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Kontigleichung

$$\frac{Dm}{Dt} = 0$$

Integralform

$$\hookrightarrow \quad \underbrace{\frac{\partial}{\partial t}\int_V \rho \, dV}_{\text{lokale Änd}} + \overbrace{\int_S \rho \vec{u}\cdot\vec{n} \, dS}^{\text{Flußdichte}} = 0$$

V = Kontrollvolumen

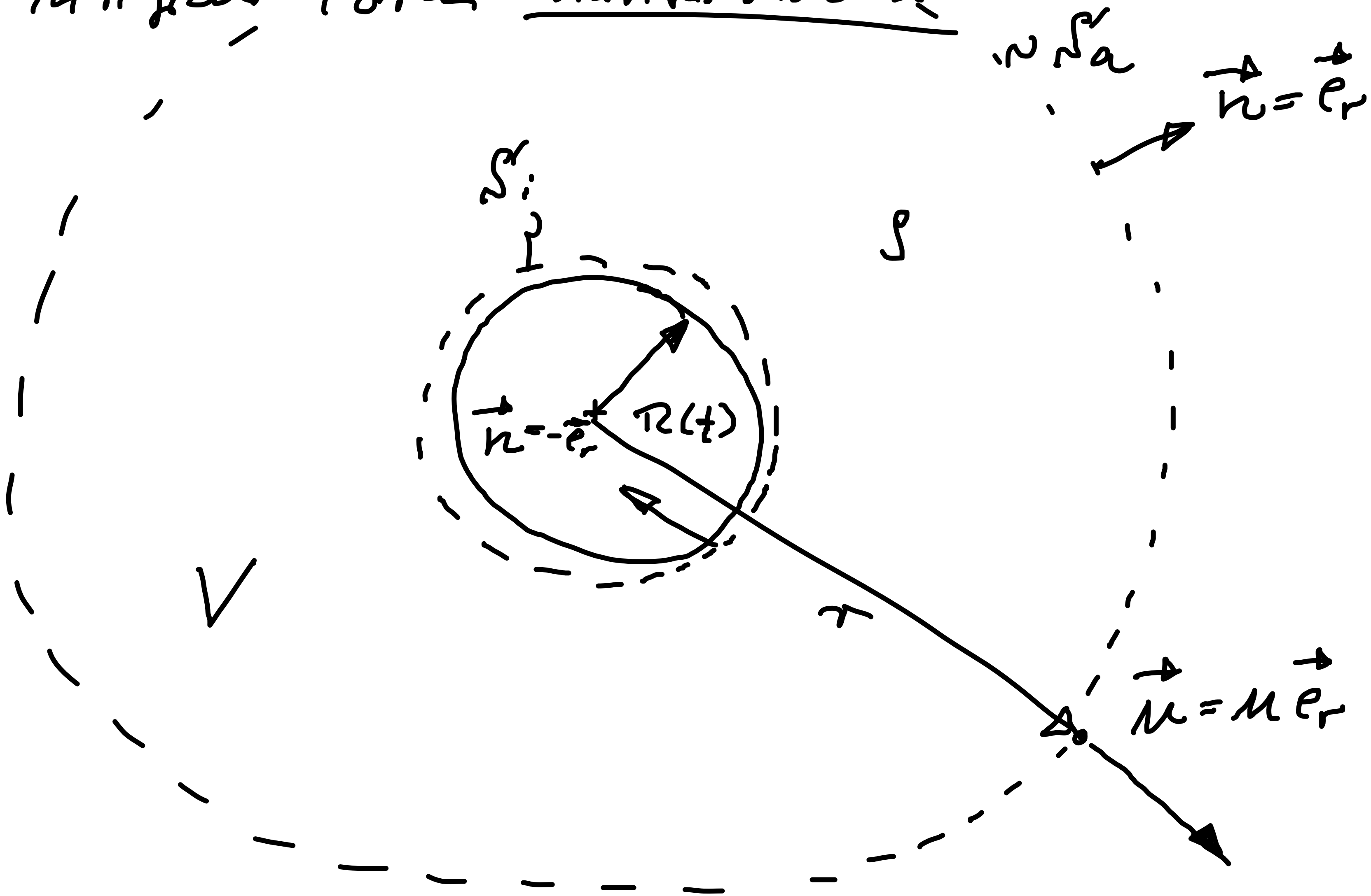
S = geschlossene Oberfläche des Kontrollvolumens

