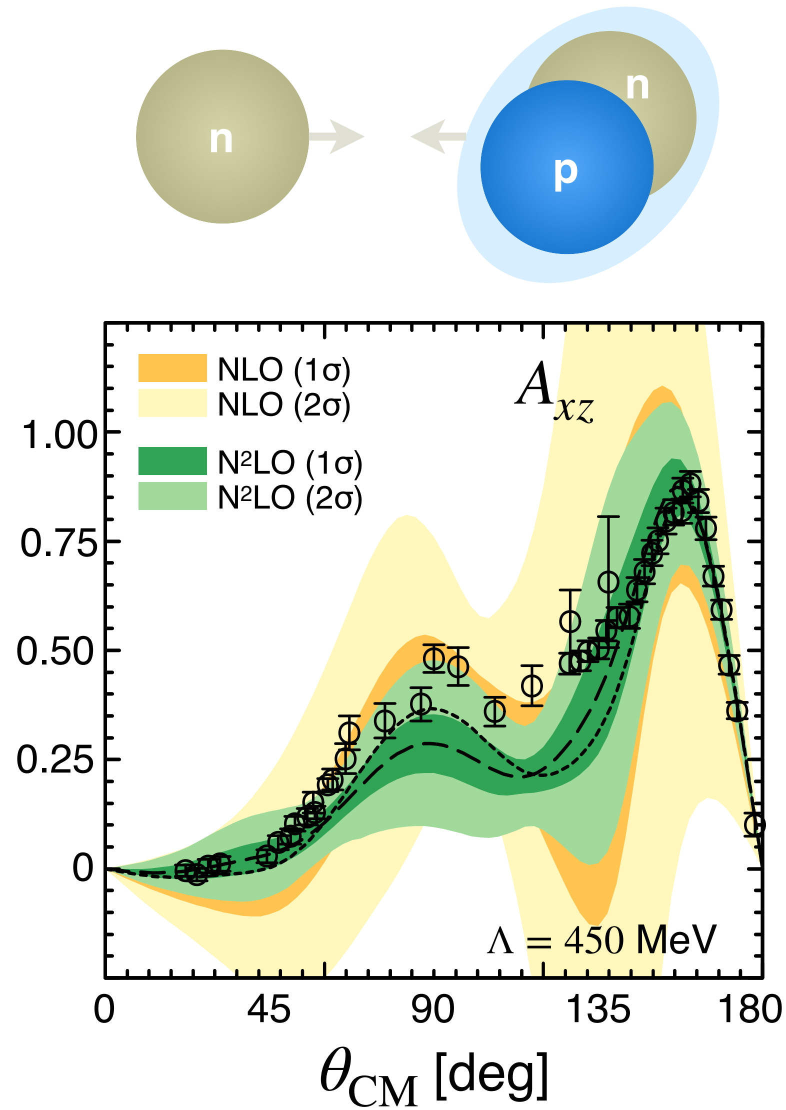






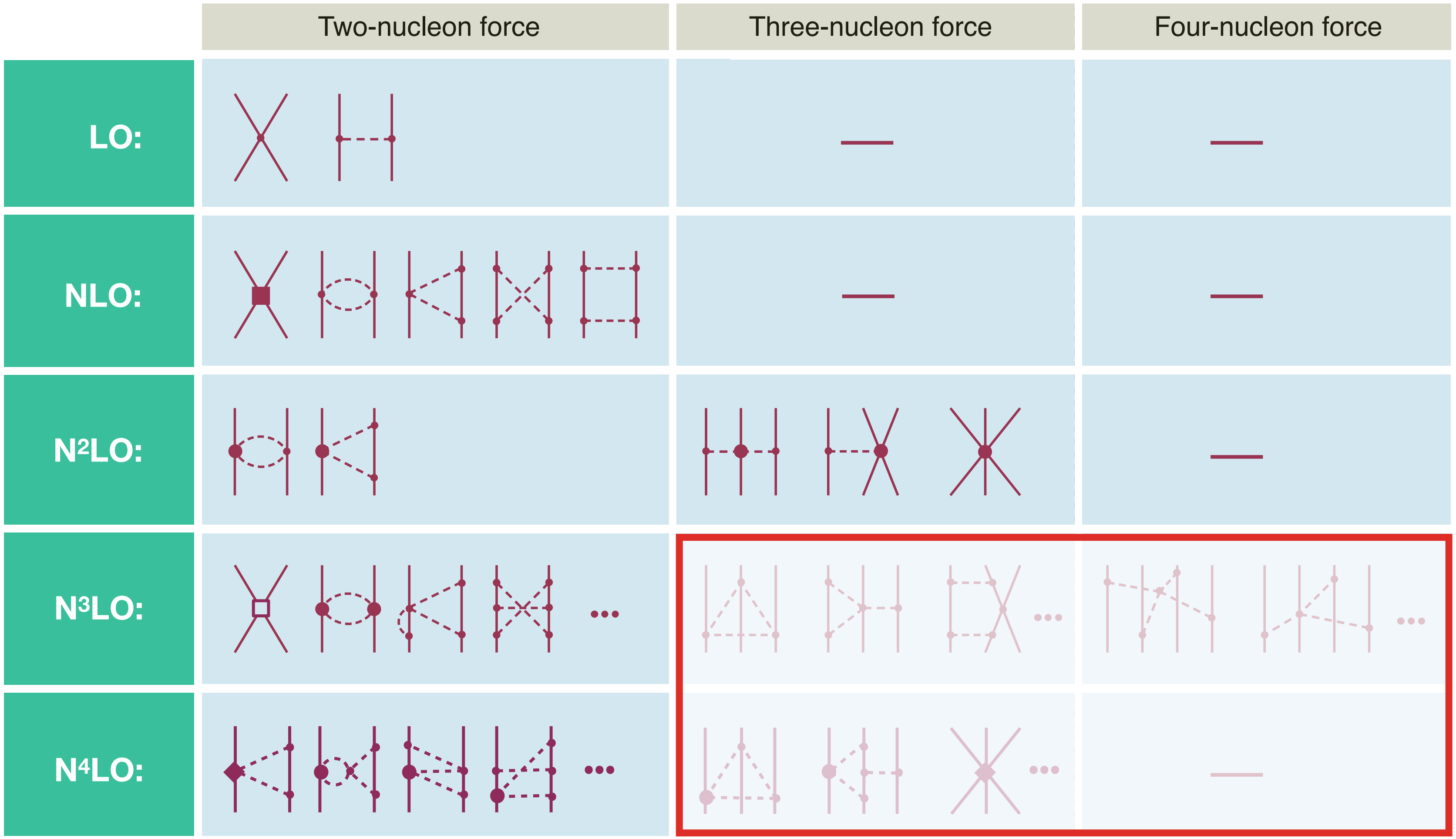
Chiral architecture beyond 2N?





