Nuclear ground-state properties (of exotic nuclei)

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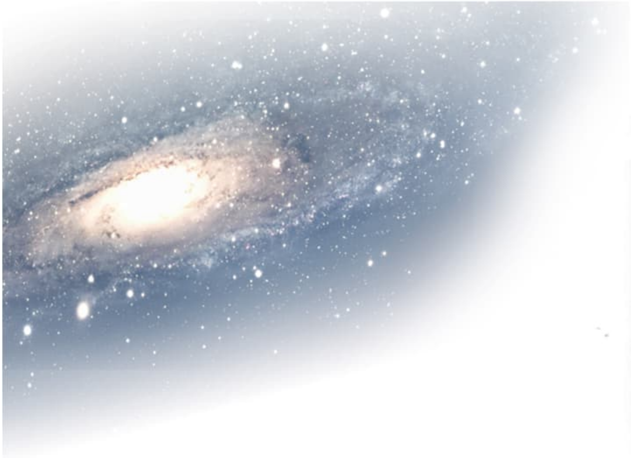
Reading for (hardcore) enthusiasts

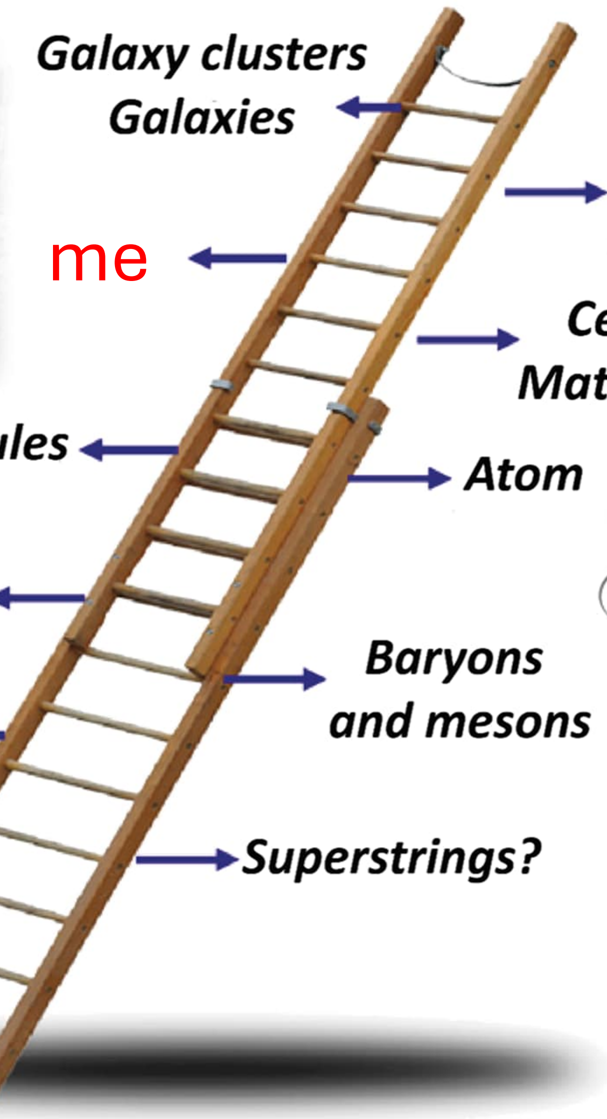
Some (recent) useful review/overview papers and book chapters

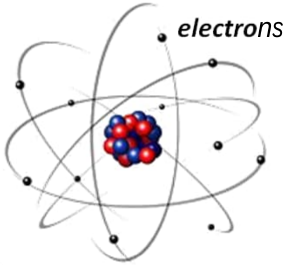
- Laser spectroscopy for the study of exotic nuclei
  X.F. Yang, S.J. Wang, S.G. Wilkins, R.F. Garcia Ruiz, Prog. in Part. and Nucl. Phys. 129 (2023) 104005
- Nuclear Charge Radii
  W. Nörtershäuser and I.D. Moore, Handbook of Nuclear Physics, Springer Nature Singapore Pte Ltd. (2023)
- Spins and Electromagnetic Moments of Nuclei
  R.P. de Groote and G. Neyens, Handbook of Nuclear Physics, Springer Nature Singapore Pte Ltd. (2023)
- Nuclear structure studies by collinear laser spectroscopy
  A. Koszorus, R.P de Groote, B. Cheal, P. Campbell, I.D. Moore, Eur. Phys. J. A 60 (2024) 20

- High-accuracy mass spectrometry with stored ions
  K. Blaum, Physics Reports 425 (2006) 1
- Masses of exotic nuclei
  K. Blaum, S. Eliseev and S. Goriely, Handbook of Nuclear Physics, Springer Nature Singapore Pte Ltd. (2023)
- Precision atomic physics techniques for nuclear physics with RIBs
  K. Blaum, J. Dilling and W. Nörtershäuser, Physica Scripta T152 (2013) 014017

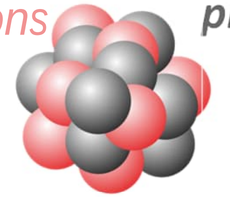
Setting the scale for our dear nucleus

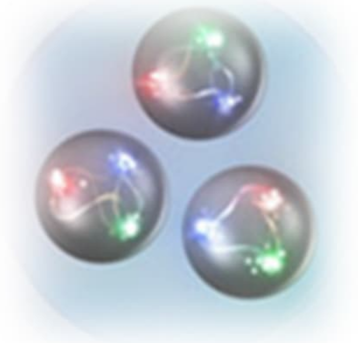


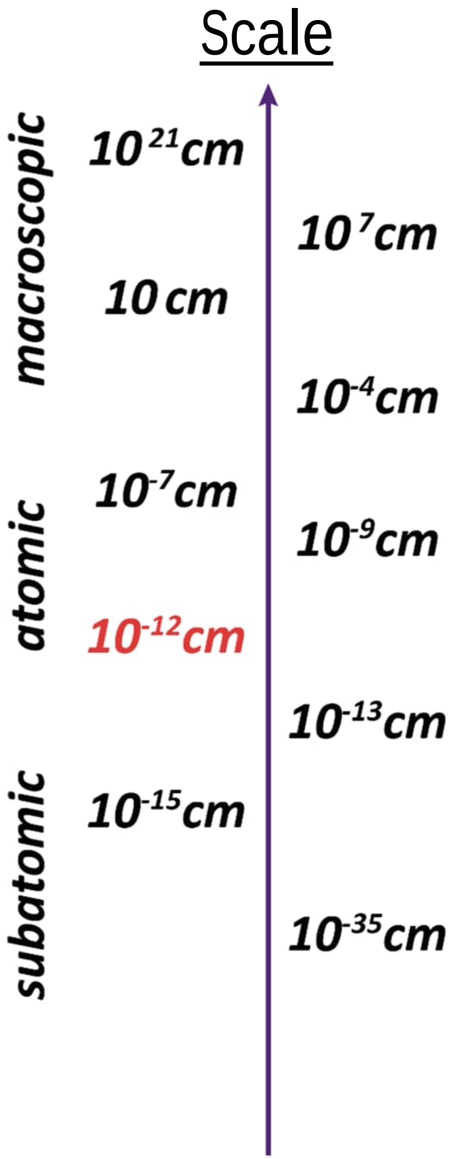




~1 femtometer (fm) = 0.000000000000001 m







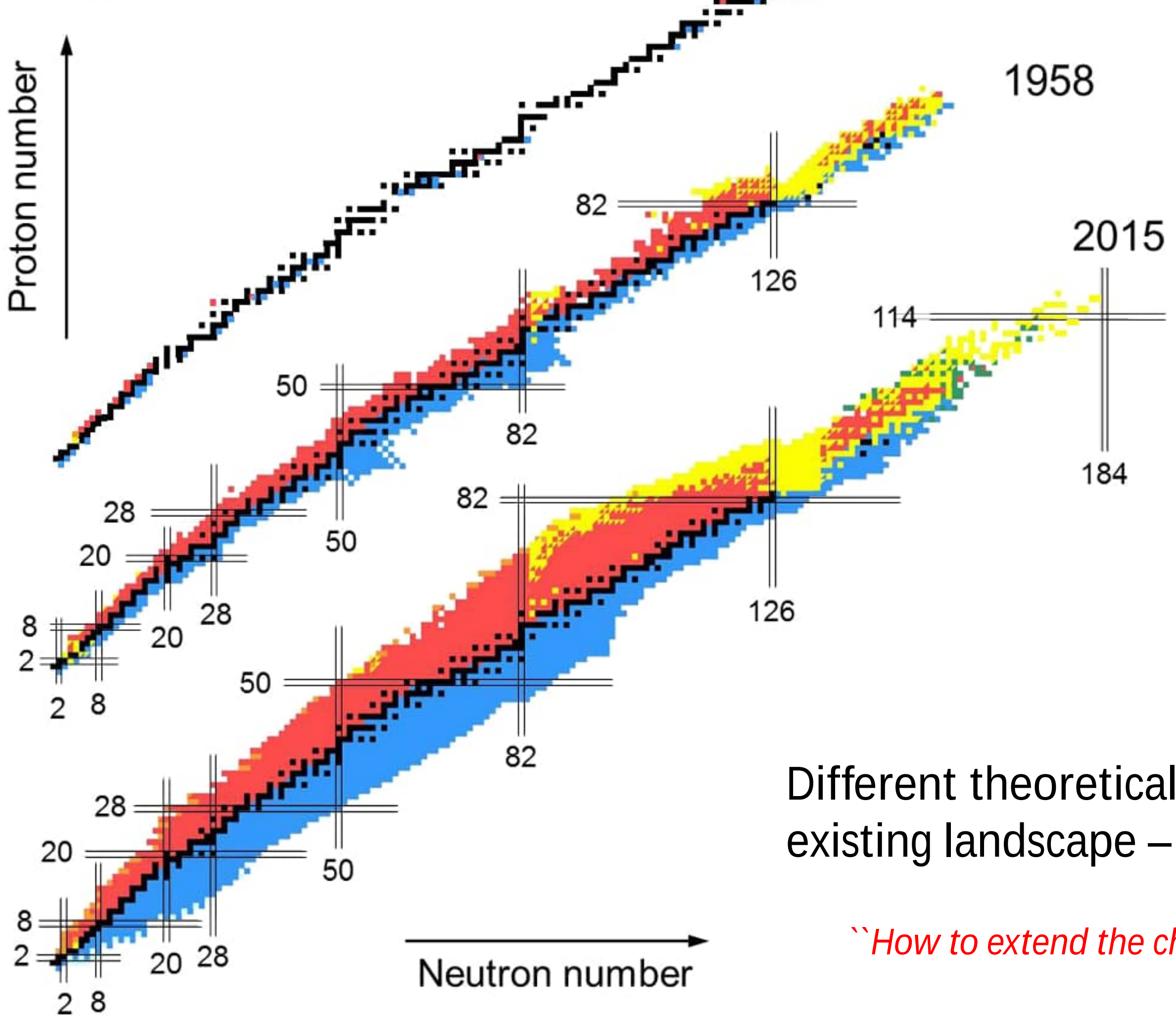
The chart of nuclides

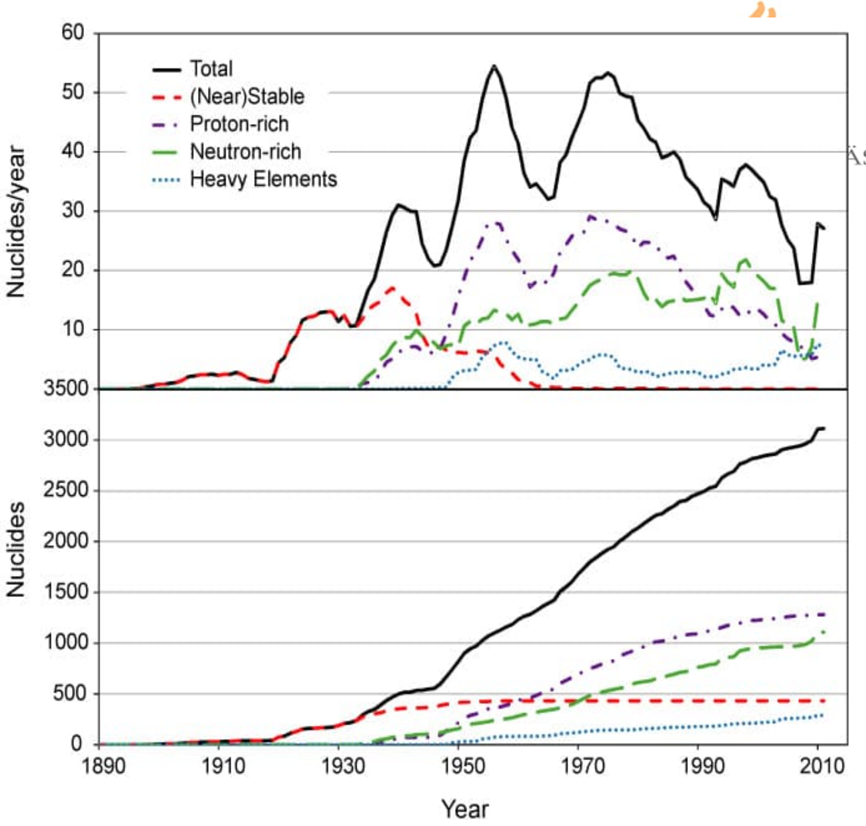
Figure from: M. Thoennessen, Rep. Prog. Phys. 76 (2013) 056301

Protons

Neutrons

~1 femtometer (fm) = 0.000000000000001 m





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journal homepage: www.elsevier.com/locate/adt

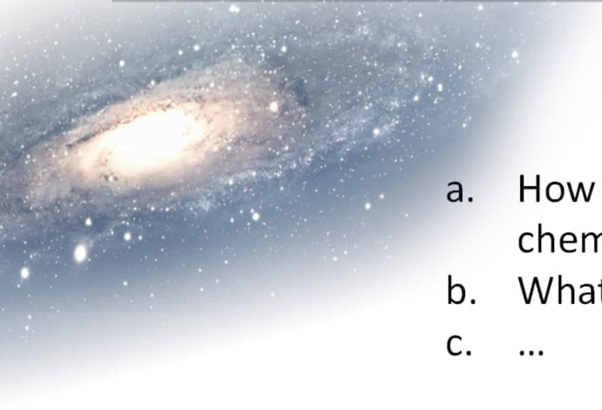
The limits of the nuclear landscape explored by the relativistic continuum Hartree–Bogoliubov theory

X.W. Xia[a], Y. Lim[b,c], P.W. Zhao[d,e], H.Z. Liang[f], X.Y. Qu[a,g], Y. Chen[d,h], H. Liu[d], L.F. Zhang[d], S.Q. Zhang[d], Y. Kim[c], J. Meng[d,a,i,*]

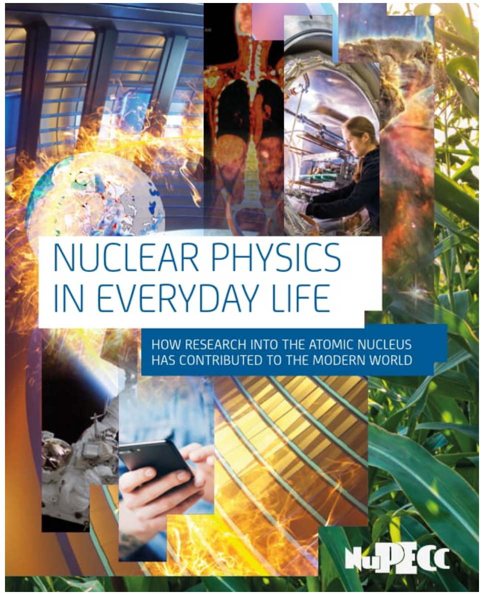
Different theoretical approaches provide estimates to the reach of the existing landscape – here, 9035 nuclei are predicted to be bound.

``How to extend the chart of nuclides?´´, G.G. Adamian et al., EPJ A 56 (2020) 47

Open questions in nuclear physics research



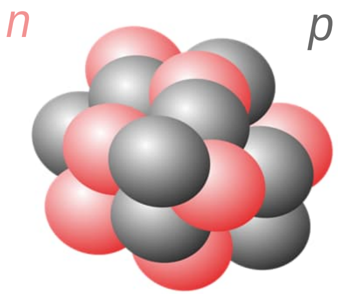
From the NuPECC Long Range Plan Draft Executive Summary 2024



Nuclear Astrophysics

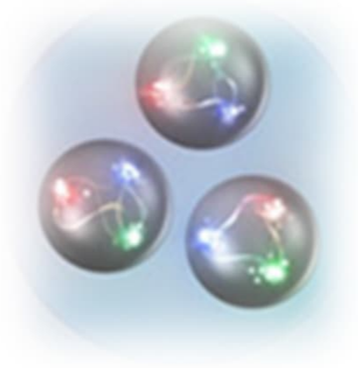
a. How can we better understand the synthesis of heavy elements and the chemical evolution of the visible universe
b. What is the role of the strong interaction in stellar objects?
c. …

Nuclear structure / nuclear matter



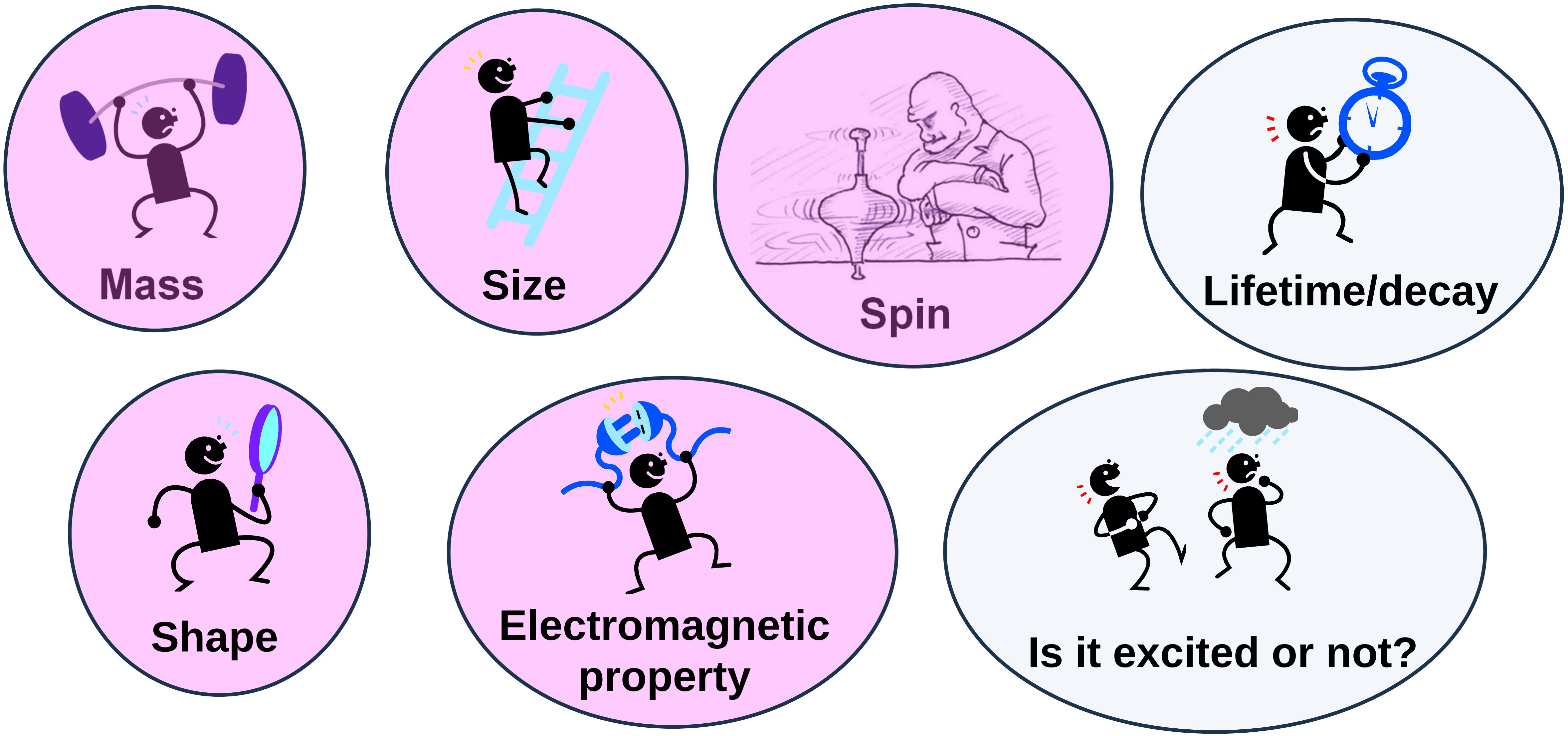
a. What shapes can nuclei take, how do nuclear shells evolve, and what role do nuclear correlations play?
b. What are the limits of the existence of nuclei, and what phenomena arise from open quantum systems?
c. What are the mechanisms behind nuclear reactions and nuclear fission?
d. …

Hadronic physics



a. How does the majority of the visible mass of the universe emerge from the almost massless quarks?
b. What are the properties of the quark-gluon plasma?
c. How do nuclei and nuclear matter emerge from the underlying fundamental interactions?

(Some) properties of a (radioactive) nucleus



Outline of lectures (let’s see about the reality! ☺)

Lecture 1: A window to the nucleus: masses and atomic spectral lines

Lecture 2: Production of radioactive ion beams

Lecture 3: Techniques of laser spectroscopy for radioactive nuclei

Lecture 4: Nuclear structure from charge radii and electromagnetic moments

Lecture 5: Mass spectrometry and a study of Ag isotopes at IGISOL

Lecture 6: Towards the heavy elements and the Nature of Time

The Atomic Mass in a nutshell

- The mass of a nucleus is a fundamental property.
- It is always less than the sum of the masses of its constituents.
- The difference is known as the nuclear binding energy.