Differentielle Form

$$\frac{D\rho}{Dt} + \rho \, \mathrm{div}\, \vec{u} = 0$$

$$\frac{D\rho}{Dt} = \frac{\partial \rho}{\partial t} + \vec{u}\cdot\nabla\rho$$

Beispiel zur Kontigleichung in integraler Form Kavitationsblase



$$S = S_i + S_a.$$

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Geg.: $R(t)$ Ges $u(r,t)$

$$\frac{D\rho}{Dt} = 0.$$

$$\underbrace{\frac{\partial}{\partial t}\int \cancel{\rho}\, dV}_{0} + \int_{S_i} \cancel{\rho}\, \underbrace{\vec{u}}_{\dot{R}\vec{e}_r} \cdot (-\vec{e}_r)\, dS + \int_{S_a} \cancel{\rho}\, \underbrace{\vec{u}}_{u\vec{e}_r} \cdot \vec{e}_r\, dS = 0.$$

$$-\dot{R}\, 4\pi R^2 + u\, 4\pi r^2 = 0$$

$$u(r,t) = \dot{R}\left(\frac{r}{R}\right)^2$$

Expandierende Blase $\hat{\approx}$ Monopol.

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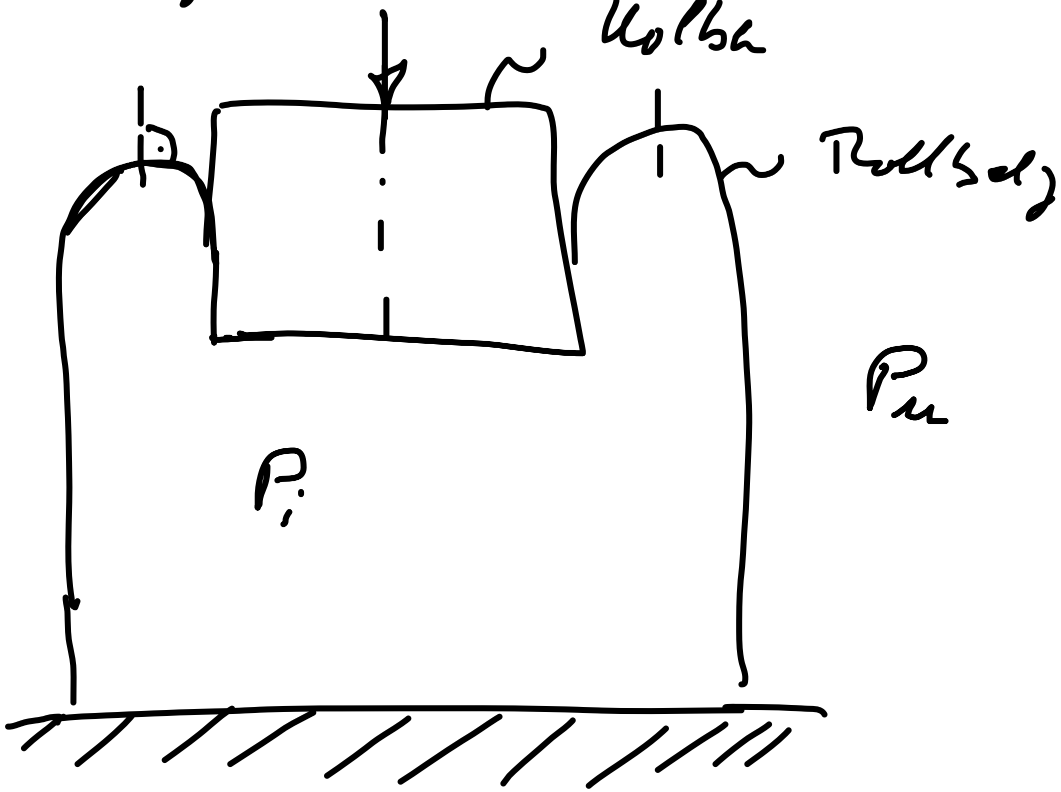
Zweites Beispiel Kontinuität in integraler Form.