Where are calculations beyond N^{2}LO?

Chiral expansion of nuclear forces



have been worked out using dimensional regularization

mixing DimReg with Cutoff violates χ-symmetry (also for current operators)

$\Rightarrow$ need to be re-derived using invariant cutoff regulator

Krebs, EE, PRC 110 (2024) 044004

Take-aways of part V

— nuclear forces can be derived from the effective Lagrangian using a variety of methods

— important to maintain consistency (nuclear potentials are scheme dependent)

— regularization of 3NFs and currents beyond tree level is nontrivial

VI: Gradient flow method

How to introduce a cutoff regulator in the way compatible
With the chiral (and gauge) symmetries?

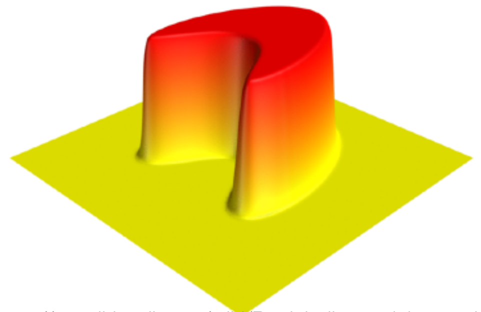
Outline

- Statement of the problem
- Chiral gradient flow
- Applications

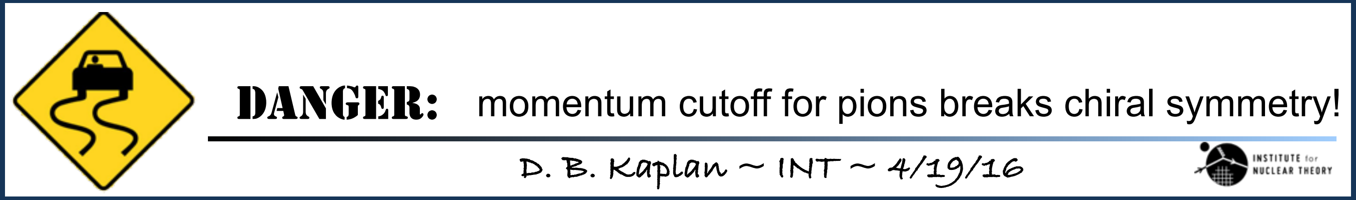
Further reading

Krebs, EE, PRC 110 (2024) 044004

Temperature evolution in a 2d metal plate

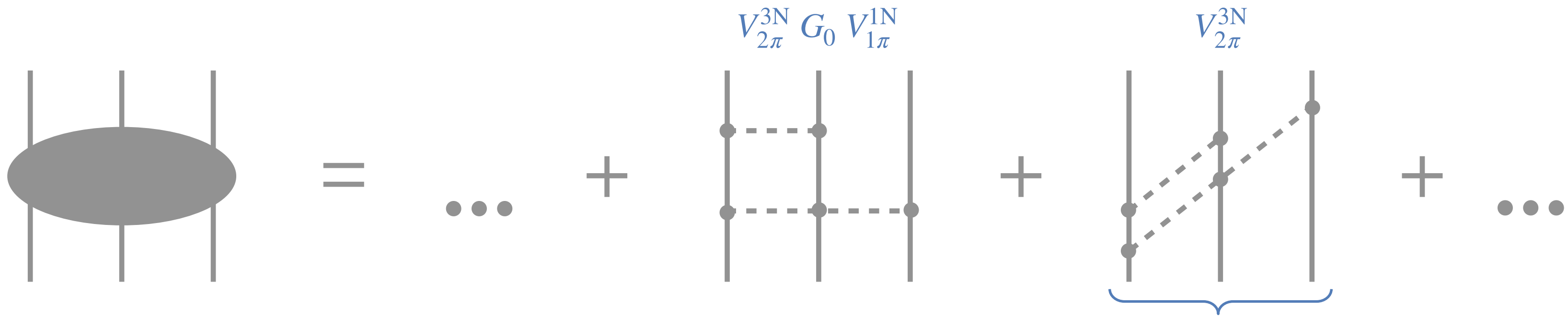


https://en.wikipedia.org/wiki/Partial_differential_equation



Essence of the problem

Faddeev equation for 3N scattering:



derived using DimReg Bernard et al. '08

$$-\Lambda \frac{g_A^4}{96\sqrt{2\pi^3}F_\pi^6}\left[\underbrace{\tau_1\cdot\tau_3\,(\vec{q}_3\cdot\vec{\sigma}_1)}_{\text{absorbable into } c_D} - \underbrace{\frac{4}{3}(\tau_2\cdot\tau_3-\tau_1\cdot\tau_3)(\vec{q}_2\cdot\vec{\sigma}_3)}_{\text{violates chiral symmetry}}\right]\frac{\vec{q}_3\cdot\vec{\sigma}_3}{q_3^3+M_\pi^2} + \ldots$$