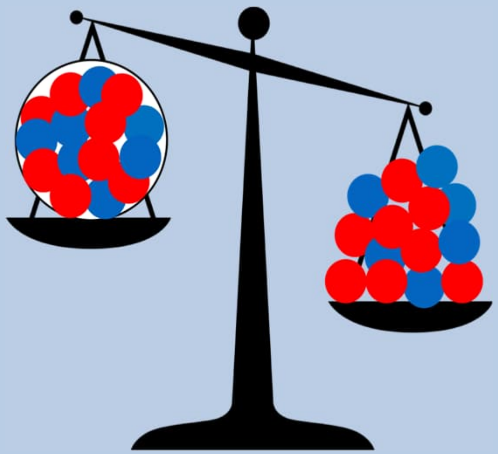
$$m_{\text{nucleus}}(\mathrm{A,Z})c^2 = (Zm_p + (A - Z)m_n)c^2 - B(\mathrm{A,Z})$$

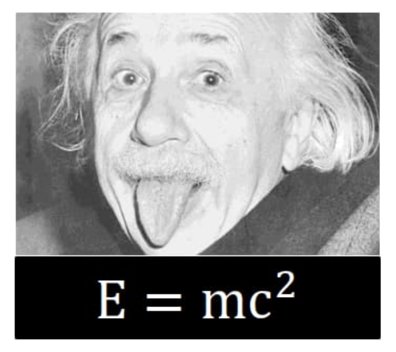
$$m_{\text{atom}}(\mathrm{A,Z})c^2 = m_{\text{nucleus}}(\mathrm{A,Z})c^2 + Zm_e c^2 - \sum_{i=1}^{Z} B_{e_i}$$

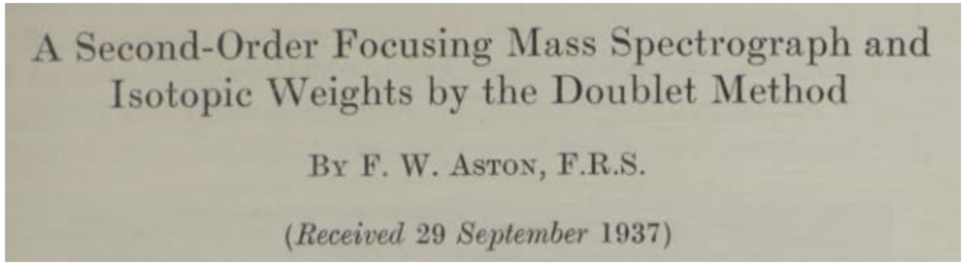
- A = mass number = neutron number + proton number
- Z = proton number (element)
- B the binding energy
  - Nuclear (~MeV/u), Atomic electrons (eV…keV)
  - Compare to absolute nucleon mass ~1 GeV

MORE BOUND = MORE BINDING ENERGY = LIGHTER NUCLEUS

Nuclear Binding Energy






A Second-Order Focusing Mass Spectrograph and Isotopic Weights by the Doublet Method

By F. W. Aston, F.R.S.

(*Received* 29 *September* 1937)

The Atomic Mass in a nutshell

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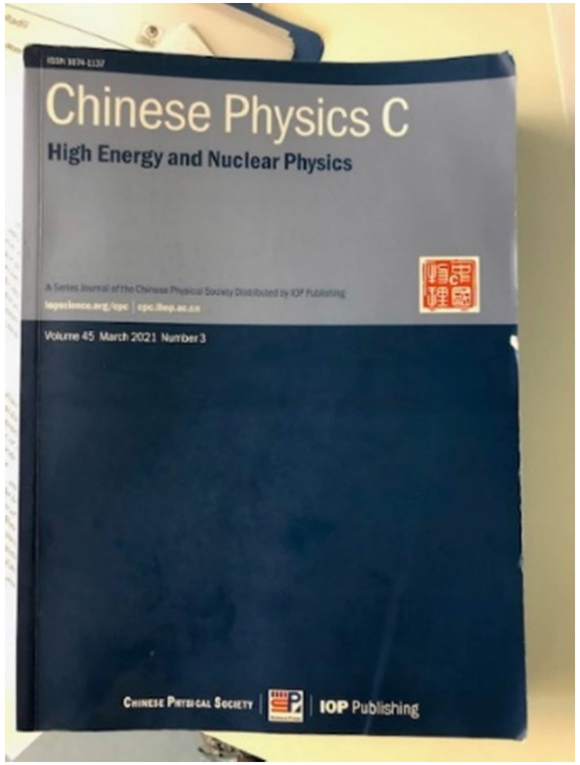
- Nuclear masses measured in terms of the *unified atomic mass unit*, u.
- Definition: mass of ^{12}C atom, $m(^{12}C)$ is exactly 12 u.
- In other words, 1 u = 1/12 x $M(^{12}C)$
- ^{12}C = 6 protons + 6 neutrons + 6 electrons
- Therefore, masses of nucleons approximately: $m_p \approx m_n \approx 1$ u
- In reality m_n = 1.00866491588(49) u > m_p = 1.0072764665789(83) u
- Electron mass: m_e = 5.485799090441(97)×10^{-4} u ($m_{p,n} \approx 2000 \times m_e$)



Generally, work with mass energies rather than masses themselves.
(1 u = 931494.10242(28) keV/c^2 CODATA2018) ≈ 1.66 x 10^{-27} kg

Usually, atomic mass tables list the mass defect or excess rather than atomic mass:

$$\Delta\left({}^{A}X\right) = \left[m(Z,N) - Au\right]c^2$$

What is the mass excess of ^{12}C?

The King James bible: the Atomic Mass Evaluation

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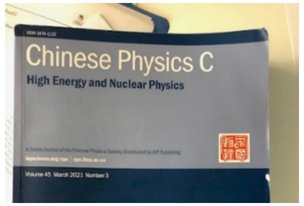
Periodically published collection of atomic masses

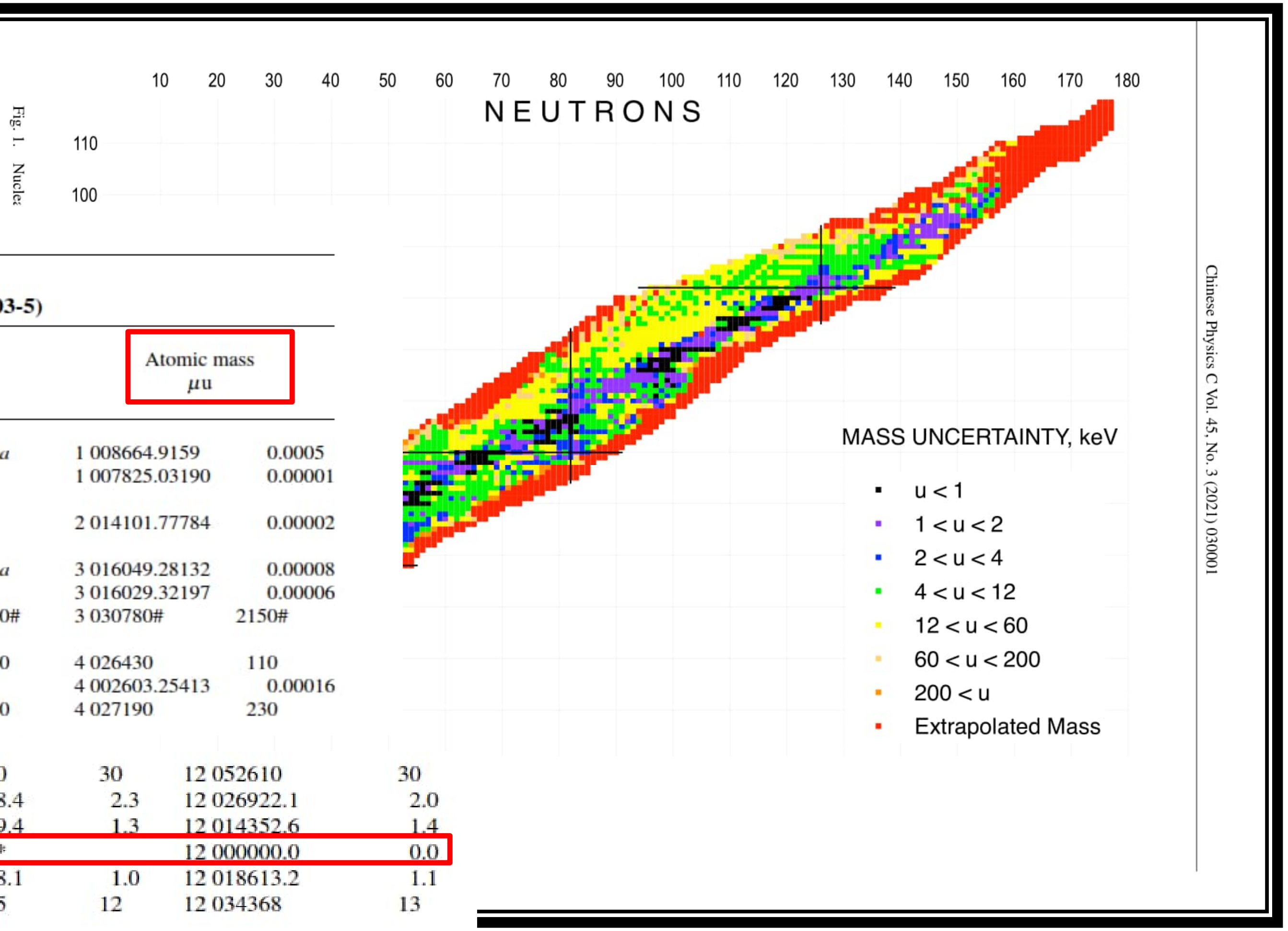


W.J. Huang et al., Chinese Phys. C 45, 030002 (2021)

Chinese Physics C Vol. 45, No. 3 (2021) 030003

Table I. The 2020 Atomic mass table (Explanation of Table on p. 030003-5)

N	Z	A	Elt.	Orig.	Mass excess (keV)		Binding energy per nucleon (keV)		Beta-decay energy (keV)			Atomic mass μu	
1	0	1	n		8071.3181	0.0004	0.0	0.0	β−	782.347	a	1 008664.9159	0.0005
0	1		H		7288.97106	0.00001	0.0	0.0		*		1 007825.03190	0.00001
1	1	2	H		13135.72290	0.00002	1112.283	a		*		2 014101.77784	0.00002
2	1	3	H		14949.81090	0.00008	2827.265	a	β−	18.592	a	3 016049.28132	0.00008
1	2		He		14931.21888	0.00006	2572.680	a		*		3 016029.32197	0.00006
0	3		Li	-pp	28670#	2000#	-2270#	670#	β+	13740#	2000#	3 030780#	2150#
3	1	4	H	-n	24620	100	1720	25	β−	22200	100	4 026430	110
2	2		He		2424.91587	0.00015	7073.916	a		*		4 002603.25413	0.00016
1	3		Li	-p	25320	210	1150	50	β+	22900	210	4 027190	230
9	3	12	Li	-n	49010	30	3791.6	2.5	β−	23930	30	12 052610	30
8	4		Be		25077.8	1.9	5720.72	0.16	β−	11708.4	2.3	12 026922.1	2.0
7	5		B		13369.4	1.3	6631.22	0.11	β−	13369.4	1.3	12 014352.6	1.4
6	6		C		0.0	0.0	7680.145	a		*		12 000000.0	0.0
5	7		N		17338.1	1.0	6170.11	0.08	β+	17338.1	1.0	12 018613.2	1.1
4	8		O	-pp	32013	12	4882.0	1.0	β+	14675	12	12 034368	13



Mass-excess uncertainties for all nuclei in ground state

Nuclear structure mass ``filters/indicators´´

The study of differences in nuclear binding energies, or mass excesses, gives information on nuclear properties. Removing a nucleon requires energy to overcome the binding energy.

These derivatives give insights into subtle changes in nuclear structure along isotopic (same *Z*) or isotonic (same *N*) lines.

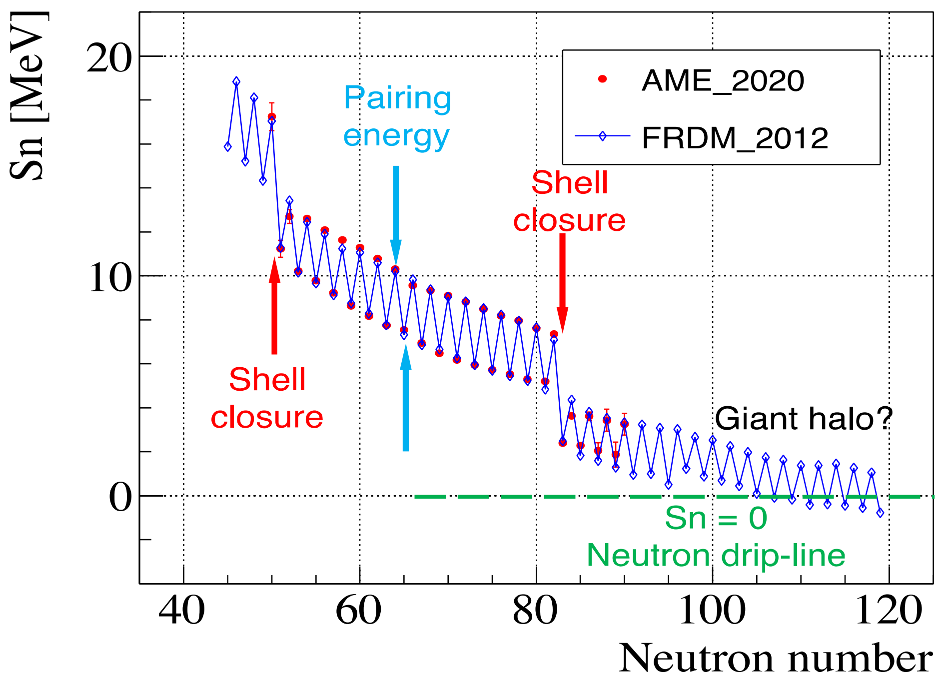
$$S_n(Z,N) = ME_n + ME(Z,N-1) - ME(Z,N)$$

$$S_{2n}(Z,N) = 2M_n + M(Z,N-2) - ME(Z,N)$$

One proton, S_p, and one neutron, S_n, separation energies determine where the driplines are located.

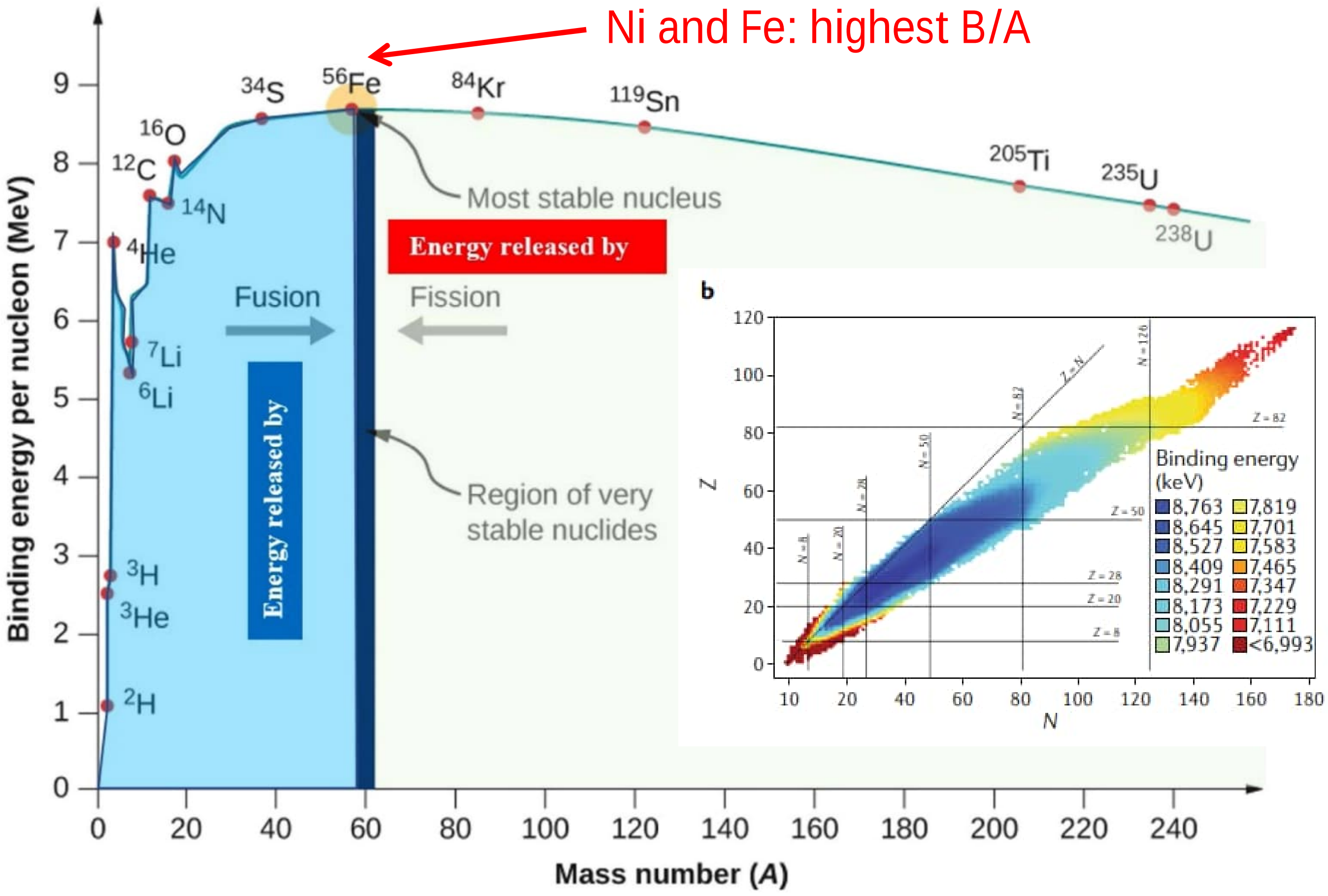
Two-neutron/proton separation energies remove the odd-even staggering effects, allowing for smoother trends to be observed, as well as indications of sudden structure changes.

tin (Z = 50) one neutron seperation energy

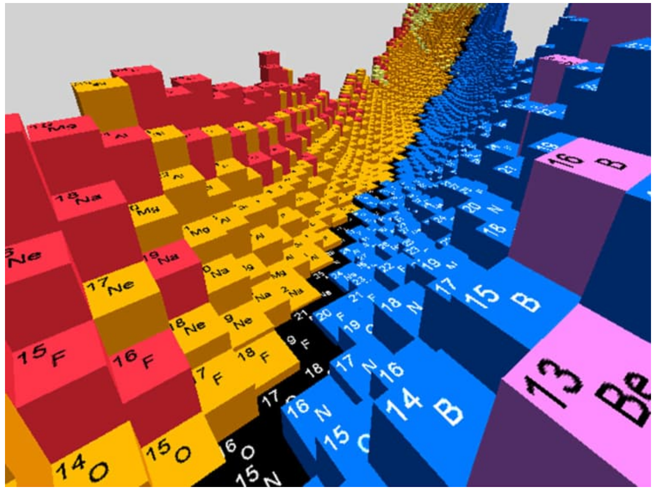


Nuclear binding energy

The energy required to dissasemble a nucleus into its constitents

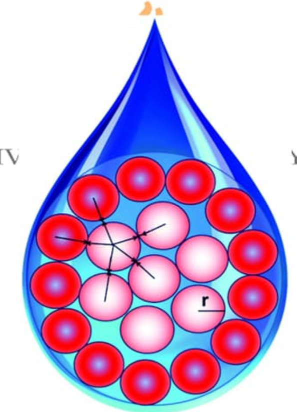


Figures from L.S. Paraschiv et al., Energy Reports 8 (2022) 342; P. Schwerdtfeger et al., The periodic table and the physics that drives it, Nature Reviews Chemistry (2020)



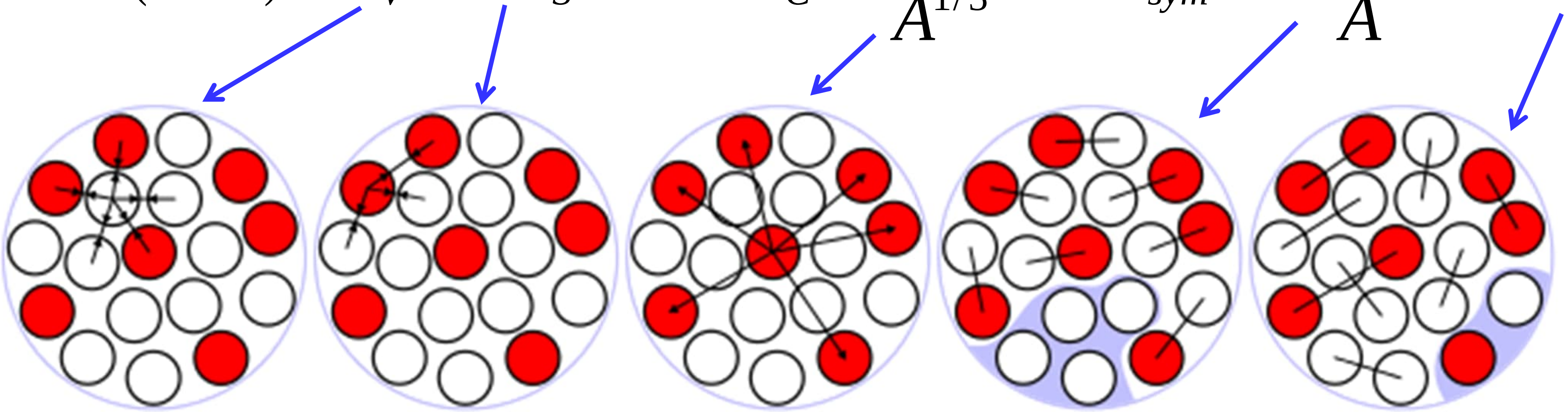
- When a nucleus is formed, the binding energy occurs due to a transformation of a mass quantity into energy.
- Nuclear stability – the nuclear landscape!
- Theory is needed to predict the limits of stability.
- Direct relationship to nuclear energy (fission & fusion) released in nuclear reactions.

