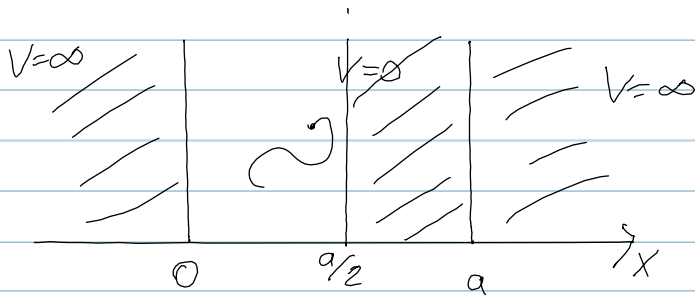


$$p_{x,1}^2 = \frac{\hbar^2 \pi^2}{a^2} = 2mE_1 \quad E_{\text{kin}} = \frac{p^2}{2m}$$

$$p_{x,1} = \pm \sqrt{2mE_1}$$

Genauigkeit: $2\sqrt{2mE_1}$

Vorlesung 22.11.2013



$$E_n = \frac{n^2 \hbar^2}{8ma^2} \quad n=1,2,3 \quad E_1 = \frac{\hbar^2}{8ma^2}$$

Aufenthaltswahrscheinlichkeit ψ_1^2

Impuls $p_x = \pm \sqrt{2mE_1}$

$$p_x = \pm \sqrt{2m \frac{\hbar^2}{8ma^2}} = \pm \sqrt{\frac{\hbar^2}{2a^2}}$$

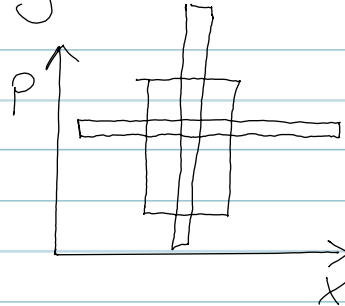
Unsicherheit $2 \frac{\hbar}{2a} = \frac{\hbar}{a}$

$\frac{a}{2}$ " $2 \frac{\hbar}{2(\frac{a}{2})} = 2 \frac{\hbar}{a}$

$\left| \Delta x \cdot \Delta p \geq \frac{\hbar}{2} \right|$ Heisenberg'sche Unschärferelation

$$\Delta x \equiv \sqrt{\langle x^2 \rangle - \langle x \rangle^2}$$

$$\Delta p \equiv \sqrt{\langle p^2 \rangle - \langle p \rangle^2}$$

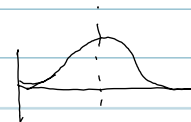


erfüllt für Teilchen im Kasten? ψ_1

$$\begin{aligned} \langle x \rangle &= \int_0^a \sqrt{\frac{2}{a}} \sin \frac{\pi x}{a} x \sqrt{\frac{2}{a}} \sin \frac{\pi x}{a} dx \\ &= \frac{2}{a} \int_0^a x \cdot \sin^2 \frac{\pi x}{a} dx \end{aligned}$$

Subst. : $\varphi = \frac{\pi x}{a} \rightarrow \frac{d\varphi}{dx} = \frac{\pi}{a}$

$$= \frac{2}{a} \frac{a}{\pi} \frac{a}{\pi} \int_0^{\pi} \underbrace{\varphi \sin^2 \varphi}_{\frac{\pi^2}{4}} d\varphi = \frac{a}{2}$$



$$\begin{aligned} \langle x^2 \rangle &= \int_0^a \sqrt{\frac{2}{a}} \sin \frac{\pi x}{a} x^2 \sqrt{\frac{2}{a}} \sin \left(\frac{\pi x}{a} \right) dx \\ &= \frac{2}{a} \int_0^a x^2 \sin^2 \frac{\pi x}{a} dx = \frac{2}{a} \frac{a^3}{\pi^3} \int_0^{\pi} \varphi^2 \sin^2 \varphi d\varphi = \frac{a^2}{3} - \frac{a^2}{2\pi^2} = \langle x^2 \rangle \end{aligned}$$

$\langle p \rangle = 0$ s.o.