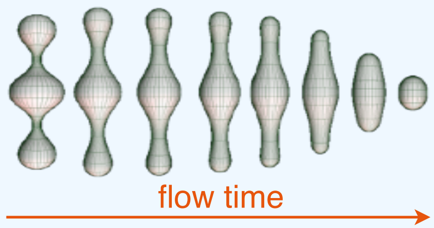
$\Rightarrow$ mixing DimReg with Cutoff regularization breaks chiral symmetry EE, Krebs, Reinert '19

$\Rightarrow$ 3NF, 4NF & MECs beyond N^{2}LO have to be re-derived using Cutoff Reg (2NF ok at fixed M_π)

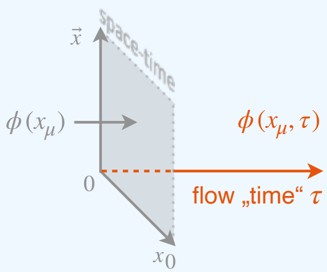
Gradient flow as a <u>symmetry-preserving</u> regulator as suggested by David Kaplan

Gradient flow

Gradient flows: methods for smoothing manifolds
(e.g., Ricci flow used in the proof of the Poincaré conjecture)



Gradient flow as a regulator in field theory

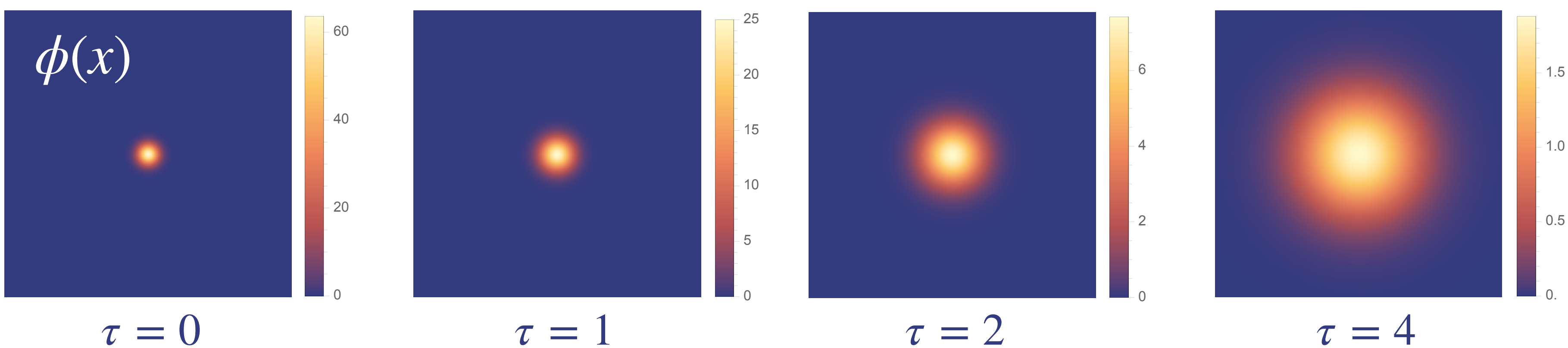


Flow equation: $\dfrac{\partial}{\partial \tau}\phi(x,\tau) = -\dfrac{\delta S[\phi]}{\delta \phi(x)}\bigg|_{\phi(x)\to\phi(x,\tau)}$

subject to the boundary condition $\phi(x,0) = \phi(x)$

Free scalar field:

$$\left[\partial_\tau - (\partial^x_\mu \partial^x_\mu - M^2)\right]\phi(x,\tau) = 0 \quad \Rightarrow \quad \phi(x,\tau) = \int d^4y\, \overbrace{\underbrace{G(x-y,\tau)}_{\textit{heat kernel}}}^{G(x,\tau)=\frac{\theta(\tau)}{16\pi^2\tau^2}e^{-\frac{x^2+4M^2\tau^2}{4\tau}}} \phi(y) \quad \Rightarrow \quad \tilde{\phi}(q,\tau) = e^{-\tau(q^2+M^2)}\tilde{\phi}(q)$$