The liquid droplet model



Based on the analogy of nuclear matter as a liquid. A semi-empirical macroscopic approach that treats the nucleus as a spherical and incompressible droplet of liquid of radius $r_0A^{1/3}$. The model assumes a homogeneous charge and density in the nucleus.

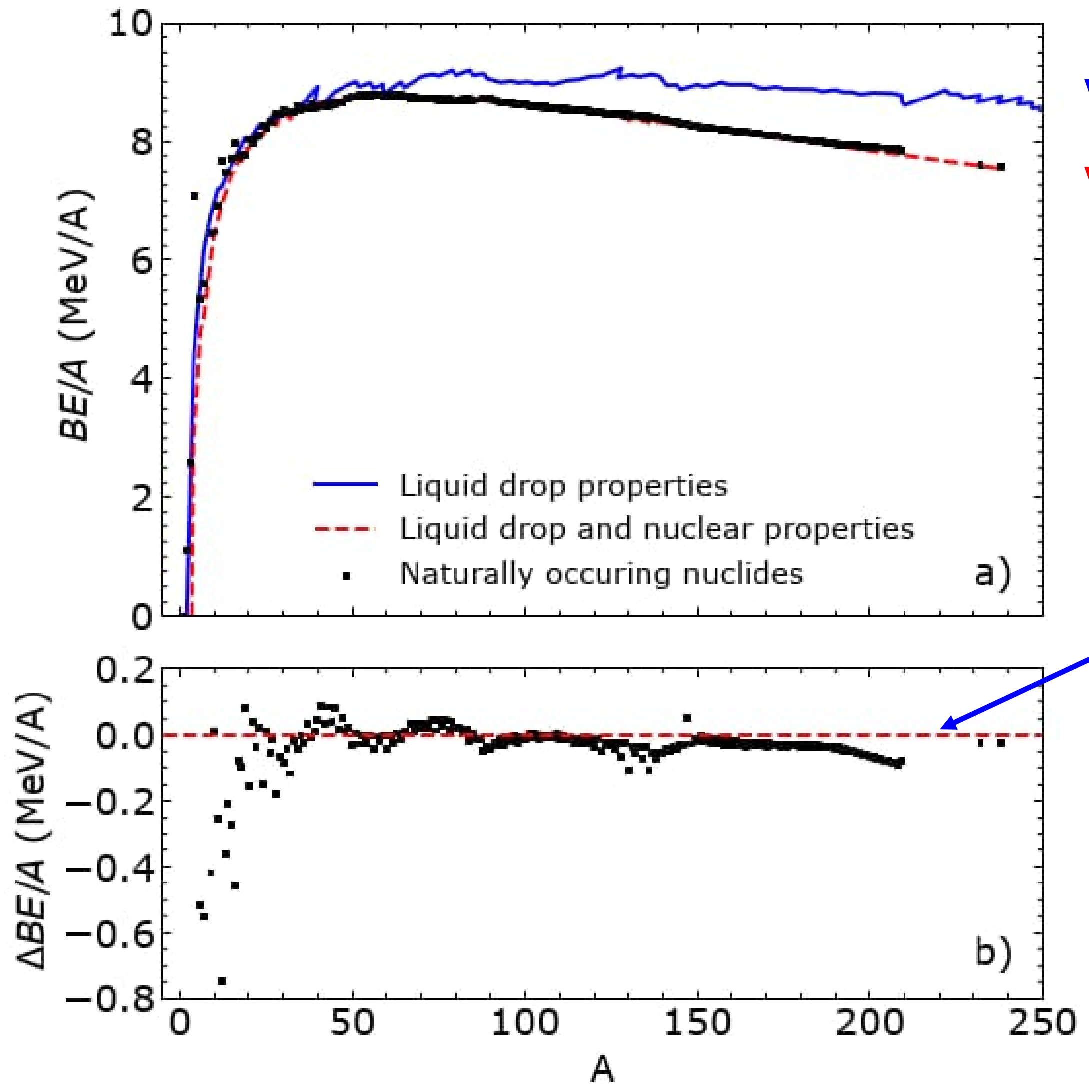
$$B(Z,A) = a_V A - a_S A^{2/3} - a_C \frac{Z(Z-1)}{A^{1/3}} - a_{sym}\frac{(A-2Z)^2}{A} + \delta$$



+ve for even-even nuclei
0 for odd A nuclei
-ve for odd-odd nuclei

a.k.a. Weizsäcker formula, Bethe-Weizsäcker (mass) formula

The liquid droplet model



Volume + surface + Coulomb terms

Volume + surface + Coulomb + Asymmetry + Pairing terms

Difference $\Delta = BE/A_{ldm} - BE/A_{exp}$ from nuclear binding energies tabulated in Atomic Mass Evaluation 2020.

Impressive reproduction of the evolution of binding energies with A.

- We know however that this picture misses relevant structure, notably shell effects…

The nuclear shell model

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The Nobel Prize in Physics 1963

Photo from the Nobel Foundation archive.

Eugene Paul Wigner

Prize share: 1/2

Photo from the Nobel Foundation archive.

Maria Goeppert Mayer

Prize share: 1/4

Photo from the Nobel Foundation archive.

J. Hans D. Jensen

Prize share: 1/4

``for their discoveries concerning nuclear shell structure´´

On the "Magic Numbers" in Nuclear Structure

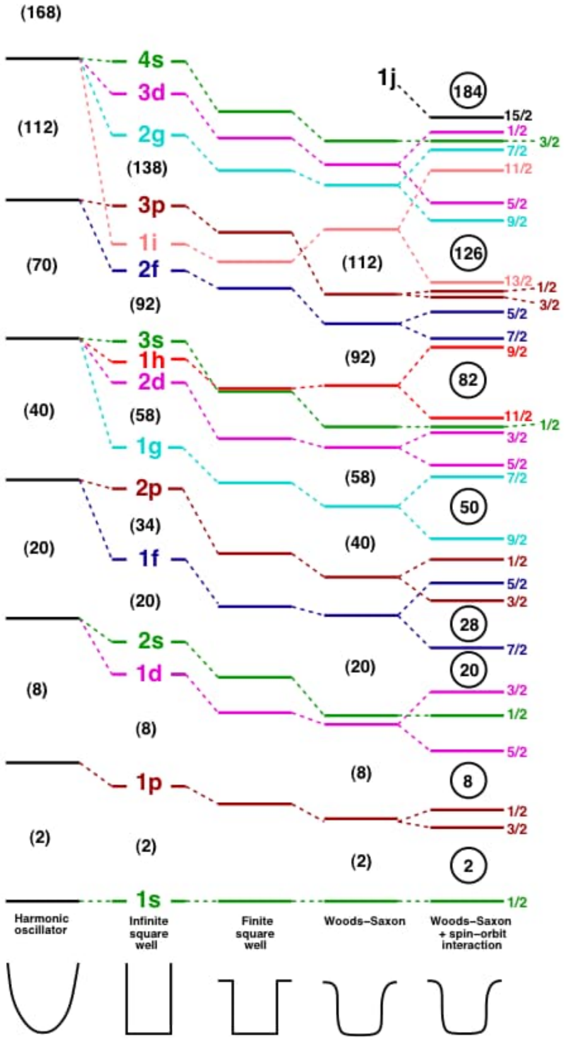
OTTO HAXEL
Max Planck Institut, Göttingen
J. HANS D. JENSEN
Institut f. theor. Physik, Heidelberg
AND
HANS E. SUESS
Inst. f. phys. Chemie, Hamburg
April 18, 1949

A SIMPLE explanation of the "magic numbers" 14, 28, 50, 82, 126 follows at once from the oscillator model of the nucleus,[1] if one assumes that the spin-orbit coupling in the Yukawa field theory of nuclear forces leads to a strong splitting of a term with angular momentum l into two distinct terms $j=l\pm\frac{1}{2}$.

On Closed Shells in Nuclei. II

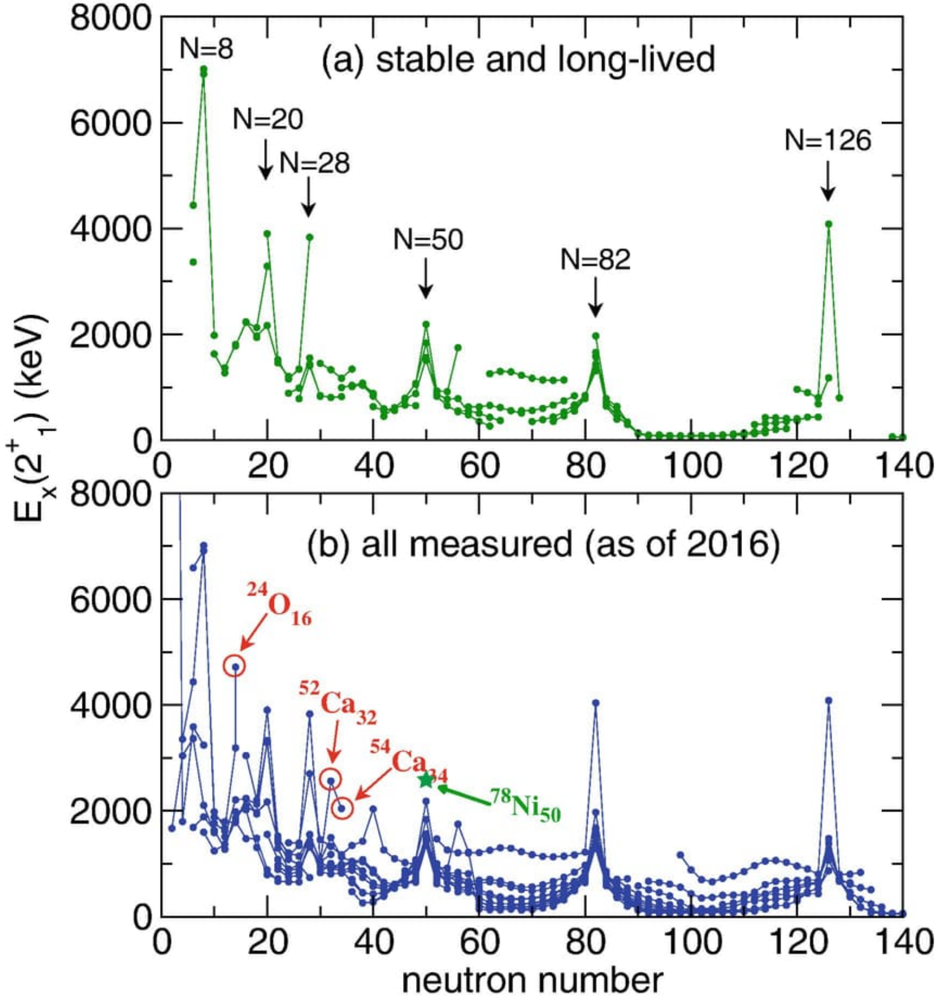
MARIA GOEPPERT MAYER
Argonne National Laboratory and Department of Physics, University of Chicago, Chicago, Illinois
February 4, 1949

THE spins and magnetic moments of the even-odd nuclei have been used by Feenberg[1,2] and Nordheim[3] to determine the angular momentum of the eigenfunction of the odd particle. The tabulations given by them indicate that spin orbit coupling favors the state of higher total angular momentum. If strong spin-orbit coupling, increasing with angular momentum, is assumed, a level assignment different from either Feenberg or Nordheim is obtained. This assignment encounters a very few contradictions with experimental facts and requires no major crossing of the levels from those of a square well potential. The magic numbers 50, 82, and 126 occur at the place of the spin-orbit splitting of levels of high angular momentum.

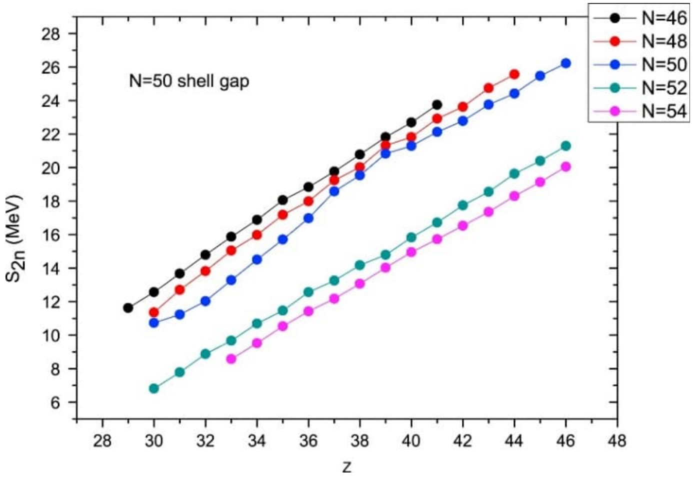


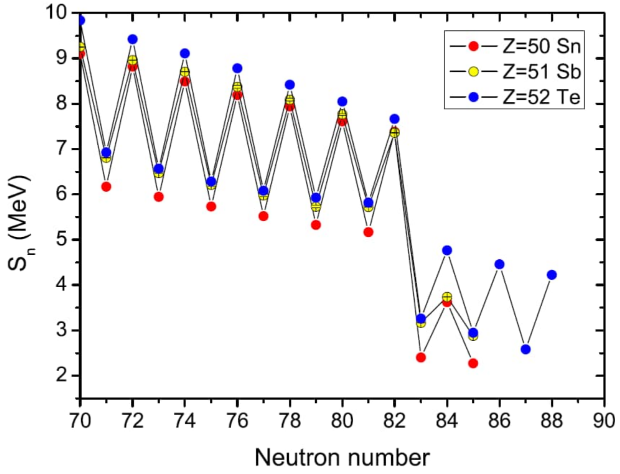
(STABLE) Magic numbers 2, 8, 20, 28, 50, 82 and 126

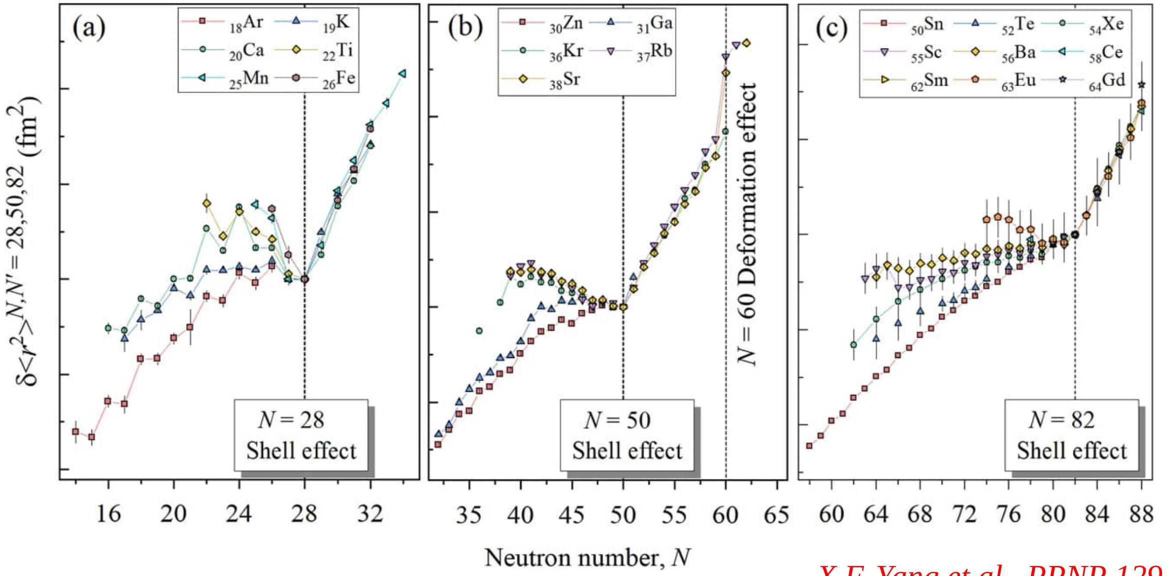
Evidence of nuclear shell structure



Otsuka et al., Rev. Mod. Phys. 92 (2020) 015002



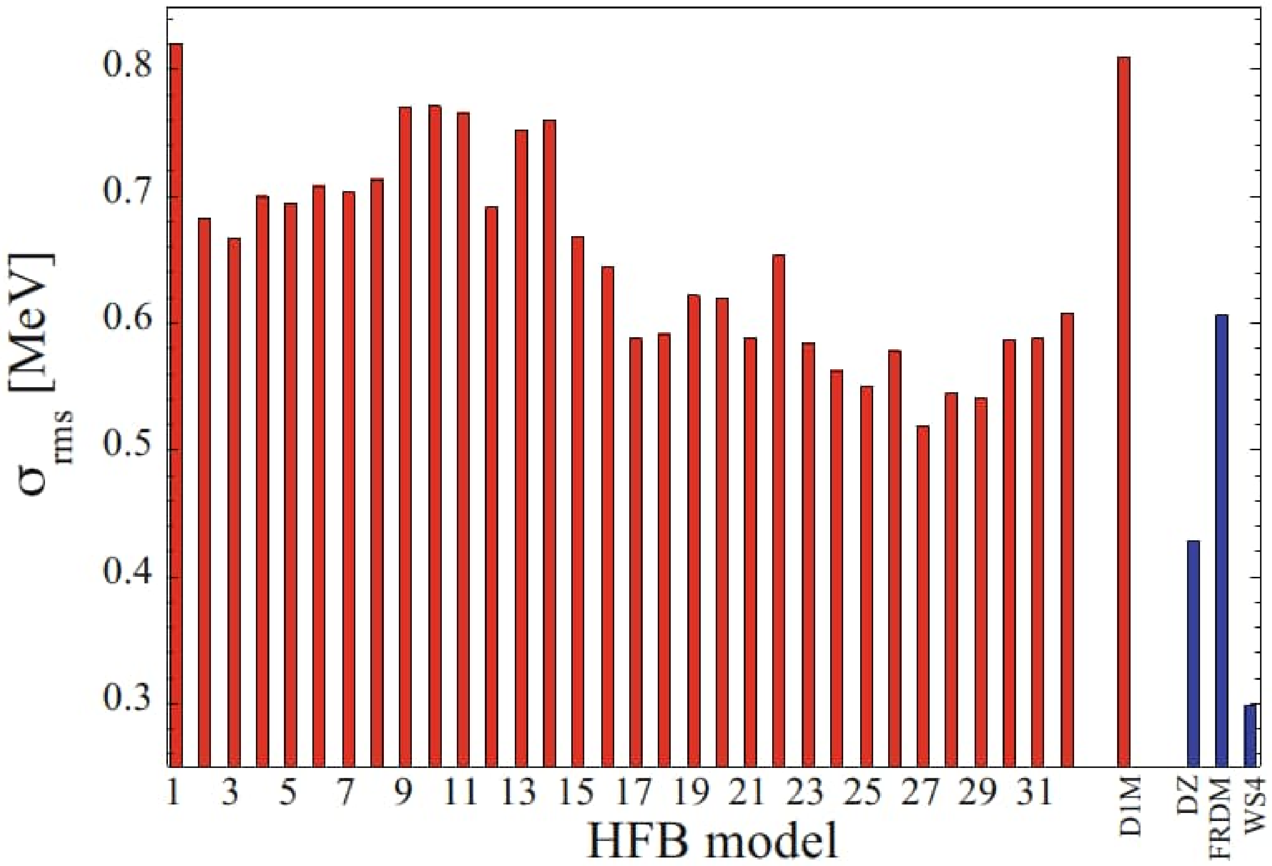




X.F. Yang et al., PPNP 129 (2023) 104005

Nuclear mass models

Representation of the rms deviation for the 32 Skyrme-HFB mass models, Gogny-HFB (D1M), the finite-range droplet model (FRDM) and Weizsäcker-Skyrme (WS). The latter two are macroscopic – microscopic mass models.