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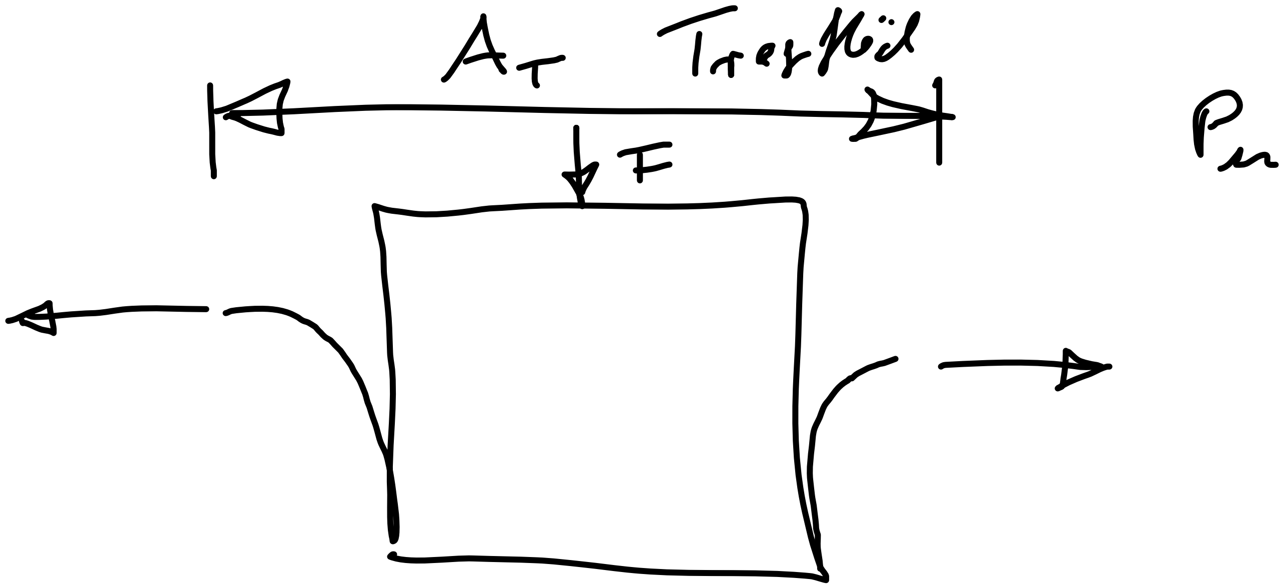

$$\left.\begin{array}{l} p_i = p_i(t) \\ \rho_i = \rho_i(t) \end{array}\right\} \text{Hydrostatik.}$$

~~ho~~ räumlich homogener Zustand.

$$\vec{F} := \int_{S_{Backe}} \vec{t}\, dS'$$

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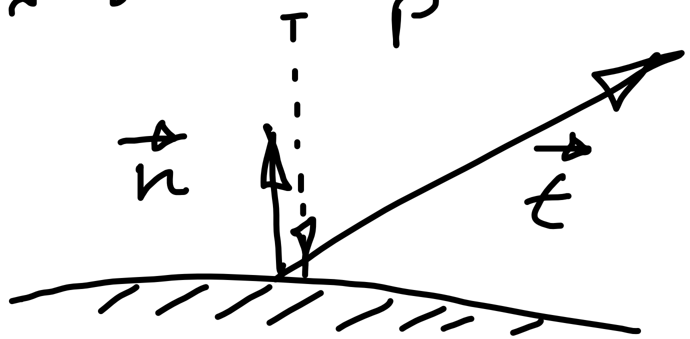
Spezialfall Hydrostatik