$\langle p^2 \rangle = \frac{\hbar^2 \pi^2}{a^2}$ s.o.

$$\begin{aligned} \Delta x \cdot \Delta p &= \left(\frac{a^2}{3} - \frac{a^2}{2\pi^2} - \frac{a^2}{4} \right)^{1/2} \left(\frac{\hbar^2 \pi^2}{a^2} - 0 \right)^{1/2} = \left(\frac{a^2}{12} - \frac{a^2}{2\pi^2} \right)^{1/2} \frac{\hbar \pi}{a} \\ &= \hbar \pi \left(\frac{1}{12} - \frac{1}{2\pi^2} \right)^{1/2} = \hbar \left(\frac{\pi^2}{12} - \frac{1}{2} \right)^{1/2} = \hbar \underbrace{\left(\frac{\pi^2 - 6}{12} \right)^{1/2}}_{> \frac{1}{2}} \geq \frac{\hbar}{2} \end{aligned}$$

c) Teilchen im 3-dim. Kasten

Anwendung: Translationszustandssumme
→ PC III

Eigenwertgl. $\left[-\frac{\hbar^2}{2m} \nabla^2 \psi = E \psi \right]$

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} = -\frac{2mE}{\hbar^2} \psi$$

Lösungsansatz: $\psi = X(x) Y(y) Z(z) = XYZ$

$$YZ \frac{\partial^2 X}{\partial x^2} + XZ \frac{\partial^2 Y}{\partial y^2} + XY \frac{\partial^2 Z}{\partial z^2} = -\frac{2mE}{\hbar^2} XYZ \quad | : XYZ$$

$$\frac{1}{X} \frac{\partial^2 X}{\partial x^2} + \frac{1}{Y} \frac{\partial^2 Y}{\partial y^2} + \frac{1}{Z} \frac{\partial^2 Z}{\partial z^2} = -\frac{2mE}{\hbar^2}$$

$$\underbrace{\frac{1}{X} \frac{\partial^2 X}{\partial x^2} + \frac{1}{Y} \frac{\partial^2 Y}{\partial y^2} + \frac{2mE}{\hbar^2}}_< = \underbrace{-\frac{1}{Z} \frac{\partial^2 Z}{\partial z^2}}_<$$

$$\left[-\frac{1}{Z} \frac{\partial^2 Z}{\partial z^2} = \frac{2mE_z}{\hbar^2} \right] = <$$

$$\frac{1}{X} \frac{\partial^2 X}{\partial x^2} + \frac{1}{Y} \frac{\partial^2 Y}{\partial y^2} + \frac{2m(E-E_z)}{\hbar^2} = 0$$

$$\frac{1}{X} \frac{\partial^2 X}{\partial x^2} + \frac{2m(E-E_z)}{\hbar^2} = -\frac{1}{Y} \frac{\partial^2 Y}{\partial y^2}$$

$$\left[-\frac{1}{Y} \frac{\partial^2 Y}{\partial y^2} = \frac{2mE_y}{\hbar^2} \right] \rightarrow \left[-\frac{1}{X} \frac{\partial^2 X}{\partial x^2} = \frac{2mE_x}{\hbar^2} \right]$$

Produktansatz von WF

$$\psi = X Y Z$$

$$\psi = \sqrt{\frac{8}{abc}} \sin \frac{n_x \pi}{a} x \sin \frac{n_y \pi}{b} y \sin \frac{n_z \pi}{c} z \quad n_x, n_y, n_z = 1, 2, 3, \dots$$