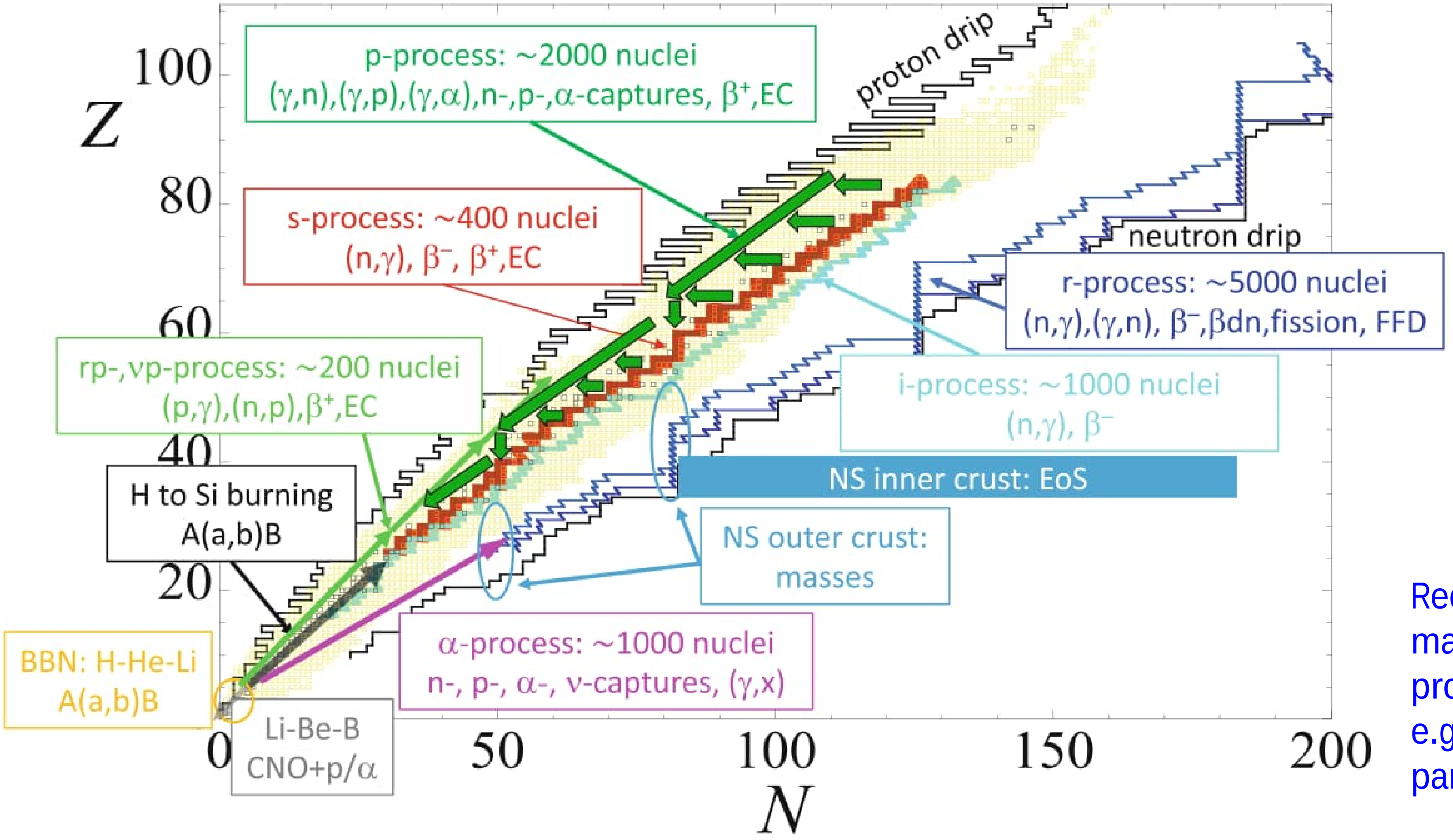


- Mass models are critically important as they are used to extrapolate to regions which have no experimental data
- Modeling of nuclear synthesis pathways in nuclear astrophysics
- Typical experimental precision ~1 keV/c^2
- RMS deviation ~0.5 – 0.7 MeV/c^2

$$\sigma_{\text{rms}} = \sqrt{\frac{1}{M}\sum_{j=1}^{M}\left(B_j^{\text{expt}} - B_j^{\text{theo}}\right)^2}$$

``Masses of exotic nuclei´´, from the Handbook of Nuclear Physics (2022)

Nuclear masses for astrophysics



``Masses of exotic nuclei´´, from the Handbook of Nuclear Physics (2022)

- Nuclear masses (binding energies) determine nucleosynthesis pathways
- Modeling the rp process:
  - X-ray bursts and light curves
  - proton captures have exponential dependence on mass
- Theoretical models are critically important, e.g., for modeling the *r*-process

Recent work illustrates only local changes in mass trends (e.g., shell effects) play a role on *r*-process abundances – not bulk properties, e.g., symmetry energy in a liquid drop parameterization

S.A. Giuliani et al., arXiv:2412.03243

Calculated and experimental S_{2n} for erbium

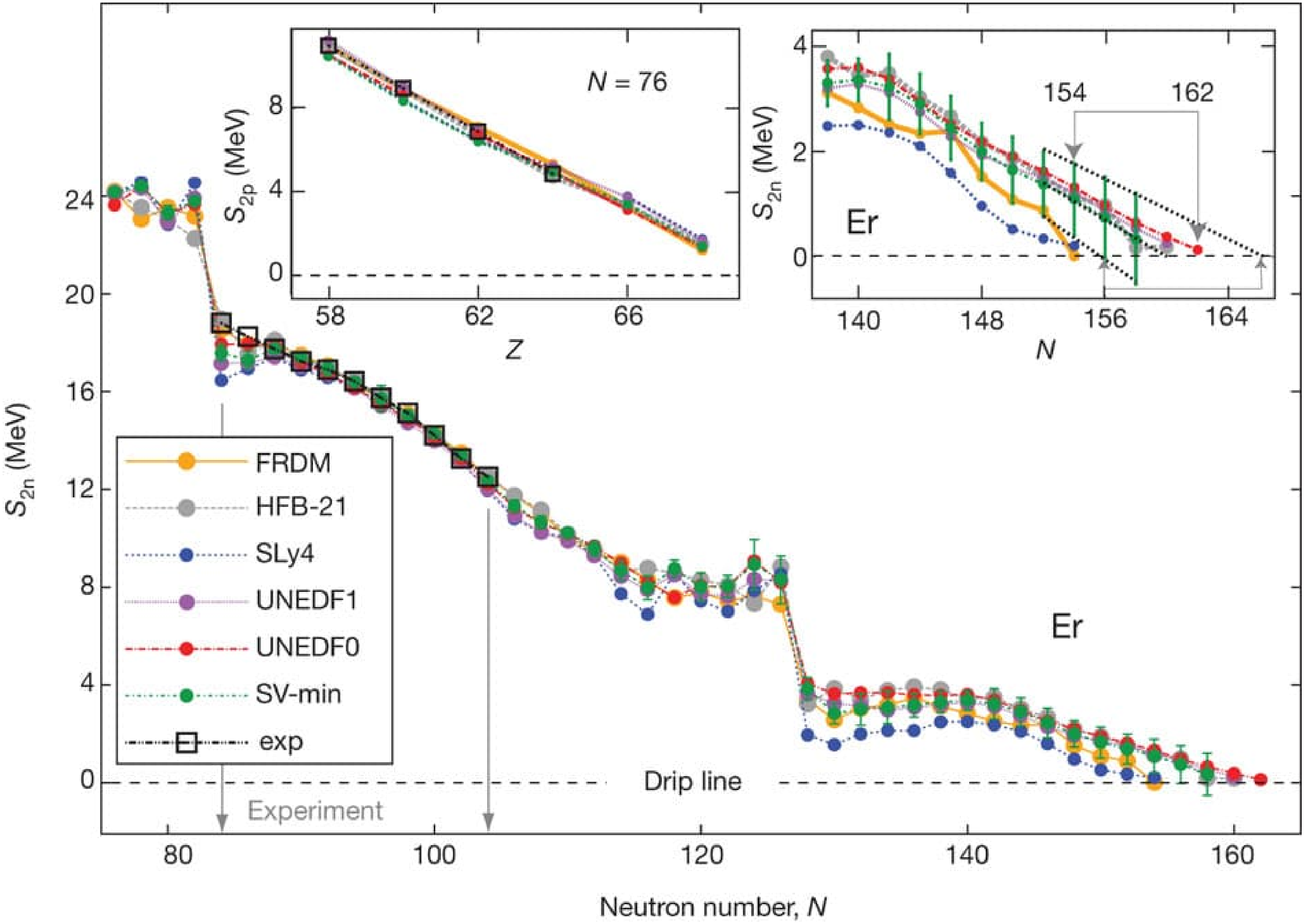


Figure from J. Erler et al., Nature 486 (2012) 509.

- Stable Er (Z = 68) isotopes from N = 94 to N = 102.
- S_{2n} = two-neutron separation energy
- S_{2n} = 0 is the neutron drip line
- S_{2p} = two-proton separation energy

$$S_{2n}(Z,N) = B(Z,N\text{-}2) - B(Z,N)$$

Measurement of atomic masses – how precise?

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Mass of each and every nuclear state is important!

- Some masses are needed with more precision, some less...

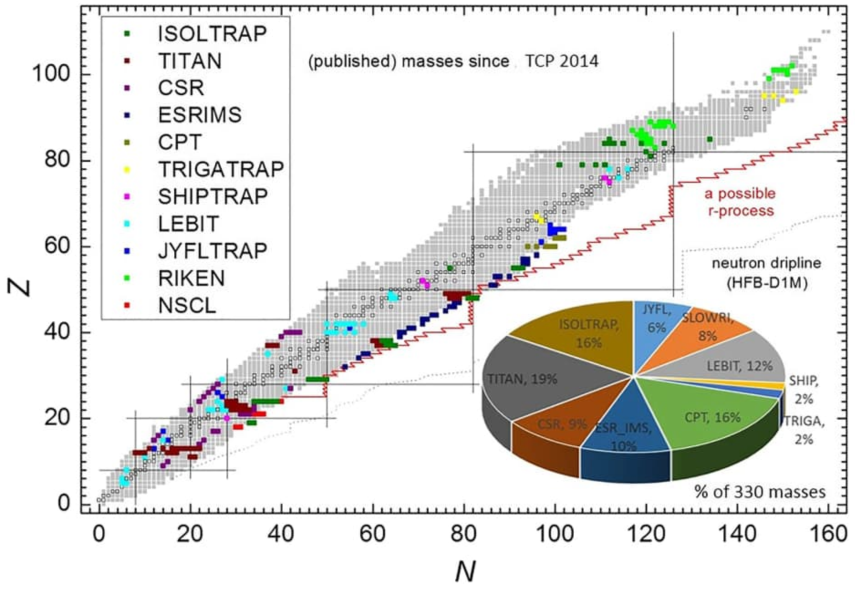
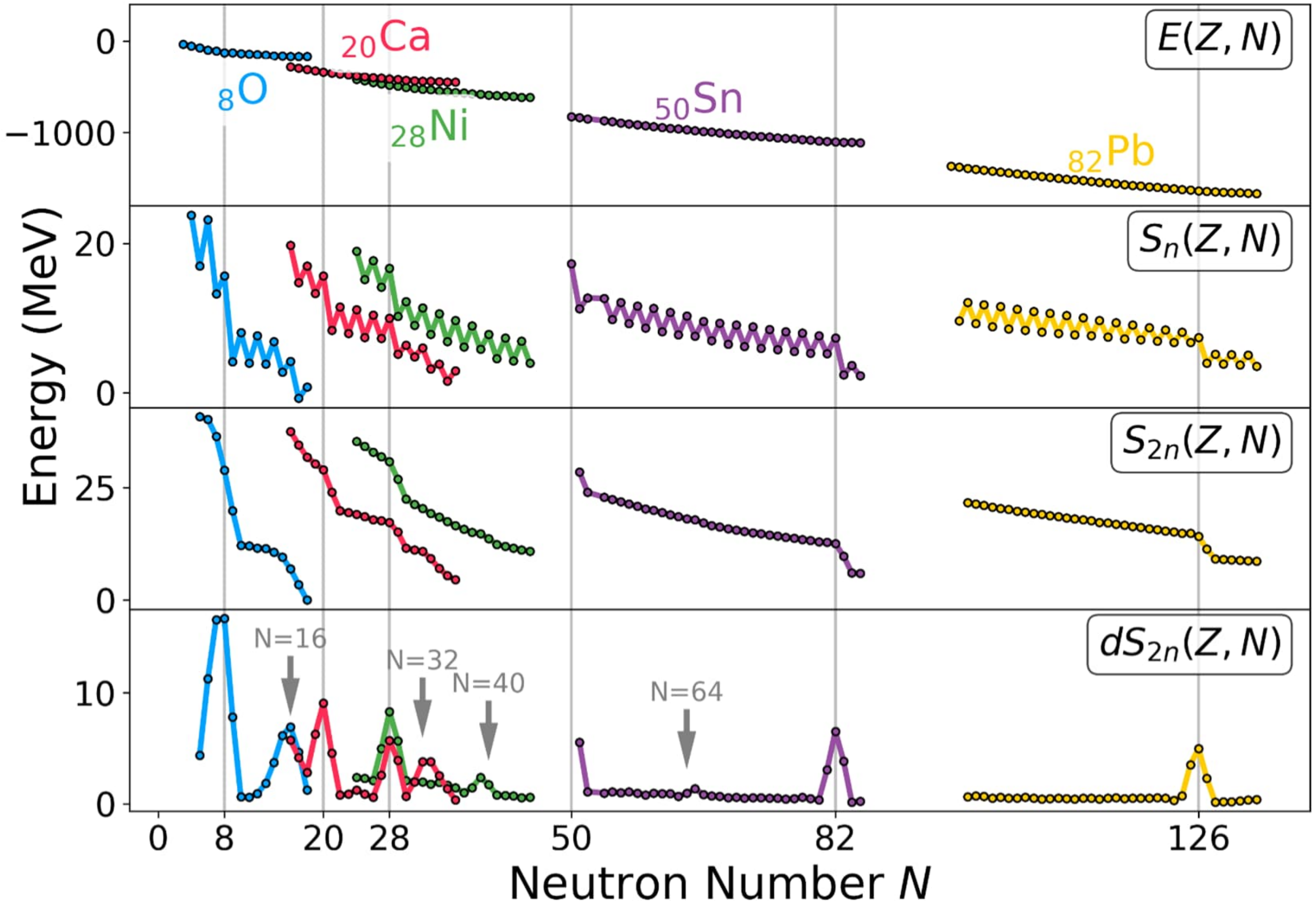


Figure from D. Lunney, Hyp. Int. 240 (2019) 48

~2500 masses experimentally determined (white boxes)

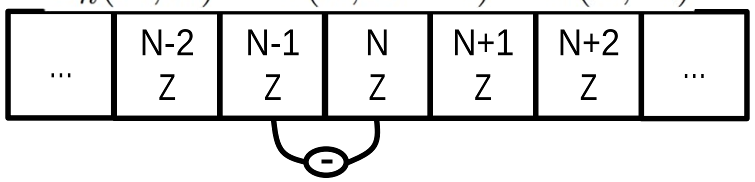
	δm/m	δm for m=100 u (μu)	δm for m=100 u (keV)
General physics & chemistry	$\leq 10^{-5}$	1000	1000
Nuclear structure physics - separation of isobars	$\leq 10^{-6}$	100	100
Astrophysics - separation of isomers	$\leq 10^{-7}$	10	10
Weak interaction studies	$\leq 10^{-8}$	1	1
Metrology - fundamental constants; Neutrino physics	$\leq 10^{-9}$	0.1	0.1
CPT tests	$\leq 10^{-10}$	0.01	0.01
QED in highly-charged ions - separation of atomic states	$\leq 10^{-11}$	0.001	0.001

Nuclear structure via mass probes

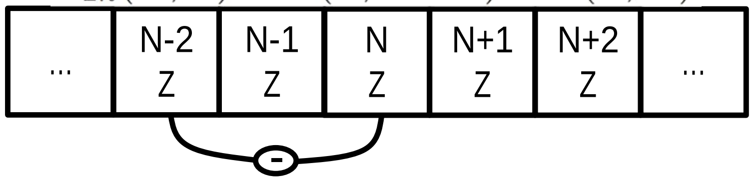


Courtesy of Lukas Nies, PLATAN 2024 conference.

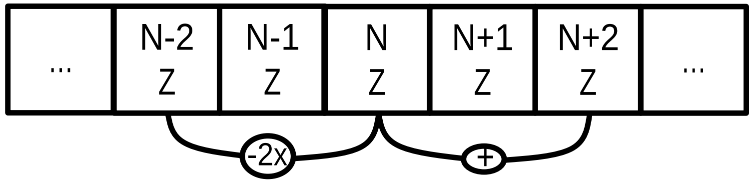
$$S_n(N, Z) = E(Z, N-1) - E(Z, N)$$



$$S_{2n}(N, Z) = E(Z, N-2) - E(Z, N)$$



$$dS_{2n}(Z, N) = S_{2n}(Z, N) - S_{2n}(Z, N+2)$$
$$= E(Z, N-2) - 2E(Z, N) + E(Z, N+2)$$



A short historical note on atomic spectroscopy

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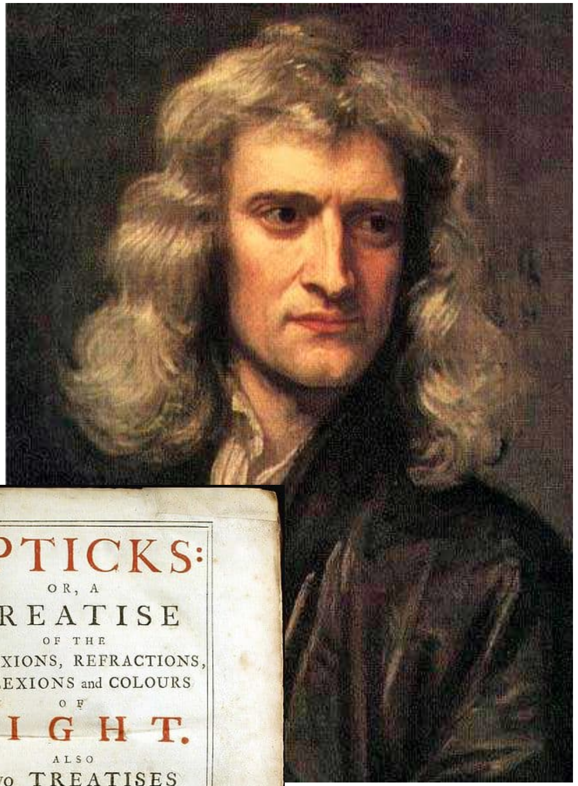
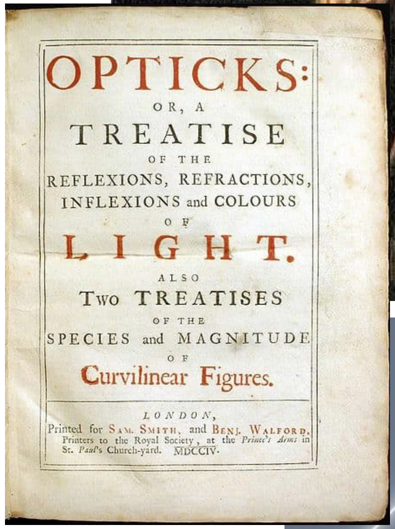
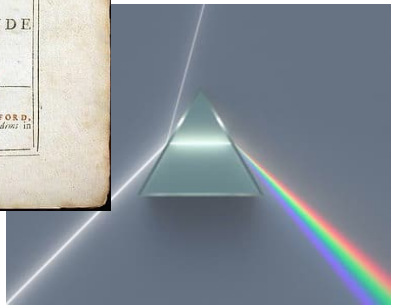
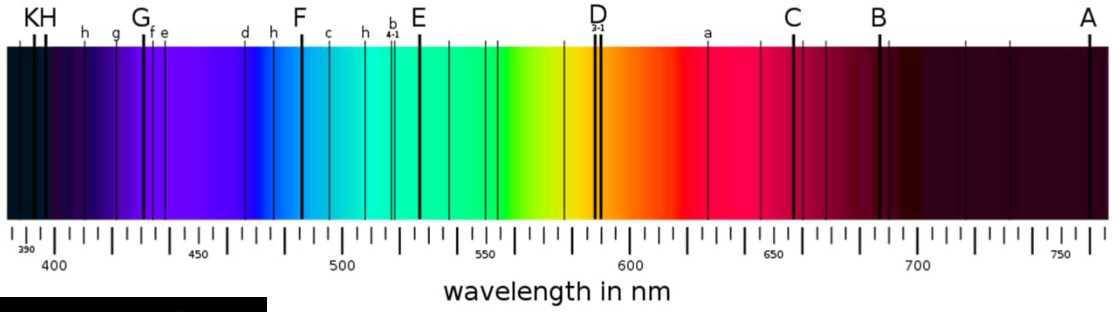
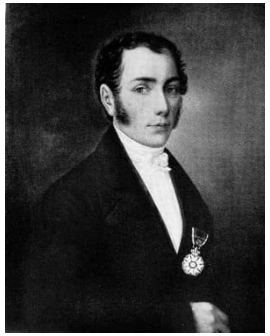
- 1704: release of Newton's "Opticks". Sun's light can be dispersed into a "spectrum"

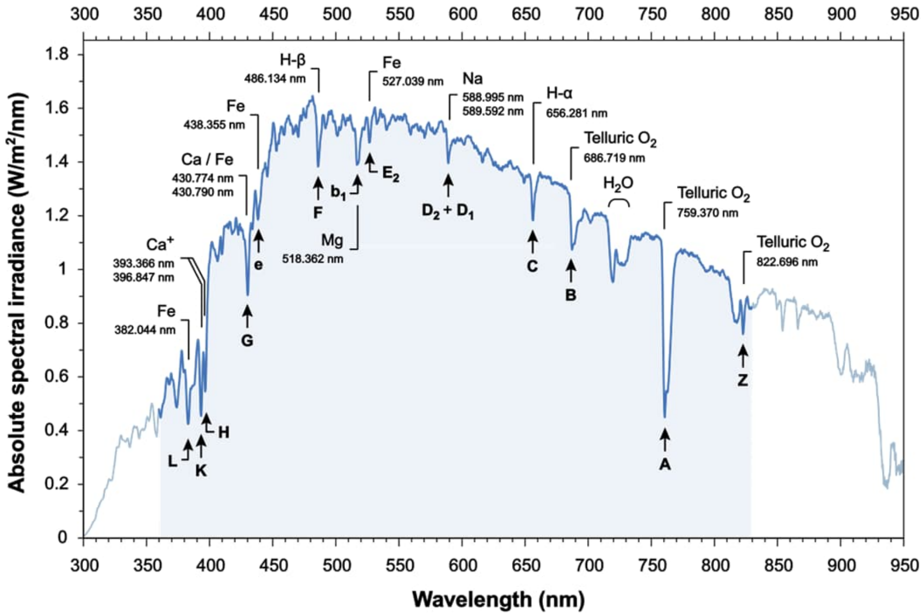



OPTICKS:
OR, A
TREATISE
OF THE
REFLEXIONS, REFRACTIONS, INFLEXIONS and COLOURS
OF
LIGHT.
ALSO
Two TREATISES
OF THE
SPECIES and MAGNITUDE
OF
Curvilinear Figures.
LONDON,
Printed for SAM. SMITH, and BENJ. WALFORD, Printers to the Royal Society, at the Prince's Arms in St. Paul's Church-yard. MDCCIV.