Preliminaries

Effective Lagrangian for Goldstone bosons

Pions transform ***nonlinearly*** under SU(2)$_L$ × SU(2)$_R$ and via the adjoint irrep. of SU(2)$_V$ (i.e., $R = L$):

$$U(\boldsymbol{\pi}) \to R U L^\dagger, \qquad U(\boldsymbol{\pi}) = \underbrace{1 + \frac{i}{F}\boldsymbol{\tau}\cdot\boldsymbol{\pi} - \frac{\boldsymbol{\pi}^2}{2F^2} - \alpha\frac{i}{F^3}\boldsymbol{\tau}\cdot\boldsymbol{\pi}\boldsymbol{\pi}^2 + \mathcal{O}(\boldsymbol{\pi}^4)}_{\text{most general parametrization of an SU(2) matrix in terms of } \pi\text{'s}}$$

SU(2) matrices

$$\underbrace{\mathcal{L}_\pi^{\rm E}}_{\text{in Euclidean space}} = \frac{F^2}{4}\mathrm{Tr}\Big[(\overbrace{\nabla_\mu U}^{\nabla_\mu U = \partial_\mu U - i r_\mu U + i U l_\mu})^\dagger \nabla_\mu U - U^\dagger \underbrace{\chi - \chi^\dagger}_{\chi = 2B(s+ip)} U\Big] = \frac{1}{2}\boldsymbol{\pi}\cdot\big(\underbrace{-\partial^2 + M^2}_{\equiv \partial_\mu\partial_\mu = \partial_0^2 + \vec{\nabla}^2}\big)\boldsymbol{\pi}^2 + \ldots$$

Effective Lagrangian for pions and nucleons

Generalization to N: $u(\boldsymbol{\pi}) := \sqrt{U}, \quad u \to R u K^\dagger = K u L^\dagger \quad \text{with} \quad K(L,R,U) = \sqrt{L U^\dagger R^\dagger} R \sqrt{U}$

Define [CCWZ '69] $N \to KN$ and introduce $u_\mu = i u^\dagger \nabla_\mu U u^\dagger, \;\; u_\mu \to K u_\mu K^\dagger, \; \ldots$

$$\Rightarrow \quad \mathcal{L}_{\pi N}^{\rm E} = N^\dagger (D_0 + g u_\mu S_\mu) N + \ldots$$

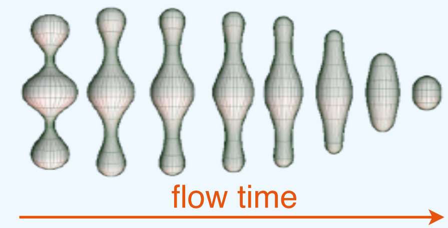
First attempt: Higher-derivative regularization of $\mathscr{L}_\pi^{\rm E}$ Slavnov '71

$$\mathcal{L}_\pi^{\rm E} \longrightarrow \mathcal{L}_{\pi,\Lambda}^{\rm E} = \frac{F^2}{4}\mathrm{Tr}\Big[\partial_\mu U^\dagger e^{-\partial^2/\Lambda^2}\partial_\mu U\Big] = \underbrace{-\frac{1}{2}\boldsymbol{\pi}\cdot\partial^2\, e^{-\partial^2/\Lambda^2}\boldsymbol{\pi}}_{\Delta_\Lambda^{\rm E} = 1/(q_0^2+\vec{q}^{\,2})\, e^{-(q_0^2+\vec{q}^{\,2})/\Lambda^2}} + \frac{1}{2F^2}\boldsymbol{\pi}\cdot\partial_\mu\boldsymbol{\pi}\, \underbrace{e^{-\partial^2/\Lambda^2}}_{\text{de-regularization...}}\boldsymbol{\pi}\cdot\partial_\mu\boldsymbol{\pi} + \mathcal{O}(\pi^6)$$

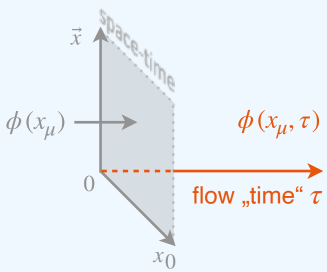
(In the absence of external sources)

Gradient flow

Gradient flows: methods for smoothing manifolds
(e.g., Ricci flow used in the proof of the Poincaré conjecture)