Spannungsvektor $\vec{t} = -p\,\vec{n}$

$$\vec{t} := \lim_{\Delta S \to 0} \frac{\Delta \vec{F}}{\Delta S}$$



$$\vec{F} = \int_{S_B} \vec{t}\, dS = \int_{S_B} (-p)\, \vec{n}\, dS = -p \int_{S_B} \vec{n}\, dS \;\Big|\cdot \vec{e}_x$$

$$F_x = \vec{F} \cdot \vec{e}_x = -p \int_S \vec{n} \cdot \vec{e}_x\, dS = -p\, S_{Proj.}$$

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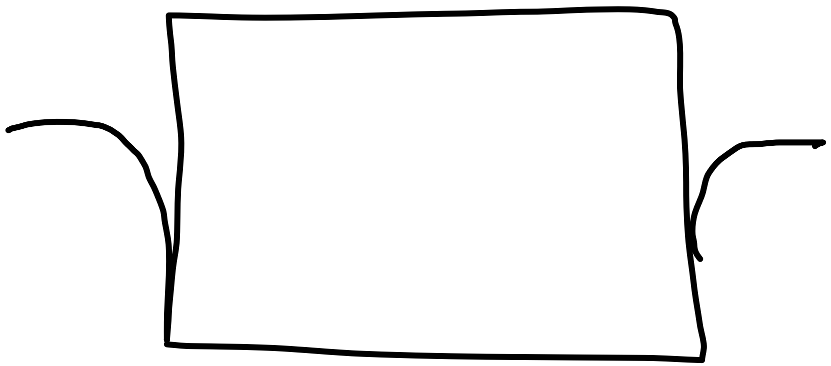