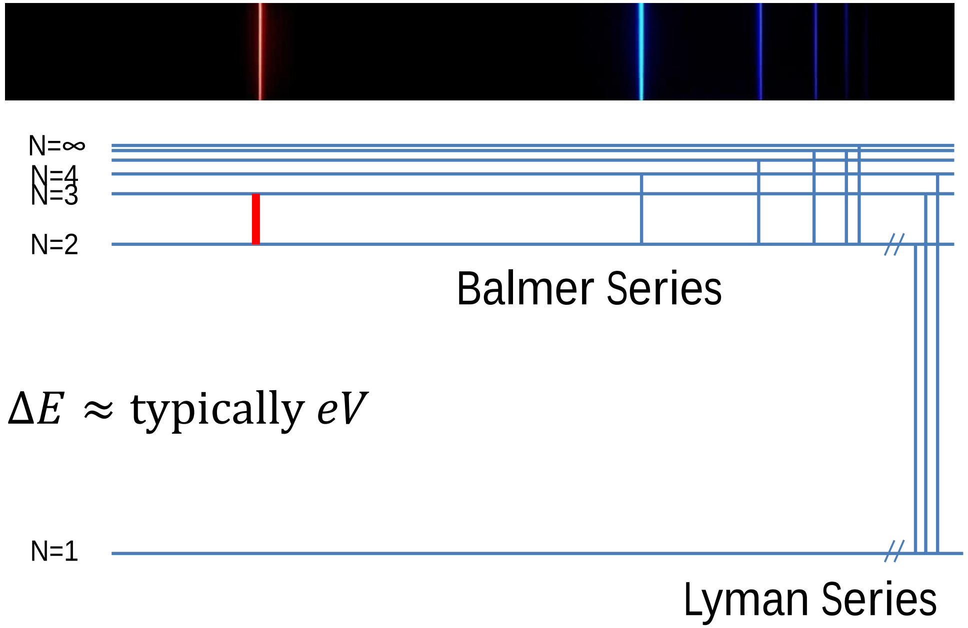






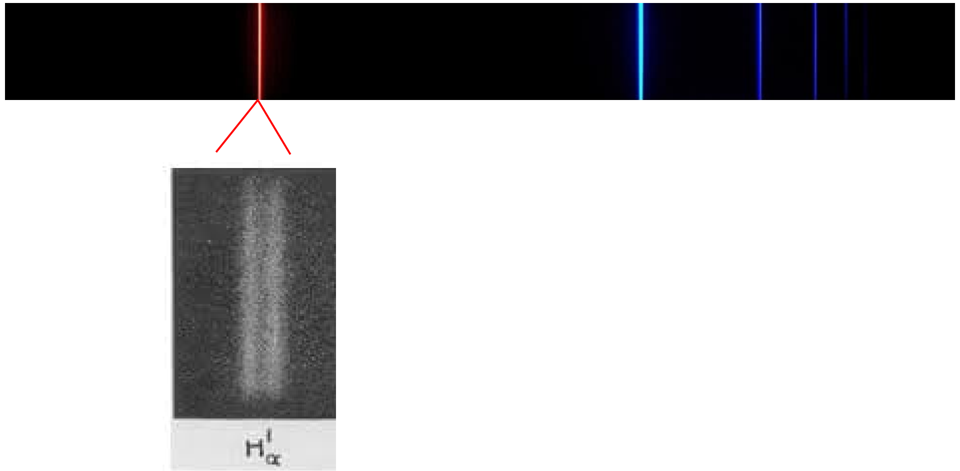


Fine structure



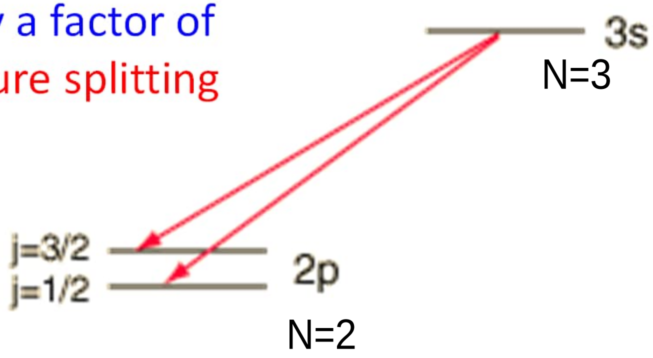
$\Delta E \approx$ typically eV

λ=656.279 nm (N=3 → N=2 in Balmer series)



Increasing the resolution by a factor of ~5000 reveals a fine structure splitting of hydrogen

$\Delta E_{FS} \approx 10^{-1} - 10^{-3} eV$



Electron angular momenta couple:

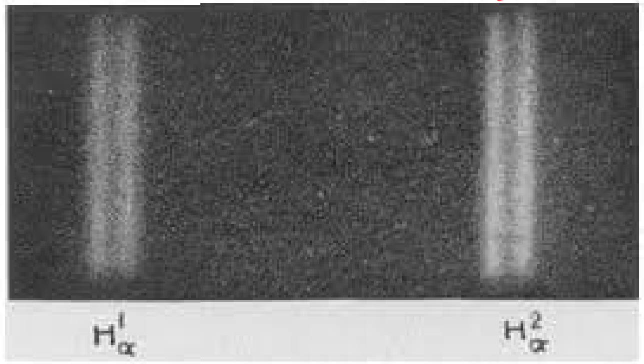
J = L+S, L+S-1, ..., |L-S|

Giving the configuration for a state: $^{2S+1}L_J$

e.g., atomic level 5d $6s^2$ has a configuration $^2D_{5/2}$

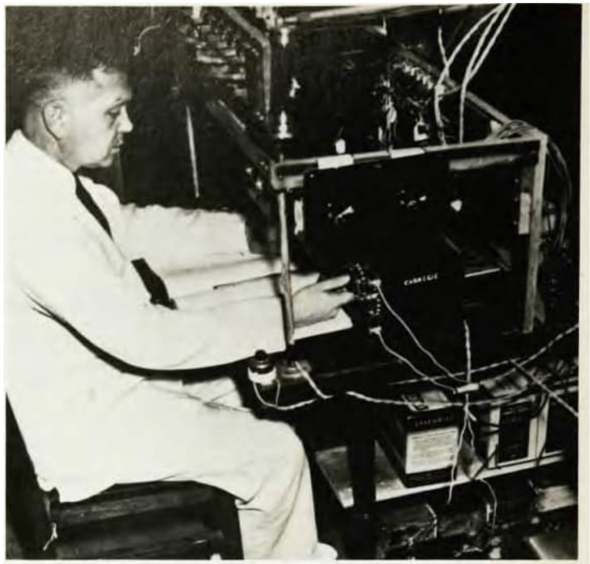
Discovery of deuterium

λ=656.279 nm (N=3 → N=2 in Balmer series)



Hydrogen Deuterium

- Harold Urey (American physical chemist) set out to discover deuterium (``heavy´´ hydrogen) in 1931, using a 6.4 m grating spectrograph which could resolve the Balmer series.
- Note the neutron was discovered in 1932
- Chemistry Nobel Prize in 1934 (``heavy´´ hydrogen)

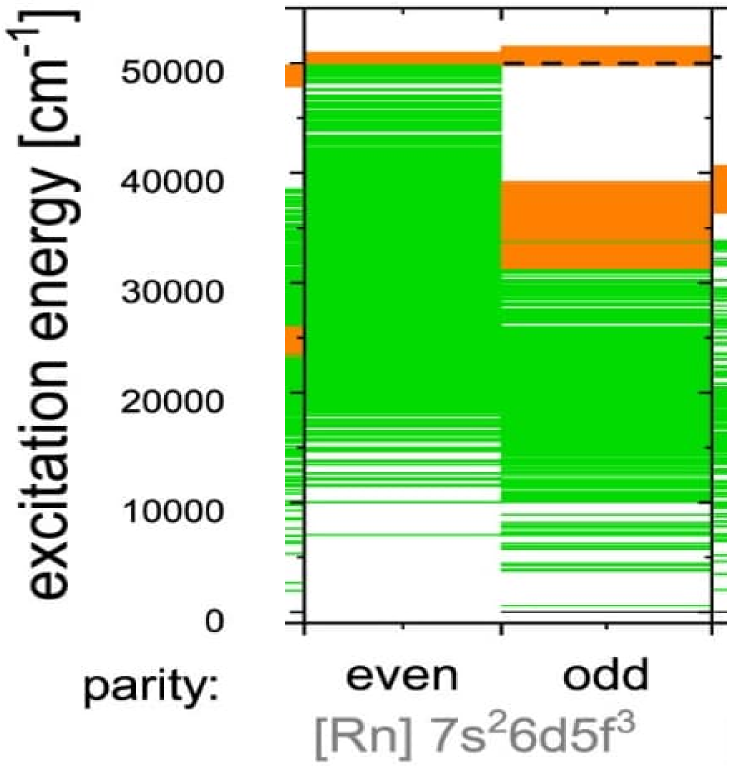


Mass spectrometer with Urey at the controls, after the discovery of deuterium. (Photograph courtesy of King Features Syndicate.)

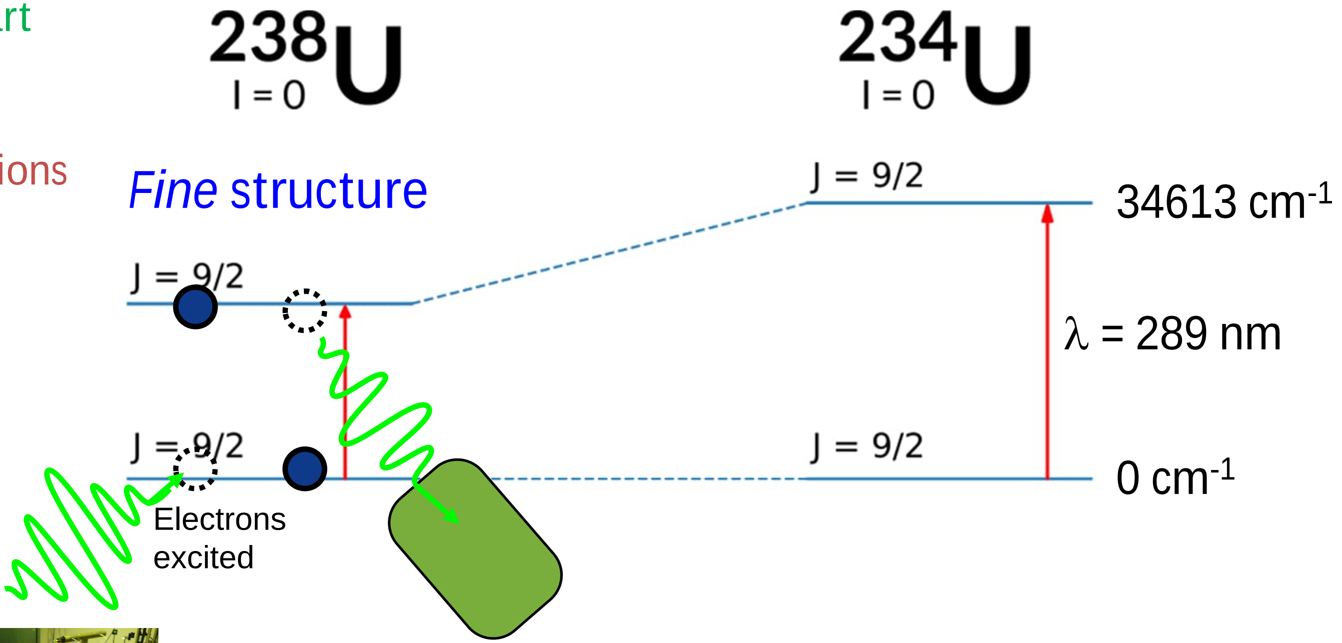
H. Urey et al,. Phys. Rev. 39 (1932) 164

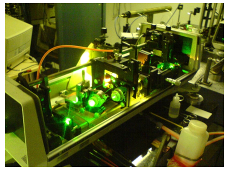
Nuclear properties via the atomic fingerprint

Uranium (Z = 92) atom



- Blaise & Wyart atomic level compilation
- Recent additions

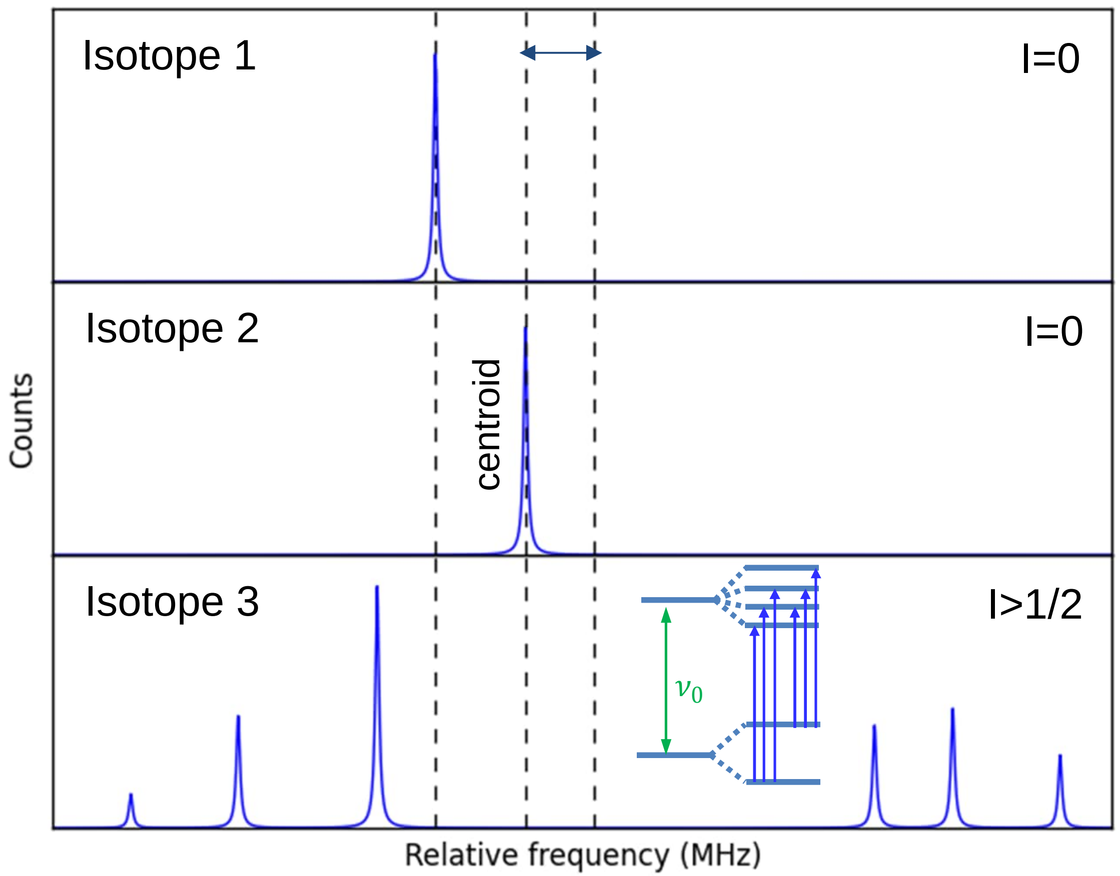




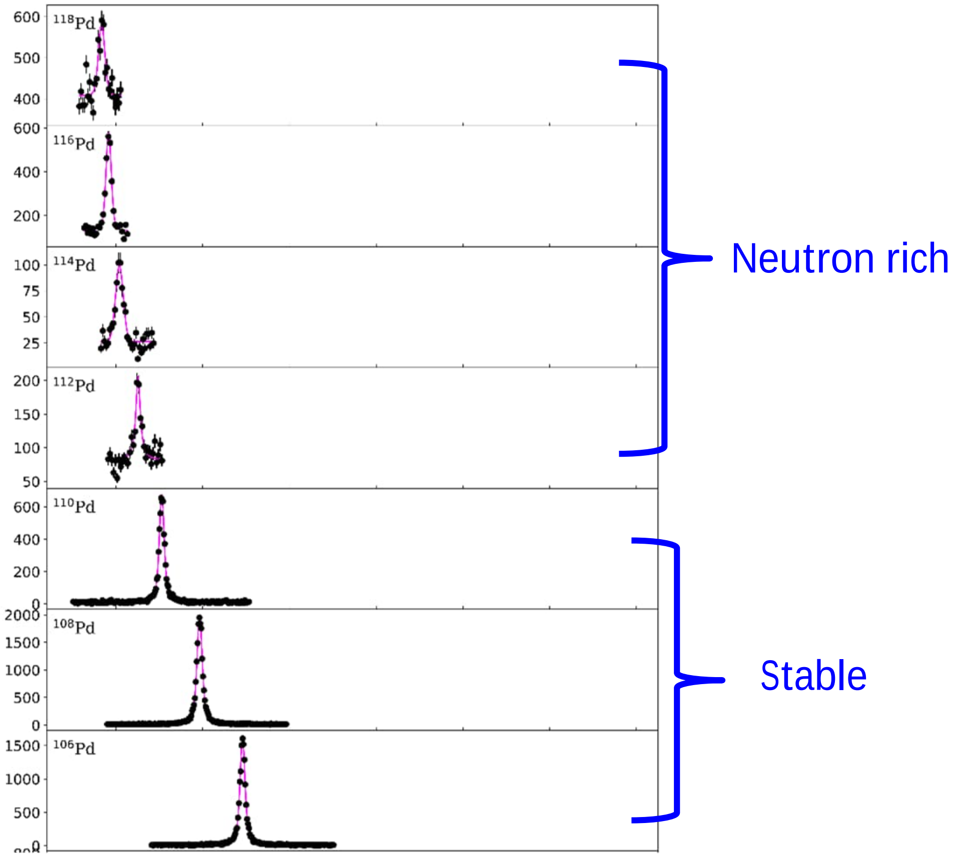
Note: the heaviest of elements, e.g., Lr with Z=103, only theoretical estimates for atomic levels exist!

1 cm^{-1} : ~30000 MHz (30 GHz)
1 eV: ~8000 cm^{-1}

Isotope shifts of electronic transitions



Even mass Pd (Z = 46) isotope shifts



The isotope shift is the frequency difference in an electronic transition between two isotopes of mass *A* and *A´*

$$\delta\nu^{AA'} = \nu^{A'} - \nu^{A}$$

Just to make a brief clarification! ☺

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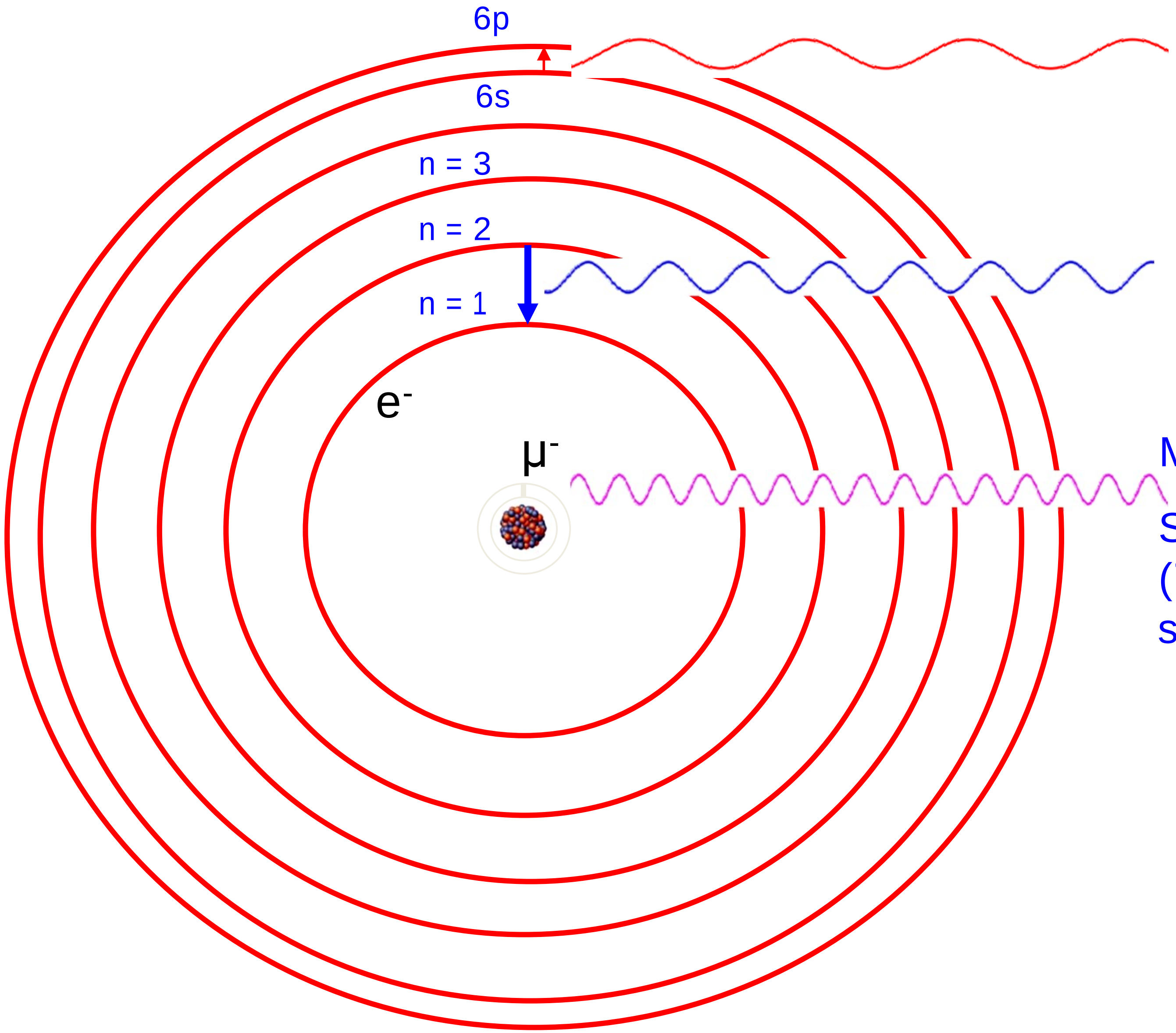


Optical transitions (3 eV)

Shift ~ 10^{-6} eV

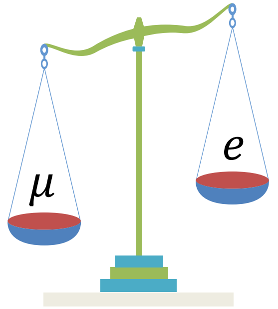
K X-rays (50 keV)

Shift ~ 0.1 eV

Muonic X-rays (1 MeV)

Shift ~ 5,000 eV
(Theory allows absolute size measurement)

$m_\mu \approx 207\, m_e$



What causes the isotope shift?