Gradient flow as a regulator in field theory



Flow equation: $\displaystyle \frac{\partial}{\partial\tau}\phi(x,\tau) = -\frac{\delta S[\phi]}{\delta\phi(x)}\bigg|_{\phi(x)\to\phi(x,\tau)}$

subject to the boundary condition $\phi(x,0) = \phi(x)$

Free scalar field:

$$\left[\partial_\tau - (\partial^x_\mu\partial^x_\mu - M^2)\right]\phi(x,\tau) = 0 \quad\Rightarrow\quad \phi(x,\tau) = \int d^4y\, \overbrace{\underbrace{G(x-y,\tau)}_{\textit{heat kernel}}}^{G(x,\tau)\,=\,\frac{\theta(\tau)}{16\pi^2\tau^2}e^{-\frac{x^2+4M^2\tau^2}{4\tau}}}\phi(y) \quad\Rightarrow\quad \tilde\phi(q,\tau) = e^{-\tau(q^2+M^2)}\tilde\phi(q)$$

YM gradient flow Narayanan, Neuberger ’06, Lüscher, Weisz ’11: $\partial_\tau A_\mu(x,\tau) = D_\nu G_{\nu\mu}(x,\tau)$ $\leftarrow$ extensively used in LQCD

Chiral gradient flow Krebs, EE, PRC 110 (2024) 044004

Generalize $U(x)$, $U(x)\to RU(x)L^\dagger$ to $W(x,\tau)$: $\partial_\tau W = -i\underbrace{w}_{\sqrt{W}}\,\overbrace{\mathrm{EOM}(\tau)}^{[D_\mu, w_\mu] + \frac{i}{2}\chi_-(\tau) - \frac{i}{4}\mathrm{Tr}\,\chi_-(\tau)}\,w, \quad W(x,0) = U(x)$

We have proven $\forall\tau$: $W(x,\tau)\in \mathrm{SU}(2)$, $W(x,\tau) \;\to\; RW(x,\tau)L^\dagger$

Solving the chiral gradient flow equation

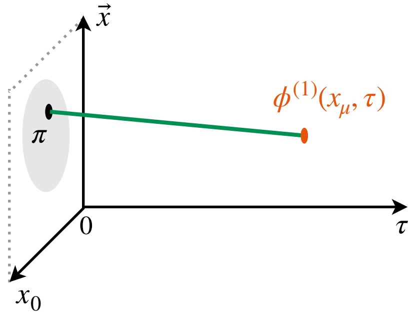
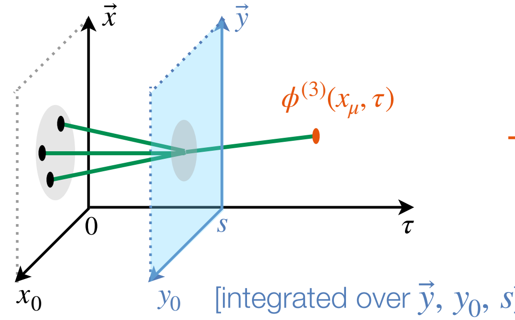
Solving the chiral gradient flow equation $\partial_\tau W = -iw\,\mathrm{EOM}(\tau)\,w$

— most general parametrization of U: $U = 1 + \frac{i}{F}\boldsymbol{\tau}\cdot\boldsymbol{\pi} - \frac{\boldsymbol{\pi}^2}{2F^2} - \alpha\frac{i}{F^3}\boldsymbol{\tau}\cdot\boldsymbol{\pi}\boldsymbol{\pi}^2 + \ldots$

— similarly, write $W = 1 + i\boldsymbol{\tau}\cdot\boldsymbol{\phi} - \boldsymbol{\phi}^2 - i\,\alpha\,\boldsymbol{\tau}\cdot\boldsymbol{\phi}\boldsymbol{\phi}^2 + \ldots$ and make an ansatz $\boldsymbol{\phi} = \sum_{n=0}^{\infty}\frac{\boldsymbol{\phi}^{(n)}}{F^n}$

$\Rightarrow$ recursive (perturbative) solution of the GF equation in $1/F$

Solving the chiral gradient flow equation

$\phi(x_\mu, \tau) \;=$  $+$  $+\;\ldots$

$$\left.\begin{aligned}\left[\partial_\tau - (\partial_\mu^x \partial_\mu^x - M^2)\right] \boldsymbol{\phi}^{(1)}(x,\tau) &= 0 \\ \boldsymbol{\phi}^{(1)}(x,0) &= \boldsymbol{\pi}(x)\end{aligned}\right\} \Rightarrow \quad \boldsymbol{\phi}^{(1)}(x,\tau) = \int d^4y \, \overbrace{G(x-y,\tau)}^{G(x,\tau) = \frac{\theta(\tau)}{16\pi^2\tau^2} e^{-\frac{x^2 + 4M^2\tau^2}{4\tau}}} \boldsymbol{\pi}(y) \quad \Rightarrow \quad \tilde{\boldsymbol{\phi}}^{(1)}(q,\tau) = e^{-\tau(q^2+M^2)} \tilde{\boldsymbol{\pi}}(q)$$