$$\underline{\underline{F = (p_i - p_u)\,A_T}}\,.$$

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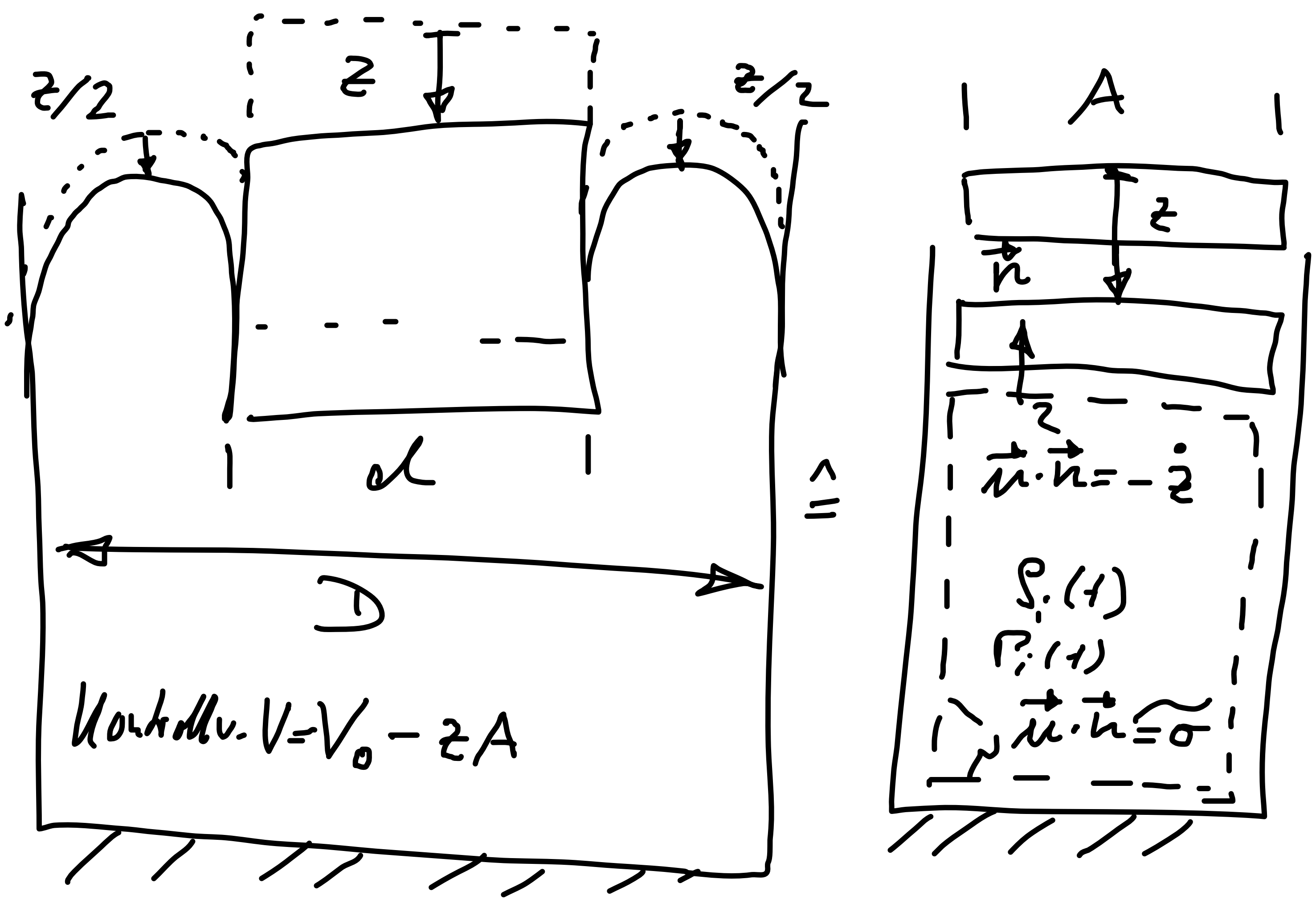

$$A = \frac{\pi}{4} d^2 + \frac{\pi}{8}\left(D^2 - d^2\right)$$

Verdrängerfläche

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$$\frac{\partial}{\partial t}\int_V \rho \, dV + \oint_S \rho \underbrace{\vec{u}\cdot\vec{n}}_{-\dot{z}} \, dS = 0.$$

$$V = V_0 - zA, \qquad S = A$$

Wichtig: 1. Das KV ist immer zeitlich fest.