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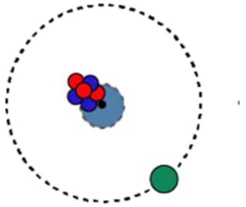
The shift in the atomic transition frequency between different isotopes of the same element arises due to changes in nuclear mass and nuclear size.

$$\delta\nu_{IS} = \delta\nu_{MS} + \delta\nu_{FS}$$

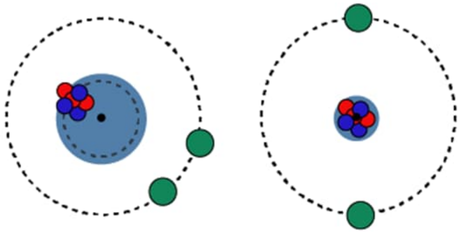
Mass shift

Normal Mass shift



$$= \nu m_e \frac{m_{A'} - m_A}{m_A\, m_{A'}}$$

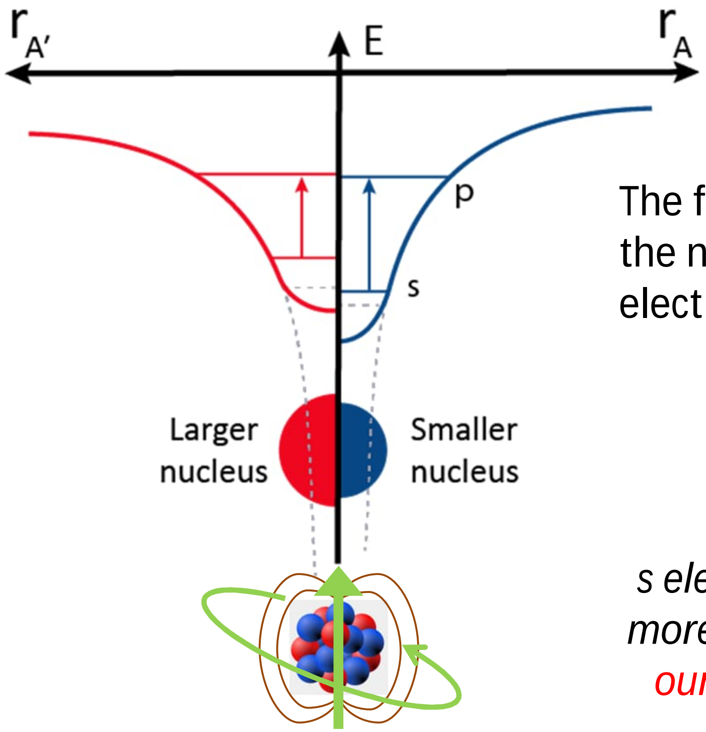
Specific Mass shift



takes into account correlations of the electron motion

Field shift

Nuclear radius: few × 10^{-15} m
Atomic radius: few × 10^{-10} m



The finite spatial extent of the nucleus perturbs the electron wavefunction.

s electrons for example are more tightly bound – guides our choice of transitions!

From the isotope shift to the nuclear size

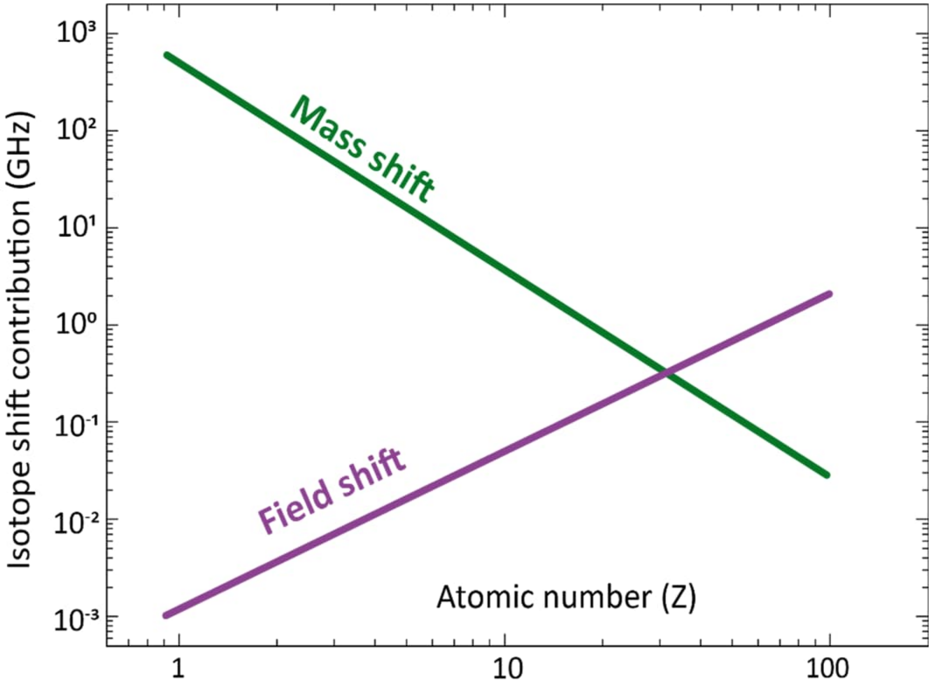
Mass shift — Field shift

$$\delta\nu_i^{A,A'} = \nu_i^{A'} - \nu_i^{A} = K_i \frac{m_{A'} - m_A}{m_A\, m_{A'}} + F_i\, \delta\langle r^2\rangle^{A,A'}$$

EXPERIMENT — THEORY

→ *Nuclear mean-square charge radii*

R



Lower Z requires high resolution techniques

Higher Z lower resolution methods

To evaluate isotope shift data:

- Mass data from Atomic Mass Evaluation (2021)
- Atomic factors (K, F) determined via atomic structure calculations: ab-initio approaches, e.g., multi-configuration Dirac Fock, many-body perturbation theory, coupled cluster…
- Typically accurate to ~10%

B. Cheal et al., Phys. Rev. A 86 (2012) 042501

An empirical method to extract atomic factors

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The King plot approach (note – this does not refer to royalty!)

If independent radius data is available for at least three different isotopes (not always the case) we can combine this information with optical isotope shift data.

Non-optical methods for charge radius (primarily restricted to stable isotopes), e.g.,

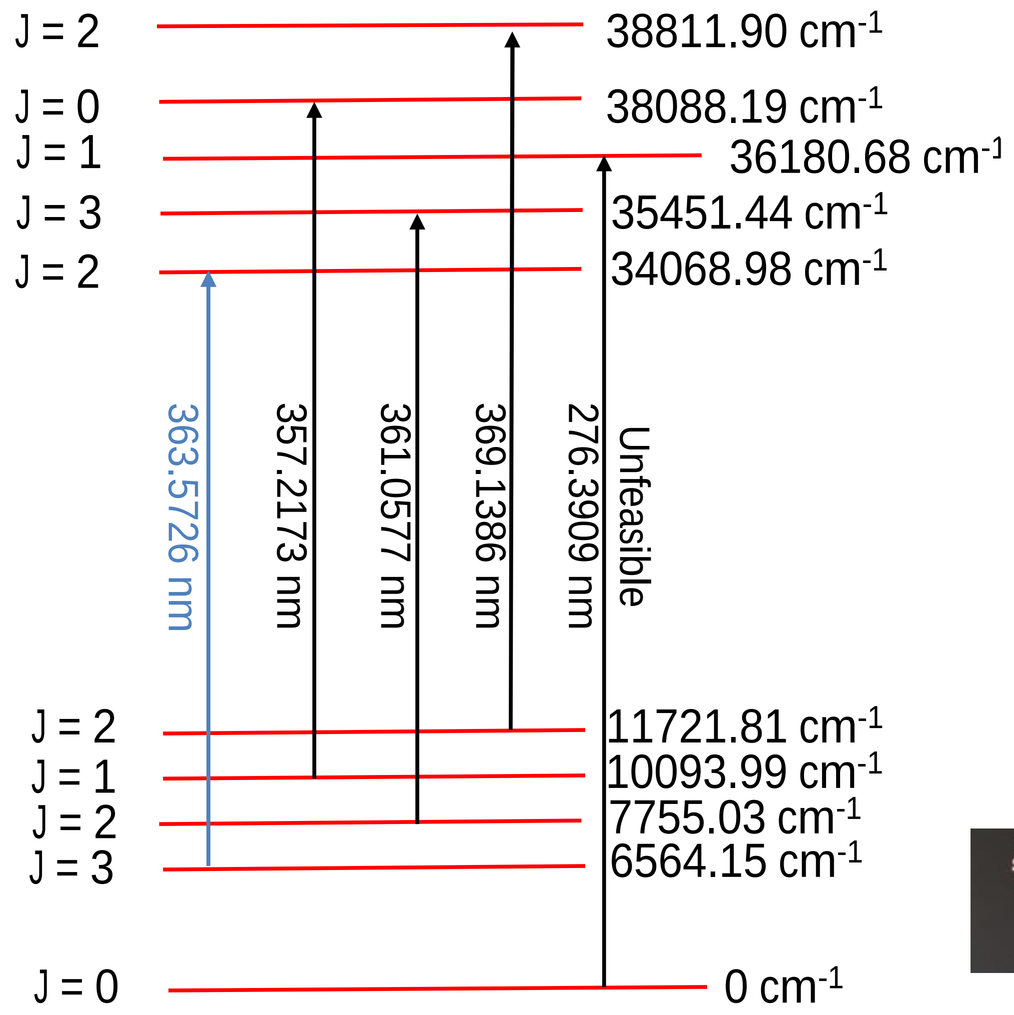
- X-ray scattering of muonic atoms
- Elastic electron scattering

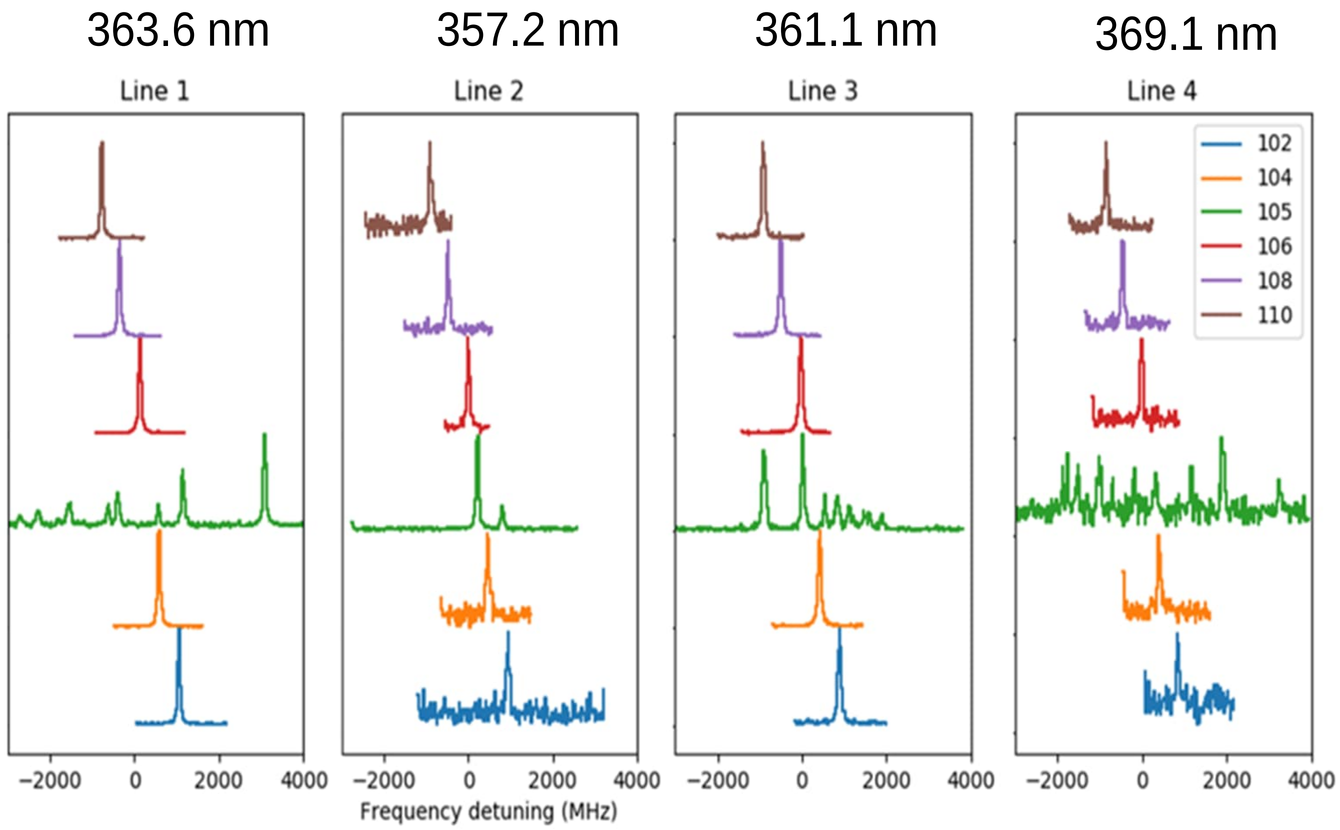
1. Take our beloved isotope shift equation: $$\delta\nu_i^{A,A'} = K_i \frac{m_{A'} - m_A}{m_A\, m_{A'}} + F_i\, \delta\langle r^2\rangle^{A,A'}$$

2. Multiply by a modification factor: $$\frac{m_A m_{A'}}{m_{A'} - m_A}$$

3. Take the modified expression and fit with a straight line: $$\delta\nu_{i,mod}^{A,A'} = K_i + F_i\, \delta\langle r^2\rangle_{mod}^{A,A'}$$

Example of palladium





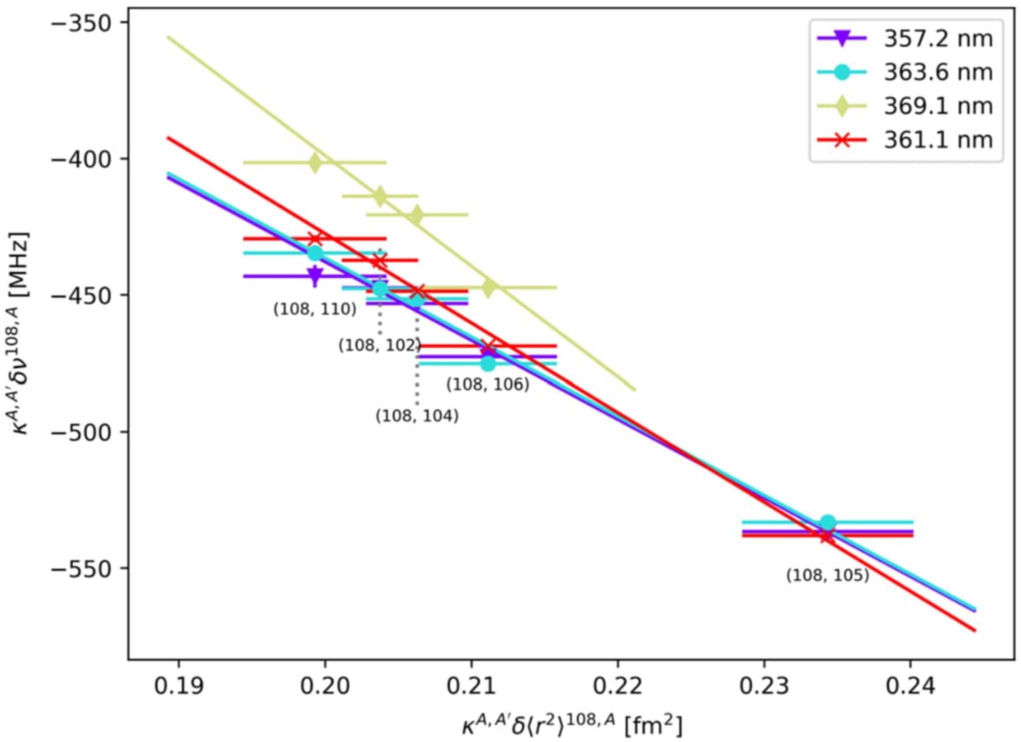
Example of palladium

Transition (nm)	Lower config.	Upper config.	Field shift (GHz/fm^2)	Mass shift (GHz.amu)
			Gradient	Intercept
363.6	$4d^9(^3D_3)5s$	$4d^9(^3P_2)5p$	-2.9(6)	845(669)
357.2	$4d^9(^3D_1)5s$	$4d^9(^3P_0)5p$	-2.9(6)	814(685)
361.1	$4d^9(^3D_2)5s$	$4d^9(^3F_3)5p$	-3.3(6)	1346(773)
369.1	$4d^9(^1D_2)5s$	$4d^9(^3F_2)5p$	-4(2)	2444(2782)

$$\delta\nu^{A,A'}_{i,mod} = K_i + F_i\,\delta\langle r^2\rangle^{A,A'}_{mod}$$

$$\delta\nu^{AA'} = \nu^{A'} - \nu^{A}$$

Modified isotope shifts from optical spectroscopy



Modified $\delta\langle r^2\rangle^{A,A'}$ from muonic X-rays

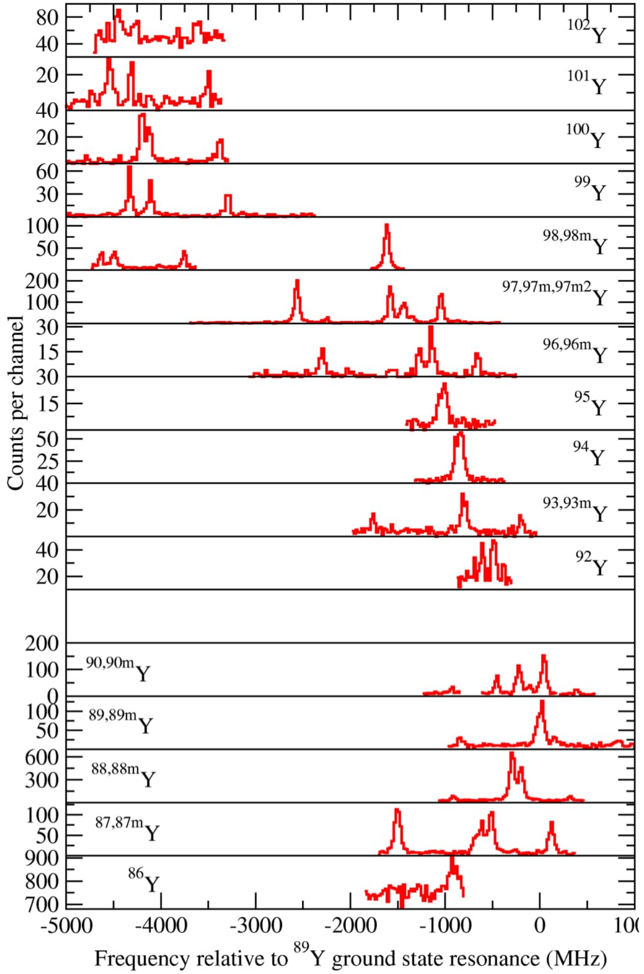
Notice:

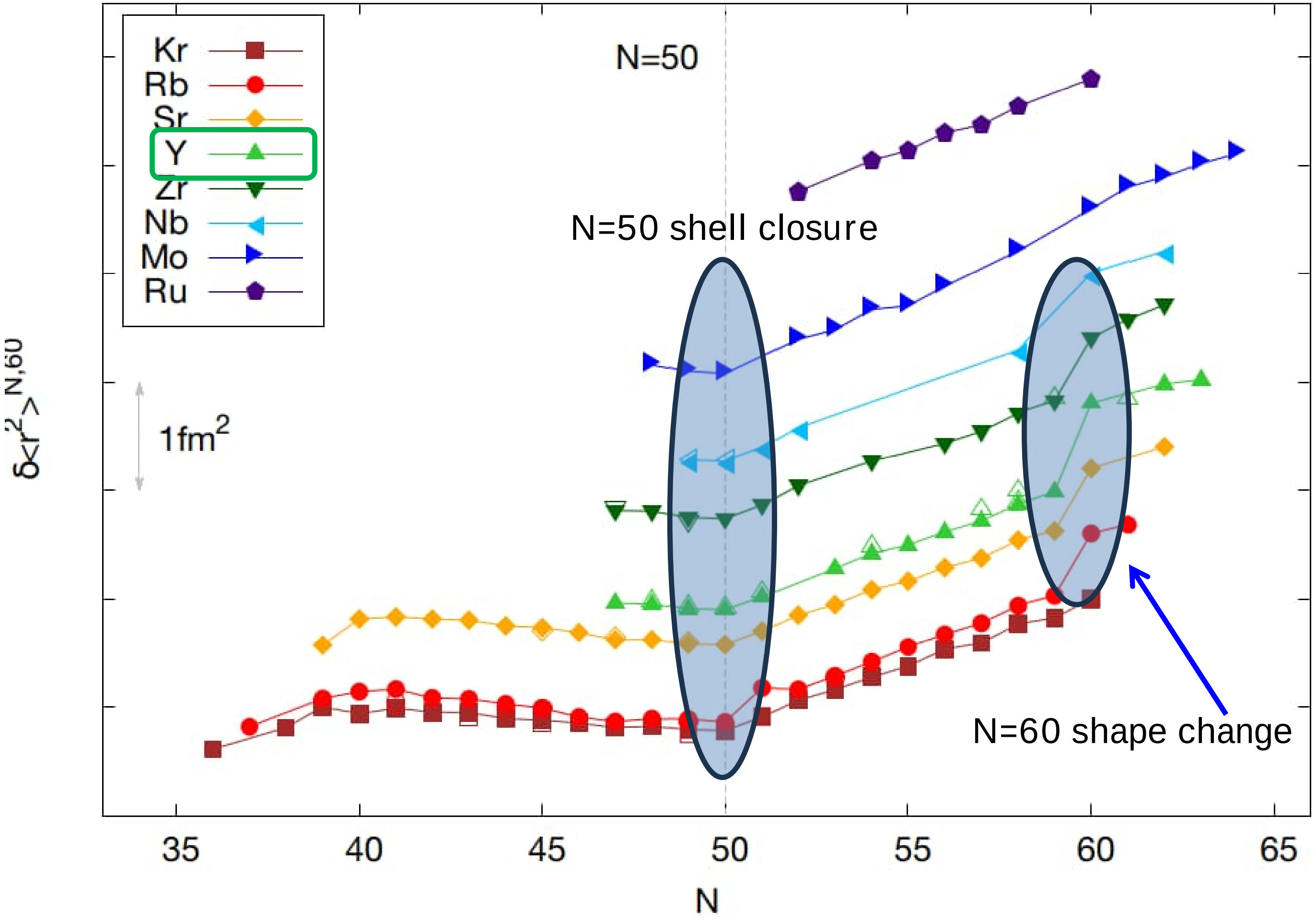
- Similar electronic configurations (atomic factors are similar)
- Transitions involve promotion of s electron into p-orbital
- These deduced factors will enable the extraction of the unknown changes in mean-square charge radii from isotope shifts.