SMS regulator for $\tau = 1/(2\Lambda^2)$

$$\left[\partial_\tau - (\partial_\mu^x \partial_\mu^x - M^2)\right] \phi_b^{(3)}(x,\tau) = \overbrace{(1-2\alpha)\partial_\mu \boldsymbol{\phi}^{(1)} \cdot \partial_\mu \boldsymbol{\phi}^{(1)} \phi_b^{(1)} - 4\alpha \, \partial_\mu \boldsymbol{\phi}^{(1)} \cdot \boldsymbol{\phi}^{(1)} \partial_\mu \phi_b^{(1)} + \frac{M^2}{2}(1-4\alpha) \boldsymbol{\phi}^{(1)} \cdot \boldsymbol{\phi}^{(1)} \phi_b^{(1)}}^{\equiv \mathrm{RHS}_b(x,\tau)}$$

$$\phi_b^{(3)}(x,0) = 0$$

$$\Rightarrow \quad \phi_b^{(3)}(x,\tau) = \int_0^\tau ds \int d^4y \, G(x-y, \tau - s) \, \mathrm{RHS}_b(y,s)$$

In momentum space, this solution takes the form:

$$\tilde{\phi}_b^{(3)}(q,\tau) = \int \prod_{i=1}^{3} \frac{d^4 q_i}{(2\pi)^4} (2\pi)^4 \delta^4(q - q_1 - q_2 - q_3) \underbrace{f_\Lambda(\{q_i\})}_{\frac{e^{-\tau(q^2+M^2)} - e^{-\tau \sum_{j=1}^3 (q_j^2 + M^2)}}{q_1^2 + q_2^2 + q_3^2 - q^2 + 2M^2}} \left[4\alpha \, q_1 \cdot q_3 - (1-2\alpha) q_1 \cdot q_2 + \frac{M^2}{2}(1-4\alpha)\right] \tilde{\boldsymbol{\pi}}(q_1) \cdot \tilde{\boldsymbol{\pi}}(q_2) \, \tilde{\pi}_b(q_3)$$

Gradient flow regularization of BChPT

— The pion Lagrangian $\mathscr{L}_\pi^{\mathrm{E}}$ is left unchanged

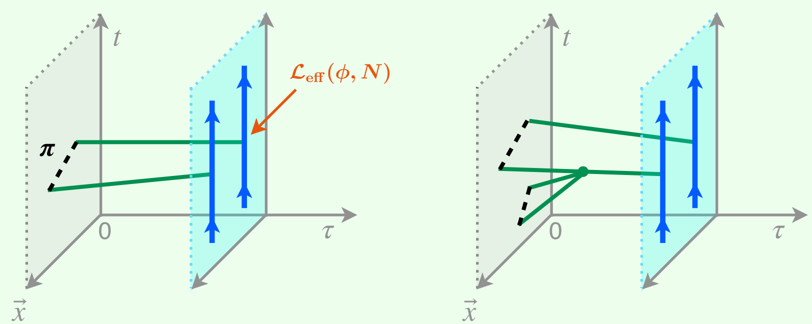
— Nucleons are ***defined*** to „live“ at a fixed $\tau > 0$.

Remember CCWZ: $u = \sqrt{U}$, $u \to \sqrt{RUL^\dagger} =: RuK^\dagger$, where $K = \sqrt{LU^\dagger R^\dagger} R \sqrt{U}$. Then: $N \to KN$.

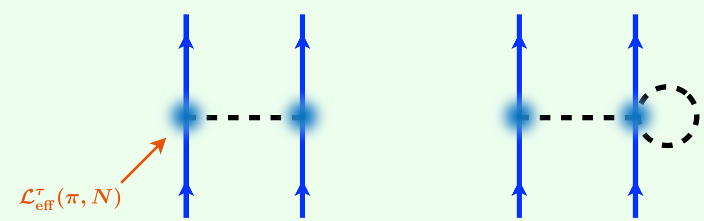
Instead, we introduce $N(\tau)$ via $N \to KN\Big|_{U \to W(\tau)}$: χ-symmetry is preserved since $W \to RWL^\dagger$.

$\Rightarrow$ BChPT using gradient flow regularization: $\mathcal{L}^{\mathrm{E}} = \mathcal{L}_\pi^{\mathrm{E}} + \mathcal{L}_{\pi N}^{\mathrm{E}}(\tau)$, $\underbrace{\mathcal{L}_{\pi N}^{\mathrm{E}}(\tau) = \mathcal{L}_{\pi N}^{\mathrm{E}}\Big|_{U \to W(\tau)}}_{\text{non-local (smeared) Lagrangian upon expressing in } \boldsymbol{\pi}\text{'s}}$

Local field theory in 5d

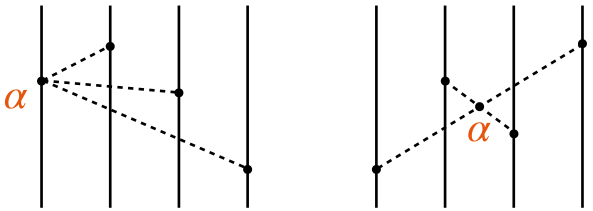


Smeared (non-local) theory in 4d



Chiral symmetry and the 4N force

unregularized



The sum of two diagrams ***must*** *be α-independent*

Unregularized expression for this 4NF EE, EPJA 34 (2007):