Energieeigenwerte

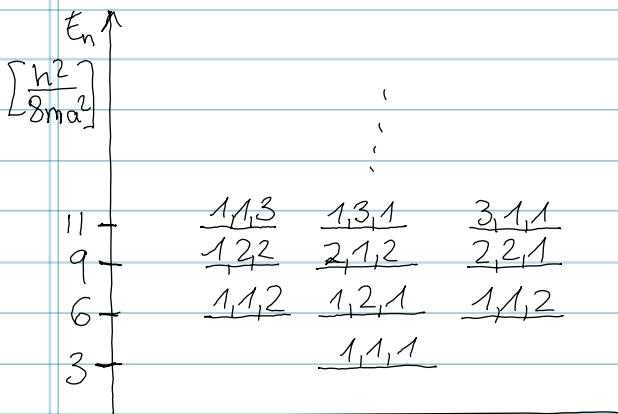
$$E = E_x + E_y + E_z$$

$$\hookrightarrow E = \frac{h^2}{8m^2} \left(\frac{n_x^2}{a^2} + \frac{n_y^2}{b^2} + \frac{n_z^2}{c^2} \right)$$

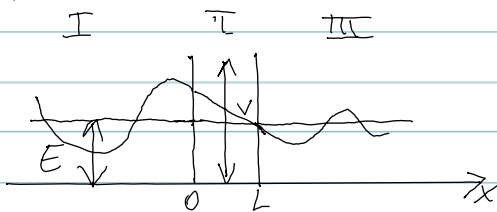
Würfel: $a=b=c$

$$E = \frac{h^2}{8ma^2} (n_x^2 + n_y^2 + n_z^2)$$

$$\text{Entartung: } E(1,1,2) = E(1,2,1) = E(2,1,1) = \frac{6h^2}{8ma^2}$$



d) Tunneleffekt



Pot. ist endlich } tunneln
 } möglich
 $E < V$

Bereich I: $V=0$

$$\frac{d^2\psi}{dx^2} = -\frac{2mE}{\hbar^2}\psi$$

$$\psi = (e^{ikx} + e^{-ikx})$$

$$+k^2 = \frac{2mE}{\hbar^2}$$

$$k = \sqrt{\frac{2mE}{\hbar^2}}$$

$$k = \sqrt{\frac{2m\hbar^2}{\hbar^2 8ma^2}}$$

$$k = \sqrt{\frac{2m\hbar^2}{2m\hbar^2}}$$

$$\rightarrow k = \frac{p}{\hbar}$$

Bereich II: $E < V$

$$\frac{d^2\psi}{dx^2} = \frac{2m(V-E)}{\hbar^2}\psi$$

$$\psi = Ee^{kx} + Fe^{-kx}$$

$$k = \sqrt{\frac{2m(V-E)}{\hbar^2}}$$

Bereich III : $V=0$

$$\psi = C' e^{ikx} + D' e^{-ikx}$$

→ Randbed. → → Koeff. (Annahme : $V \gg E$)
 $k \cdot a \gg 1$

Ziel: Transmissionswahrscheinlichkeit T
Tunnel - " -

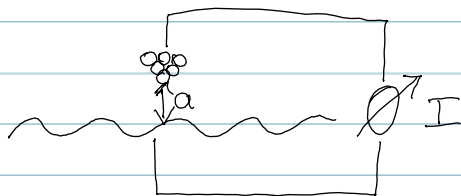
$$T = \frac{|C|^2}{|I|^2}$$

$$T = \frac{16E(V-E)}{V^2} e^{-2\kappa a} \quad \kappa = \sqrt{\frac{2m(V-E)}{\hbar^2}}$$

$T \uparrow$ $m \downarrow$ $(V-E) \downarrow$

2.5 Anwendungen

Rastertunnelmikroskopie



Tunnelstrom $I \propto T$

$$\text{Bsp: } \left. \begin{array}{l} a_1 = 0,5 \text{ nm} \\ a_2 = 0,6 \text{ nm} \end{array} \right\} \frac{I(a_2)}{I(a_1)} = \frac{T(a_2)}{T(a_1)} = e^{-2\kappa(a_2-a_1)} = 0,23$$