S. Geldhof…IM et al., Hyp. Int. 241 (2020) 41

From the isotope shift to the nuclear size

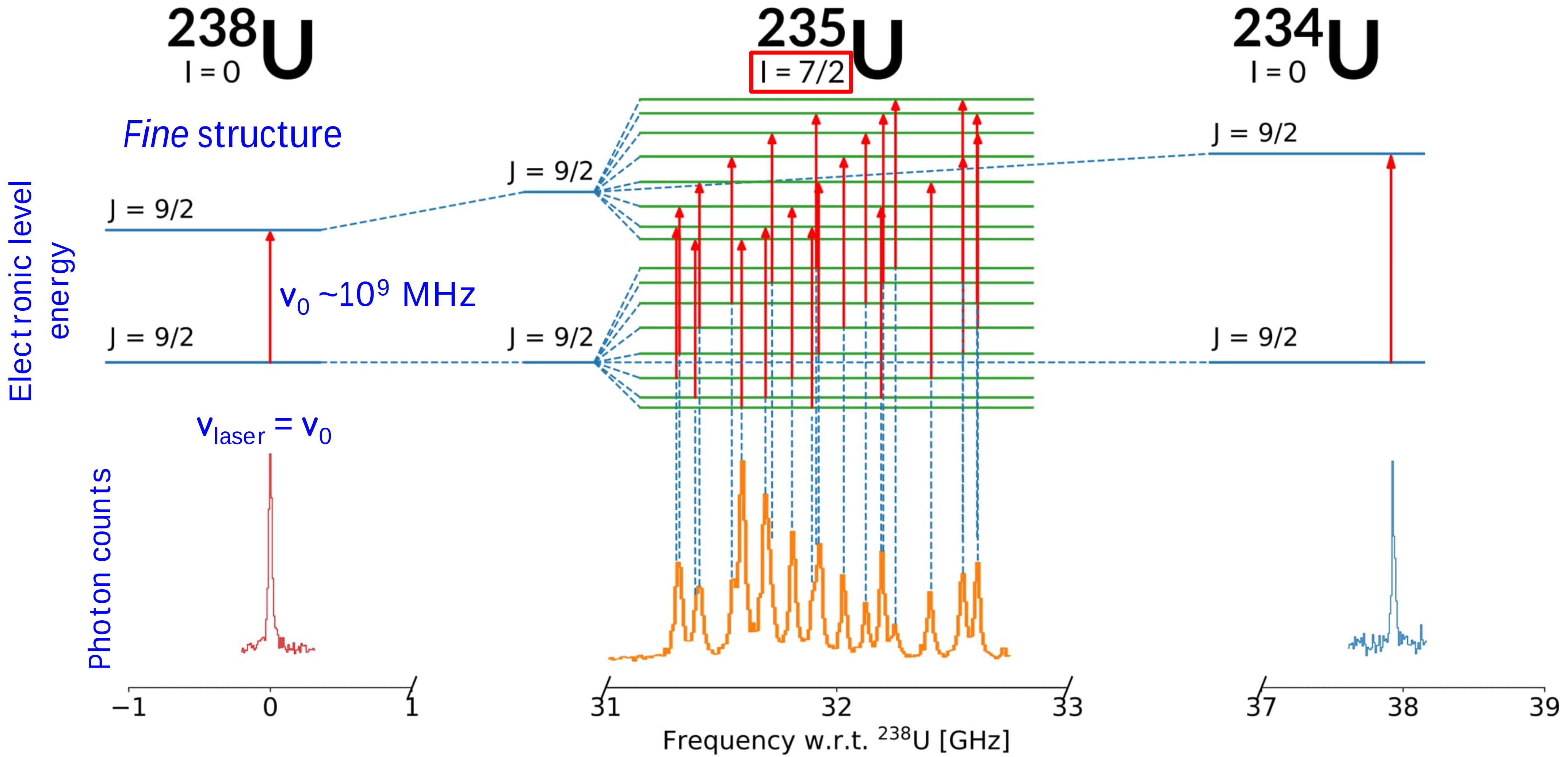
Raw data: optical spectra of Y isotopes





More about this in Lecture 4

Let’s return to the atomic structure of uranium…



Hyperfine interaction (in free atoms)

Hyperfine interaction = the interaction of nuclear magnetic and electric moments with electromagnetic fields (which are produced at the nucleus by the orbiting electrons)

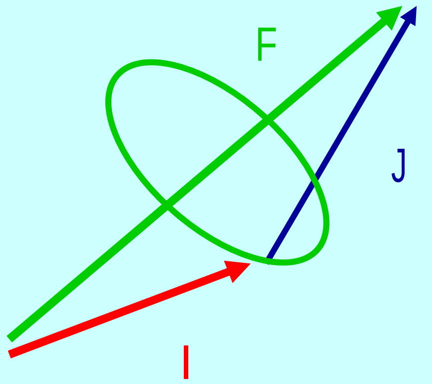
Lets consider the effect on an atomic orbit of spin J (where J = L + S) – our fine structure orbital.

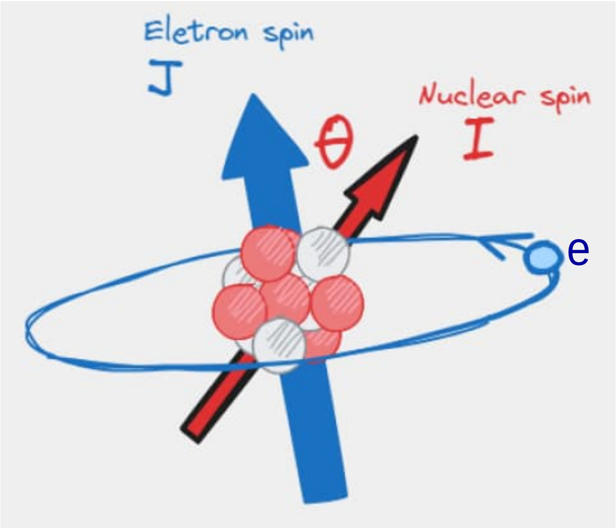
The atomic and nuclear spins couple to form the total angular momentum

F = I + J

Each state J has several F-states:

$$\vec{F} = \vec{I} + \vec{J}$$





$$H_{hf} = \sum_k T_N^k \cdot T_e^k$$

$$F = I + J, I + J - 1, \ldots, |I - J|$$

States of the same I and J but coupled to different angular momenta F have slightly different energies

Nuclear magnetic dipole moment

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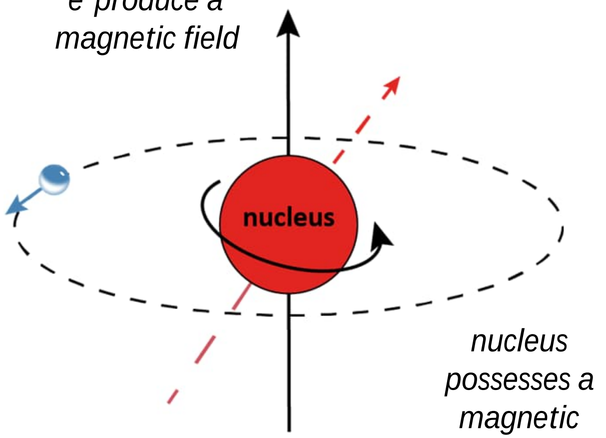
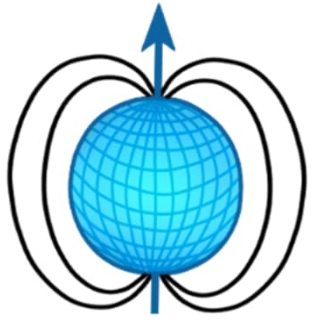
From the definition:

$$\hat{\mu}_I = \sum_i (g_l^i \mathbf{l_i} + g_s^i \mathbf{s_i}) = g\mathbf{I}\mu_N$$

We can see there are contributions from orbiting charge and intrinsic spin





Protons:	$g_l = +1$	$g_s = +5.586$
Neutrons:	$g_l = 0$	$g_s = -3.826$

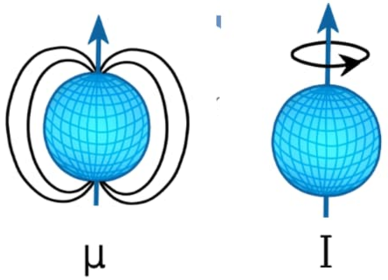
(These are values for a *free* proton/neutron)

The magnetic dipole moment of a state of spin I = expectation value of the z-component of the dipole operator μ:

$$\mu(I) \equiv \langle I, m = I | \hat{\mu}_z | I, m = I \rangle = g I \mu_N$$

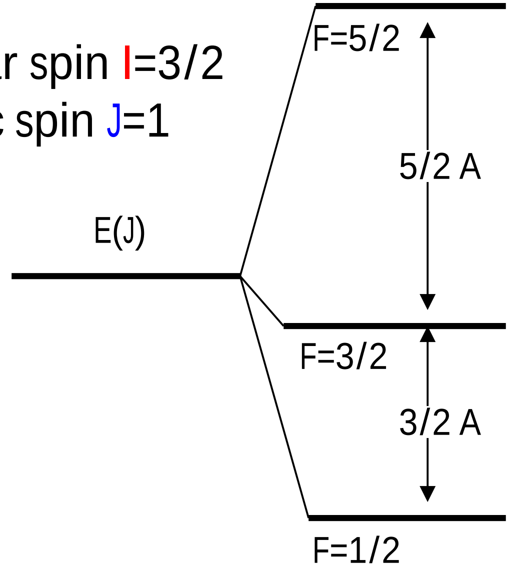
Measuring a magnetic moment is an excellent way to learn more about the wavefunction/configuration of the nucleus.

The magnetic dipole interaction



^{201}Hg

Nuclear spin I=3/2
Atomic spin J=1



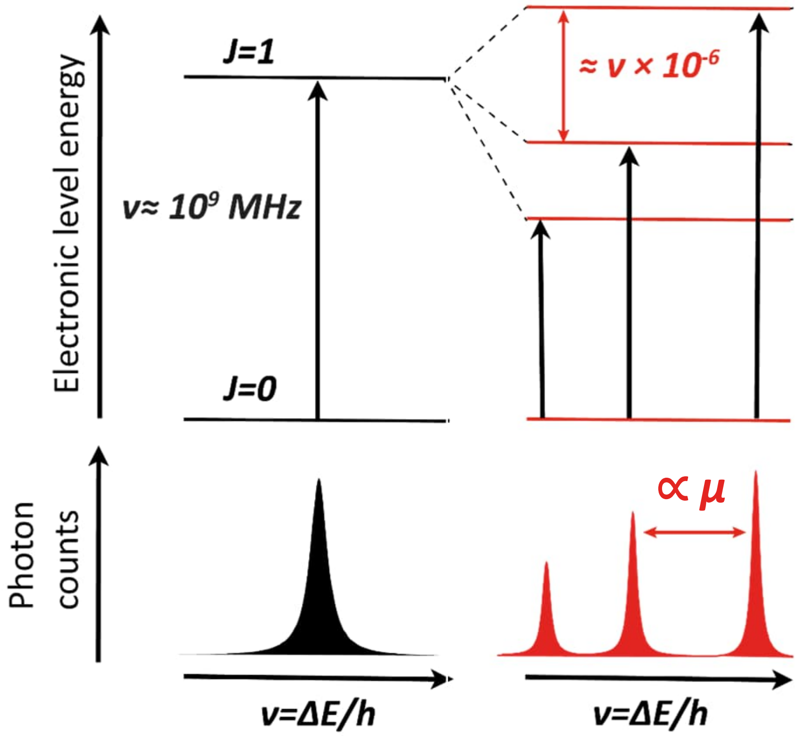
The interaction energy depends on angle θ

$$E = -\boldsymbol{\mu} \cdot \mathbf{B_e} = -\mu B_e \cos\theta$$

$$A = \frac{\mu_I B_e(0)}{I \cdot J},$$

$B_e(0)$ = magnetic field at nucleus

A = magnetic dipole HFS parameter



The original fine structure level E(J) is perturbed so that the final energy due to the magnetic hyperfine effect:

$$E(F) = \frac{A}{2}C$$

where $C = F(F+1) - I(I+1) - J(J+1)$

Access to nuclear spin I (number of hyperfine components) and μ_I

The electric quadrupole moment

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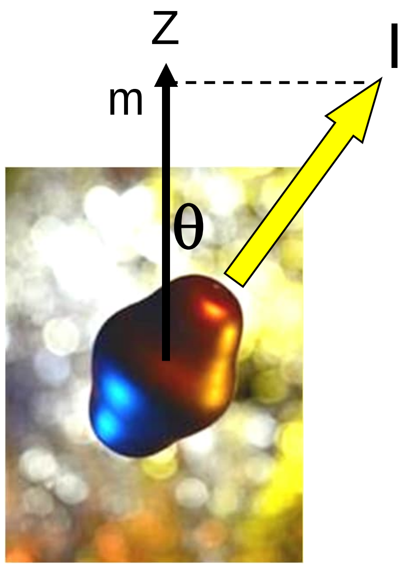
The electric quadrupole moment provides a measure of the deviation of charge distribution from sphericity:



$$eQ = \int_0^\infty \rho_n(\mathbf{r})(3z^2 - r^2)\,\mathrm{d}\tau$$

A spherical nucleus would have zero Q

Experiments measure the maximum "projection" of the intrinsic quadrupole moment along the quantization axis, the ``spectroscopic Q_s´´.

Using angular momentum algebra, we get

$$Q_s = Q_0 \frac{3K^2 - I(I+1)}{(I+1)(2I+3)}$$

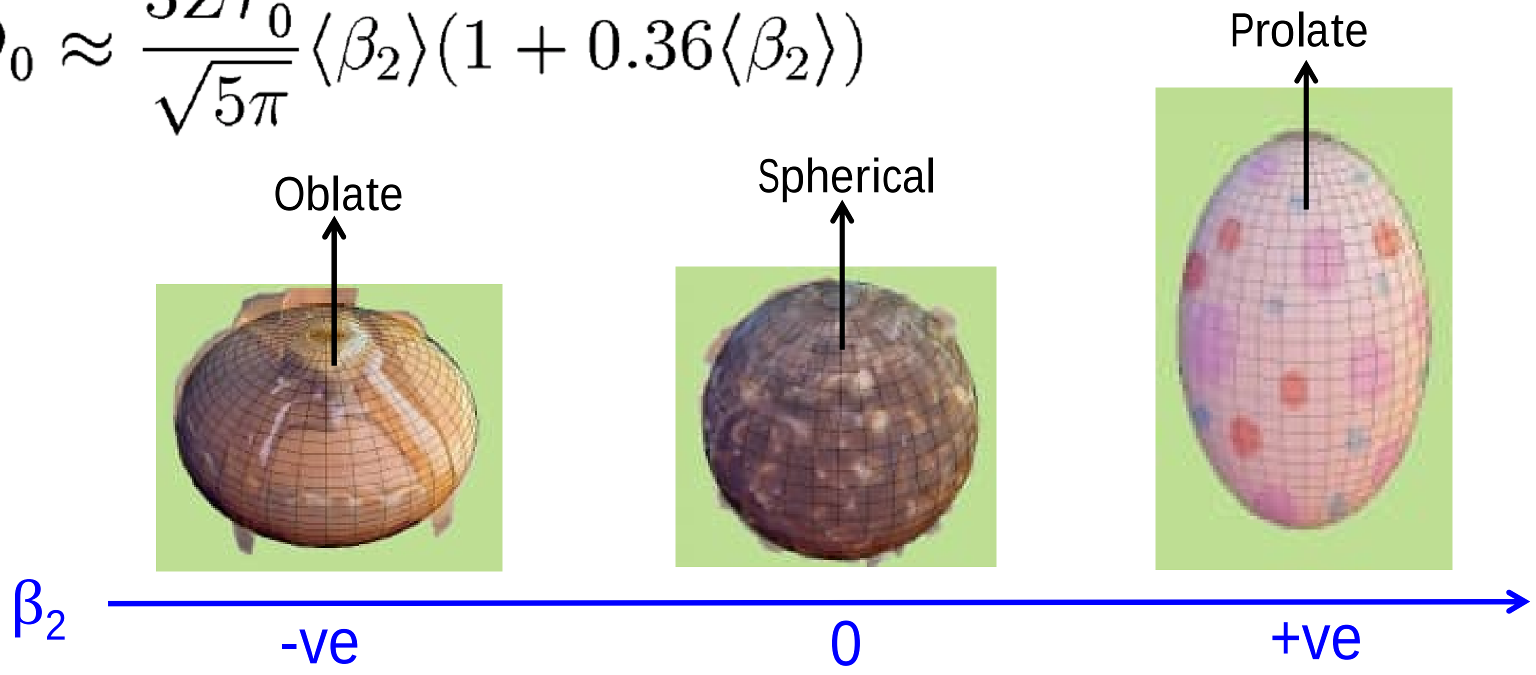
this assumes a well-defined deformation axis (not always a good approximation)

Note for nuclear spin I=0 and I=1/2 the spectroscopic quadrupole moment vanishes even if the intrinsic shape is deformed.

The electric quadrupole moment

If we assume the nucleus adopts a basic shape (ellipsoid), then it is possible to relate the intrinsic Q moment to the quadrupole deformation parameter β_2

$$Q_0 \approx \frac{3Zr_0^2}{\sqrt{5\pi}}\langle\beta_2\rangle(1+0.36\langle\beta_2\rangle)$$



We know however that there is no reason to assume, except in rare cases, that a real nucleus would adopt such a basic shape!

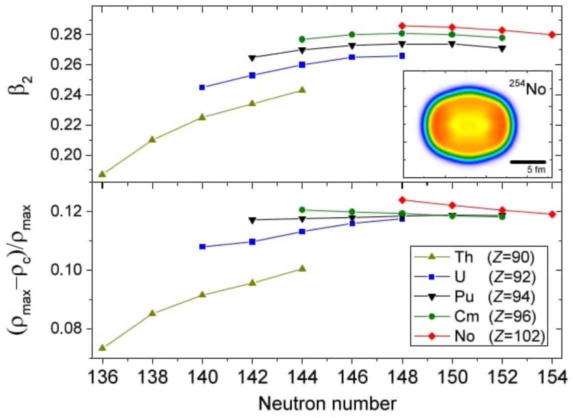
Please don't confuse models with reality!

Also, we hear a lot about nuclear deformation, but I want to stress that the nuclear shape is not direct observable!

How common is (quadrupole) deformation?

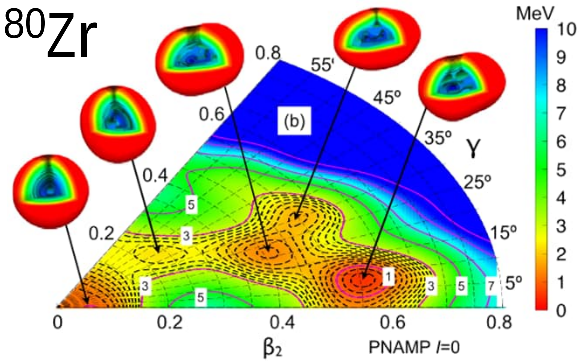
Or, we could rephrase the question: how uncommon are spherical nuclei?

^{254}No

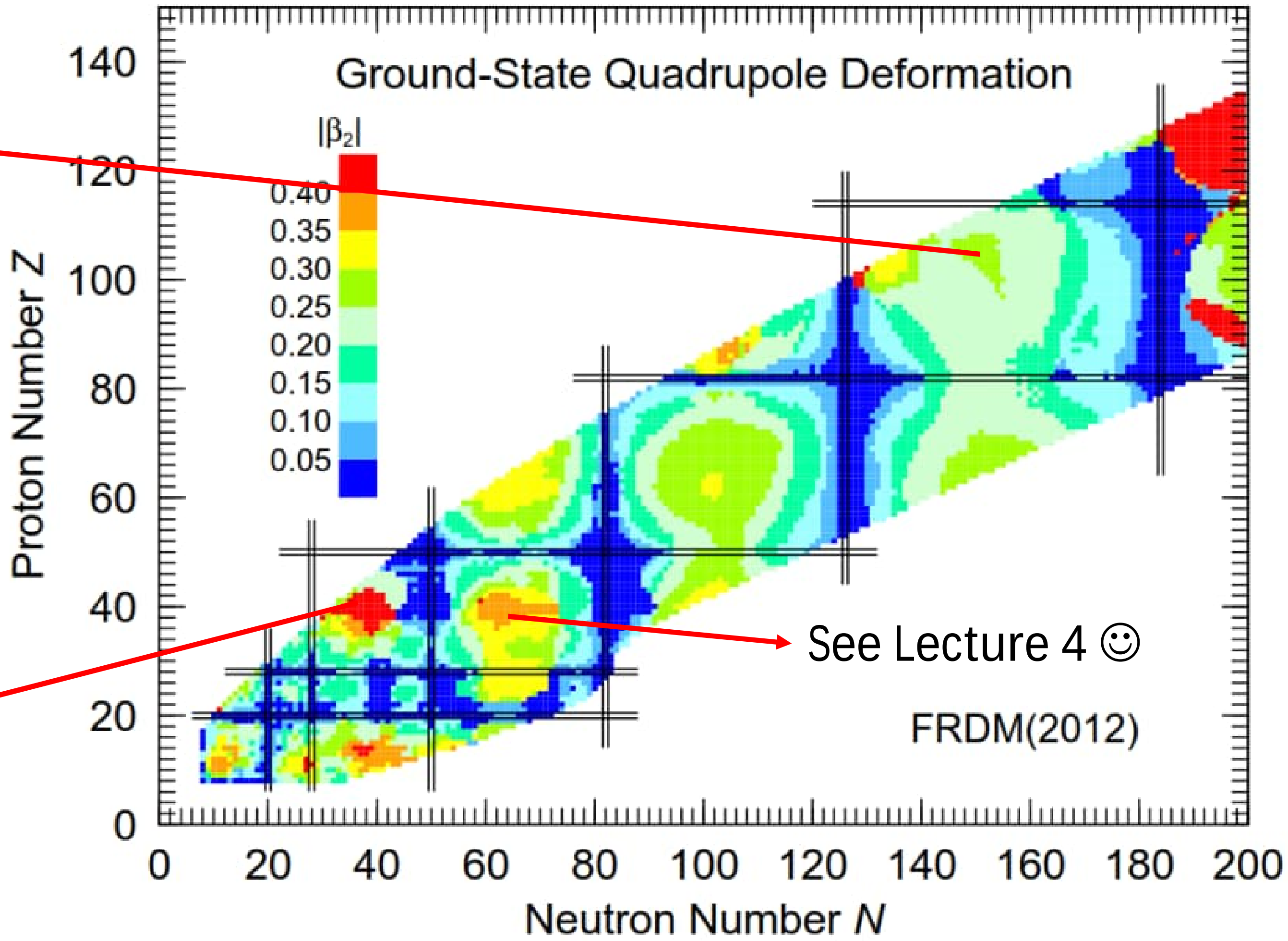


S. Raeder et al., PRL 120 (2018) 232503

^{80}Zr



Rodriguez and Egido, PLB 705 (2011) 255



See Lecture 4 ☺

Discussion pause...