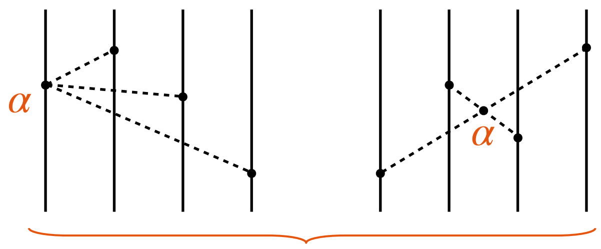
$\hat{O}_{[\sigma_i, \tau_i, \vec{q}_i]} = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \boldsymbol{\tau}_3 \cdot \boldsymbol{\tau}_4 \, \vec{\sigma}_2 \cdot \vec{q}_2 \, \vec{\sigma}_3 \cdot \vec{q}_3 \, \vec{\sigma}_4 \cdot \vec{q}_4$

$$V^{4N} = -\frac{g^4}{64F^6} \frac{\hat{O}_{[\sigma_i, \tau_i, \vec{q}_i]}}{(\vec{q}_2^{\,2} + M^2)(\vec{q}_3^{\,2} + M^2)(\vec{q}_4^{\,2} + M^2)} \, \vec{\sigma}_1 \cdot \vec{q}_{12}$$

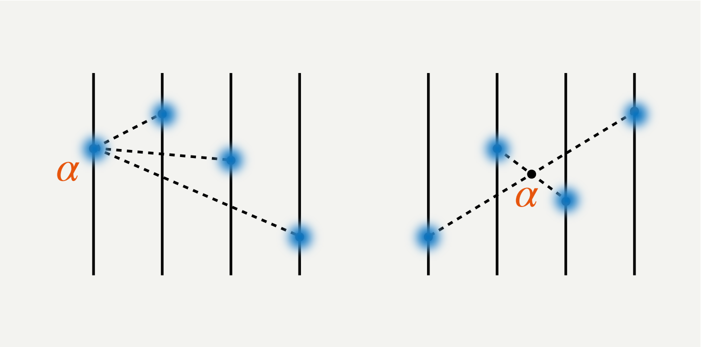
$$+ \frac{g^4}{128F^6} \frac{\vec{\sigma}_1 \cdot \vec{q}_1 \, \hat{O}_{[\sigma_i, \tau_i, \vec{q}_i]}}{(\vec{q}_1^{\,2} + M^2)(\vec{q}_2^{\,2} + M^2)(\vec{q}_3^{\,2} + M^2)(\vec{q}_4^{\,2} + M^2)} \left(M^2 + \vec{q}_{12}^{\,2}\right) + 23 \text{ perm.}$$

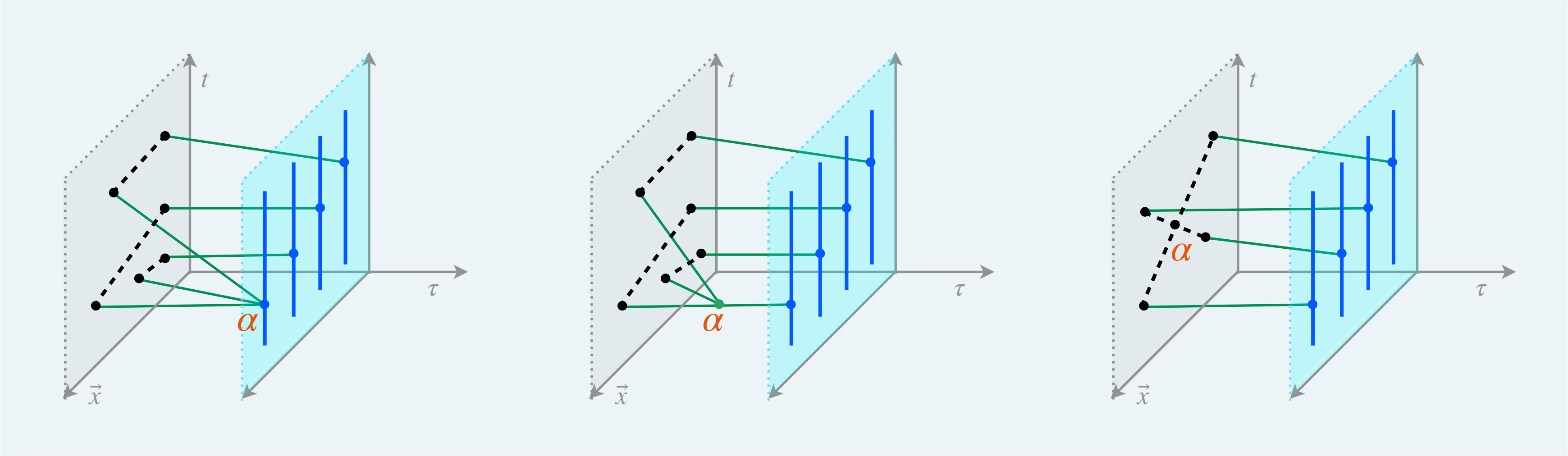
Chiral symmetry and the 4N force

unregularized



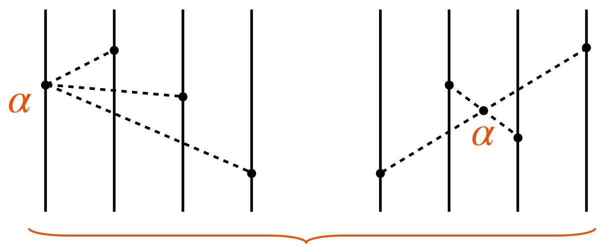
*The sum of two diagrams **must** be α-independent*