Can you think of any other methods that might be used to probe nuclear deformation?

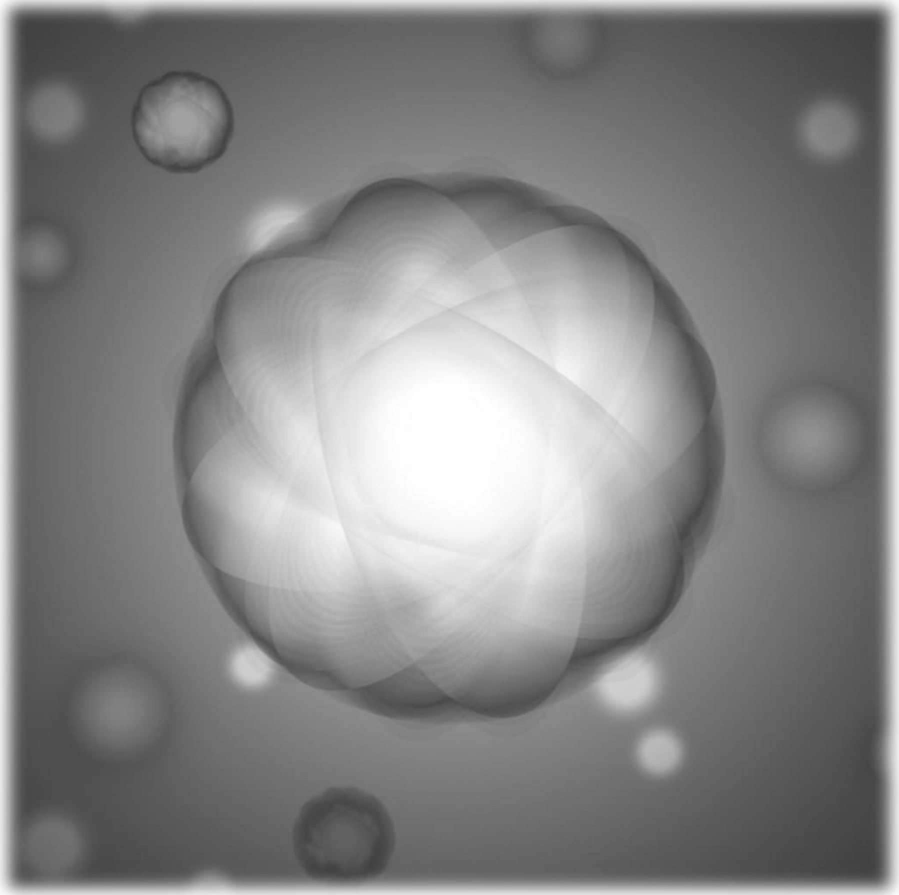


Figure from D. Verney, ``History of the concept of nuclear shape´´, Eur. Phys. J. A 61 (2025) 82

Discussion pause...

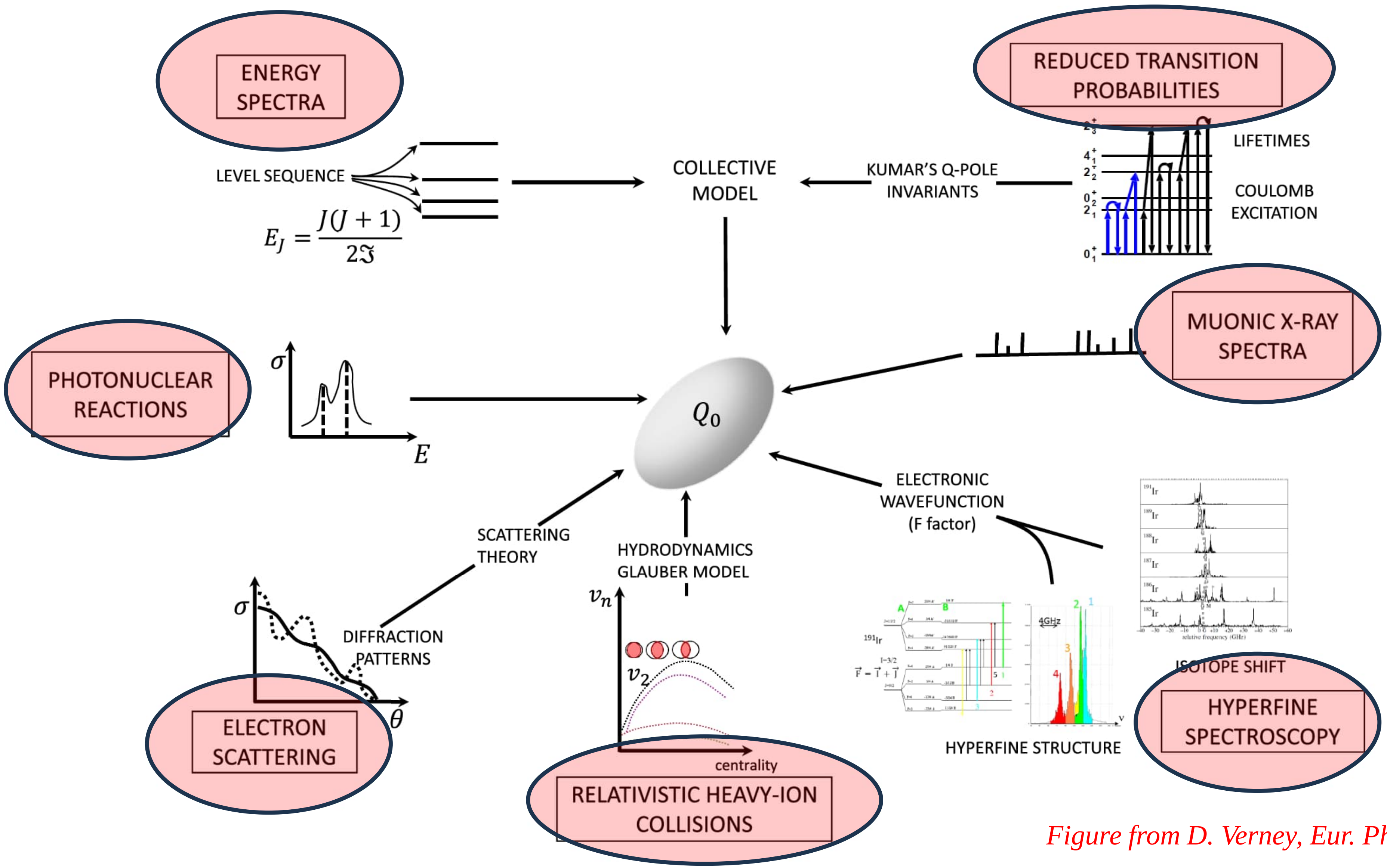
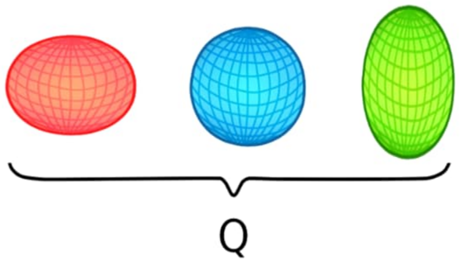


Figure from D. Verney, Eur. Phys. J. A 61 (2025) 82

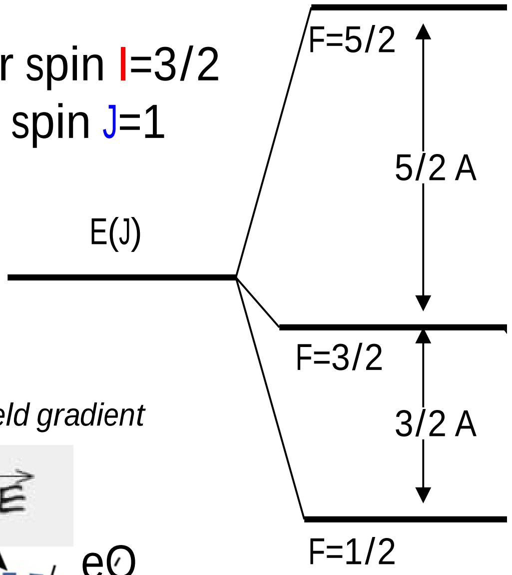
The electric quadrupole interaction



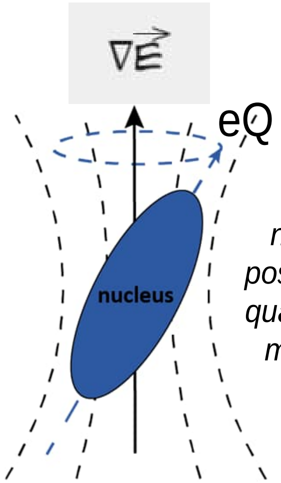
A further perturbation of the atomic level, $\propto B$

^{201}Hg

Nuclear spin I=3/2
Atomic spin J=1



Electric field gradient

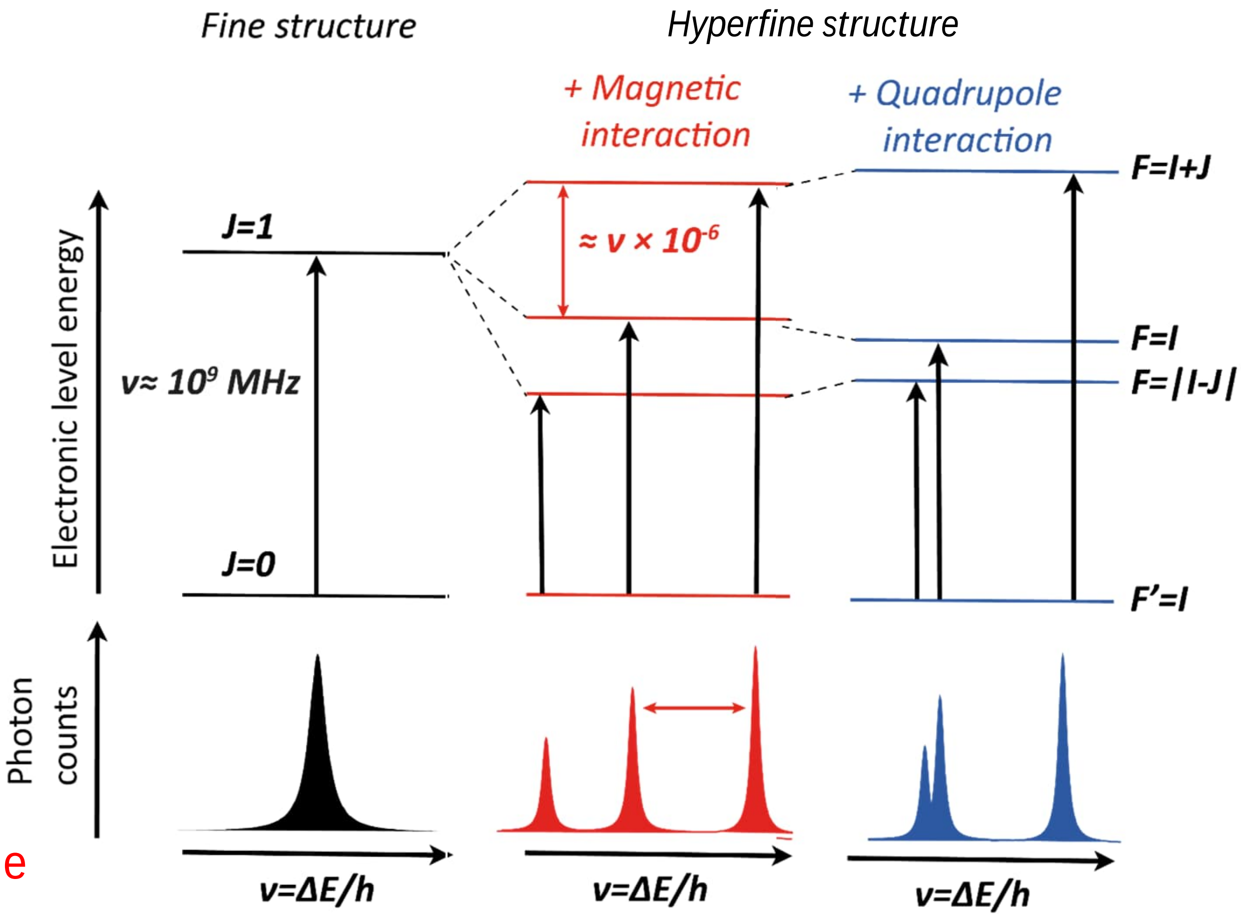


nucleus possesses a quadrupole moment

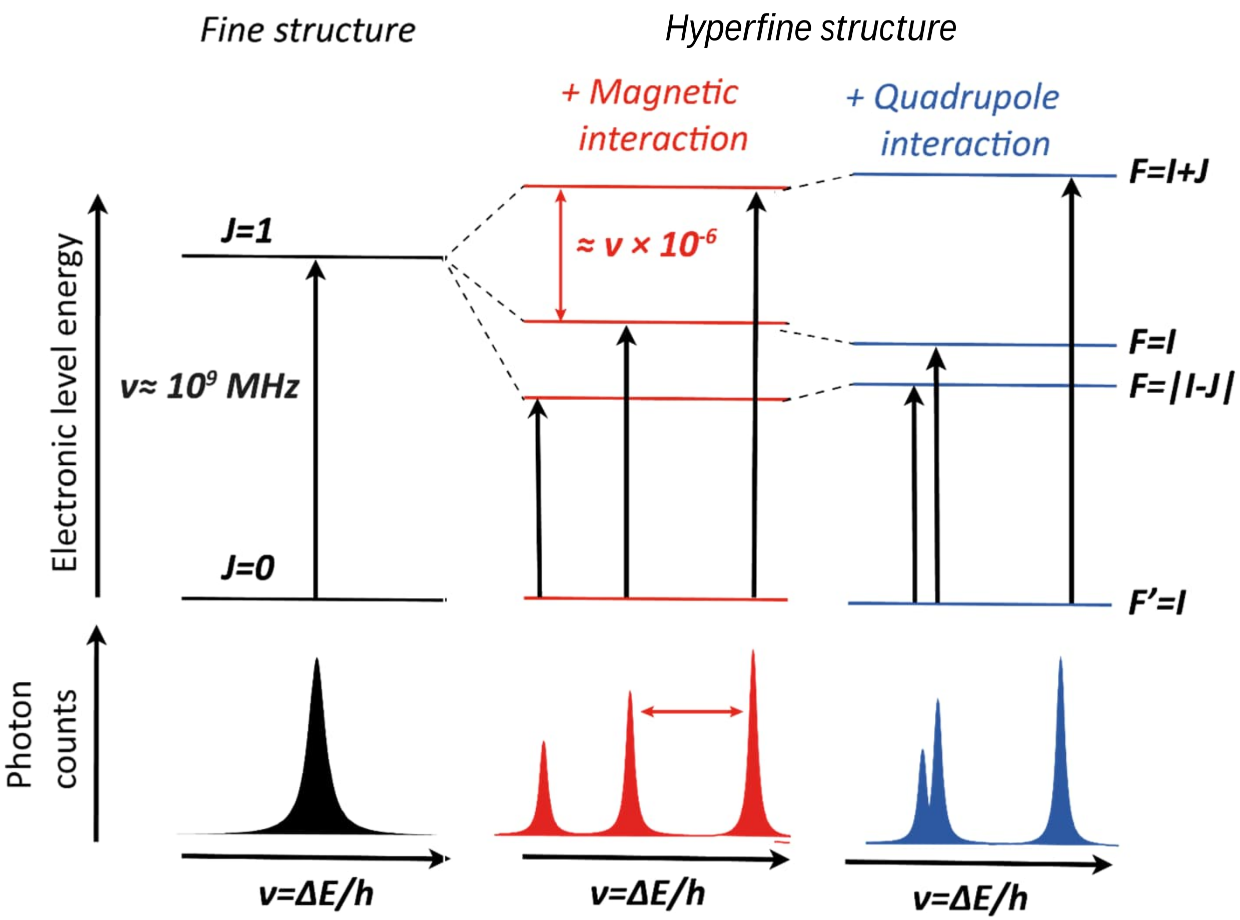
$$B = eQ_s V_{JJ}(0),$$
$$V_{JJ}(0) = \text{electric field gradient at nucleus}$$

Access to Q_s

B = electric quadrupole HFS parameter



Let’s summarize…



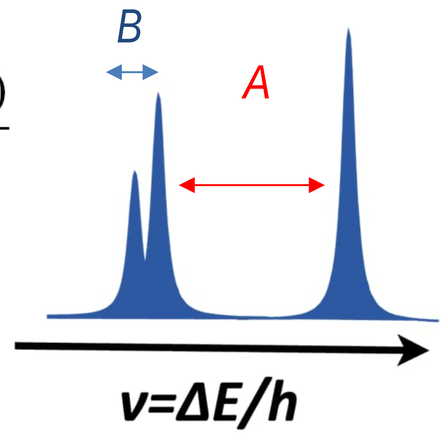
- The hyperfine interaction manifests as a splitting and perturbation of the atomic fine structure lines.
- The energy of the F state is given by:

$$E(F) = \frac{A}{2}C + B\frac{\frac{3}{4}C(C+1) - I(I+1)J(J+1)}{2(2I-1)(2J-1)I \cdot J} +$$

where $C = F(F+1) - I(I+1) - J(J+1)$

- Experimentally we extract the *A* and *B* hyperfine coefficiencts from either the upper or lower fine structure state:

$$A = \frac{\mu_I B_J(0)}{I\,J}$$



$$B = eQ_s V_{JJ}(0)$$

In reality (umm, experiment) what we are measuring

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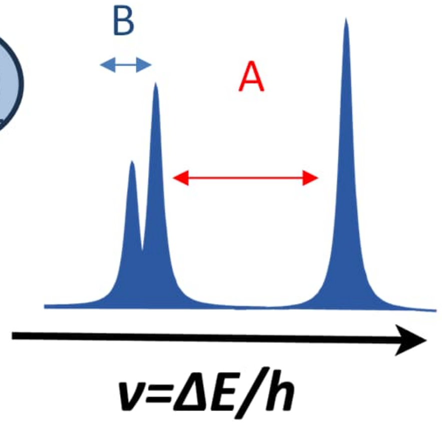
$$A = \frac{\mu_I B_J(0)}{I\,J}$$



$$B = eQ_s V_{JJ}(0)$$

- To extract the hyperfine parameters *A* and *B* from experimental measurements, the magnetic field B_J and electric field gradient V_{jj} need to be extracted.
- This can be done in principle using atomic theory
- A more elegant way is to use reference nuclei with known moments. B_J and V_{jj} depend only on electronic structure.

<u>Nuclear magnetic moment</u>

$$\frac{A_1}{A_{ref}} = \frac{\mu_1}{\mu_{ref}} \frac{I_{ref}}{I_1}$$

Reference nuclei (stable isotopes) with accurately known moments

$$\mu_1 = \frac{I_1}{I_{ref}} \frac{A_1}{A_{ref}} \mu_{ref}$$

A_1, A_{ref} from fitting our optical spectra
μ_{ref} from e.g., NMR.

<u>Nuclear quadrupole moment</u>

$$B = eQV_{JJ}(0)$$

$$Q_s = Q_{s,ref} \frac{B}{B_{ref}}$$

B, B_{ref} from fit
$Q_{s,ref}$ from e.g., NQR.

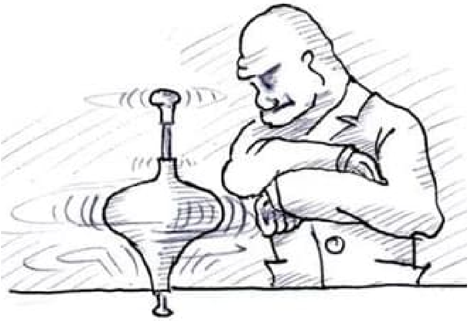
Take home messages from Lecture 1

Mass

Size

Shape

Electromagnetic property



Spin

- The nuclear mass and binding energy contain a wealth of nuclear structure information.

- The isotope shift is a gateway to changes in the nuclear size (charge radii).

  - close connection with shell structure, deformation, neutron-proton pairing… (Lecture 4)

- A measurement of hyperfine structure gives access to magnetic dipole and electric quadrupole moments (single particle and collective properties)

  - These observables are very sensitive probes of the configuration of the nuclear wave function (Lecture 4)