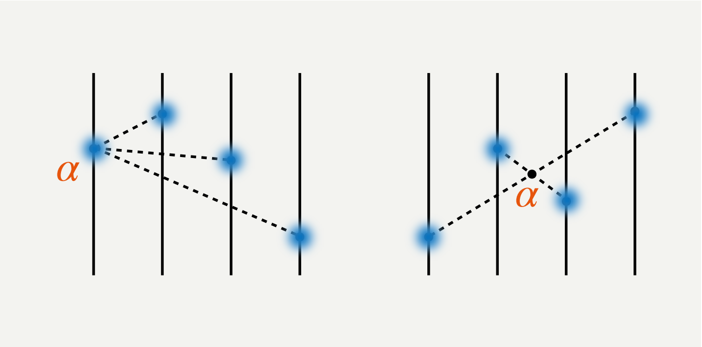


Chiral symmetry and the 4N force

unregularized



*The sum of two diagrams **must** be α-independent*



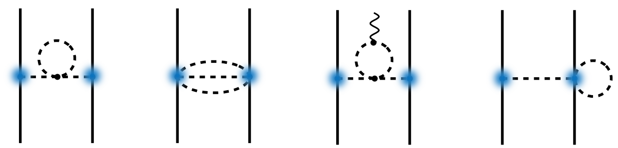
Regularized expression (ready to use in the A-body Schrödinger equation):

$$
\begin{aligned}
V_\Lambda^{4N} \;=\; & \frac{g^4}{64F^6}\,\frac{\hat{O}_{[\sigma_i,\tau_i,\vec q_i]}}{(\vec q_2^{\,2}+M^2)(\vec q_3^{\,2}+M^2)(\vec q_4^{\,2}+M^2)}\Big[\vec\sigma_1\cdot\vec q_1\big(2g_\Lambda-4f_\Lambda^{123}+2f_\Lambda^{134}-f_\Lambda^{234}\big)-\vec\sigma_1\cdot\vec q_2\,f_\Lambda^{234}\\
&+2\vec\sigma_1\cdot\vec q_1\big(5M^2+\vec q_1^{\,2}+\vec q_2^{\,2}+\vec q_3^{\,2}+\vec q_4^{\,2}+\vec q_{34}^{\,2}\big)\frac{g_\Lambda-f_\Lambda^{134}}{2M^2+\vec q_1^{\,2}+\vec q_3^{\,2}+\vec q_4^{\,2}-\vec q_2^{\,2}}\\
&-4\vec\sigma_1\cdot\vec q_1\big(3M^2+\vec q_1^{\,2}+\vec q_2^{\,2}+\vec q_3^{\,2}+\vec q_4^{\,2}-\vec q_{34}^{\,2}\big)\frac{g_\Lambda-f_\Lambda^{124}}{2M^2+\vec q_1^{\,2}+\vec q_2^{\,2}+\vec q_4^{\,2}-\vec q_3^{\,2}}\Big]\\
&+\frac{g^4}{128F^6}\,\frac{\vec\sigma_1\cdot\vec q_1\,\hat{O}_{[\sigma_i,\tau_i,\vec q_i]}}{(\vec q_1^{\,2}+M^2)(\vec q_2^{\,2}+M^2)(\vec q_3^{\,2}+M^2)(\vec q_4^{\,2}+M^2)}\big(M^2+\vec q_{12}^{\,2}\big)\big(4f_\Lambda^{123}-3g_\Lambda\big)\;+\;23\text{ perm.},
\end{aligned}
$$

$$f_\Lambda^{ijk}=e^{-\frac{\vec q_i^{\,2}+M^2}{\Lambda^2}}\,e^{-\frac{\vec q_j^{\,2}+M^2}{\Lambda^2}}\,e^{-\frac{\vec q_k^{\,2}+M^2}{\Lambda^2}}$$

$$e^{-\frac{\vec q_1^{\,2}+M^2}{2\Lambda^2}}\,e^{-\frac{\vec q_2^{\,2}+M^2}{2\Lambda^2}}\,e^{-\frac{\vec q_3^{\,2}+M^2}{2\Lambda^2}}\,e^{-\frac{\vec q_4^{\,2}+M^2}{2\Lambda^2}}$$

(reduces to the unregularized result in the $\Lambda\to\infty$ limit)



some purely pionic loops are divergent and require e.g. DR

Take-aways of part VI

— taking nuclear potentials (and currents), derived using DimReg, and putting an additional cutoff in the Schröd. equation violates the chiral & gauge symmetries

⇒ need to re-derive nuclear potentials using a symmetry-preserving cutoff regulator

⇒ merge Gradient Flow with chiral perturbation theory

— The future: Chiral EFT with consistently regularized nuclear forces and currents (all symmetries intact!)

Thank you for your attention