2. In der Hydrostatik ist der Integrand konstant.

$$(V_0 - \underline{\underline{z}}A)\dot{\rho} - \dot{z}A\rho = 0.$$

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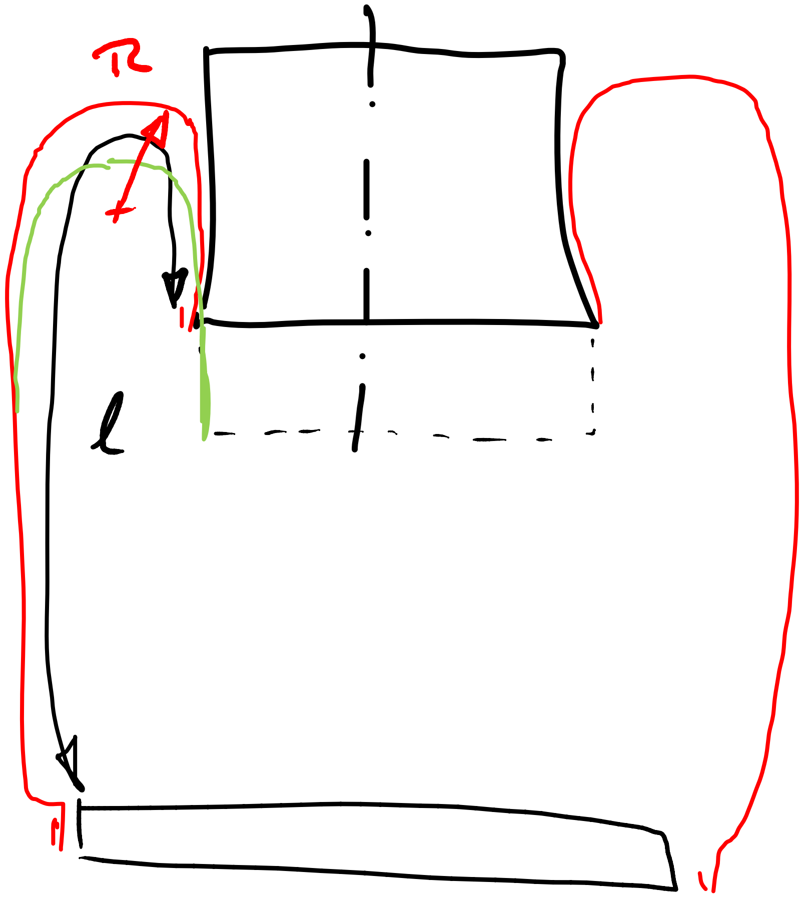

$l \approx$ const.

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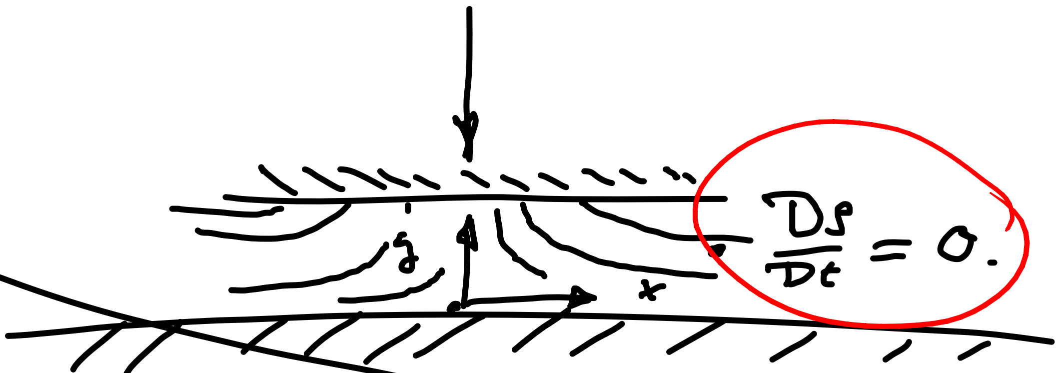
Beispiel für Konti in differentieller Form.

$u = ax$

$\vec{u} = ax\,\vec{e}_x + v(y)\,\vec{e}_y$ $\qquad \vec{x} = x\,\vec{e}_x + y\,\vec{e}_y$

?

Gegebenes Geschwindigkeitsfeld.



$\frac{D\rho}{Dt} = 0.$

$\cancel{\frac{D\rho}{Dt}}_{=0} + \cancel{\rho}\, \operatorname{div}\vec{u} = 0.$

$\rightsquigarrow \operatorname{div}\vec{u} = 0 \Leftrightarrow \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0.$

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$$\frac{\partial u}{\partial x} = a = -\frac{\partial v}{\partial y}.$$

$$v(y) = -ay + C.$$

Kinematische Randbedingung an der unteren Wand $y = 0$.

$$\vec{u} \cdot \vec{n} = v(0) \overset{!}{=} 0$$

$$\Rightarrow C = 0$$

$$\Rightarrow v(y) = -ay. \qquad u(x) = ax$$

Ebene Staupunktströmung.

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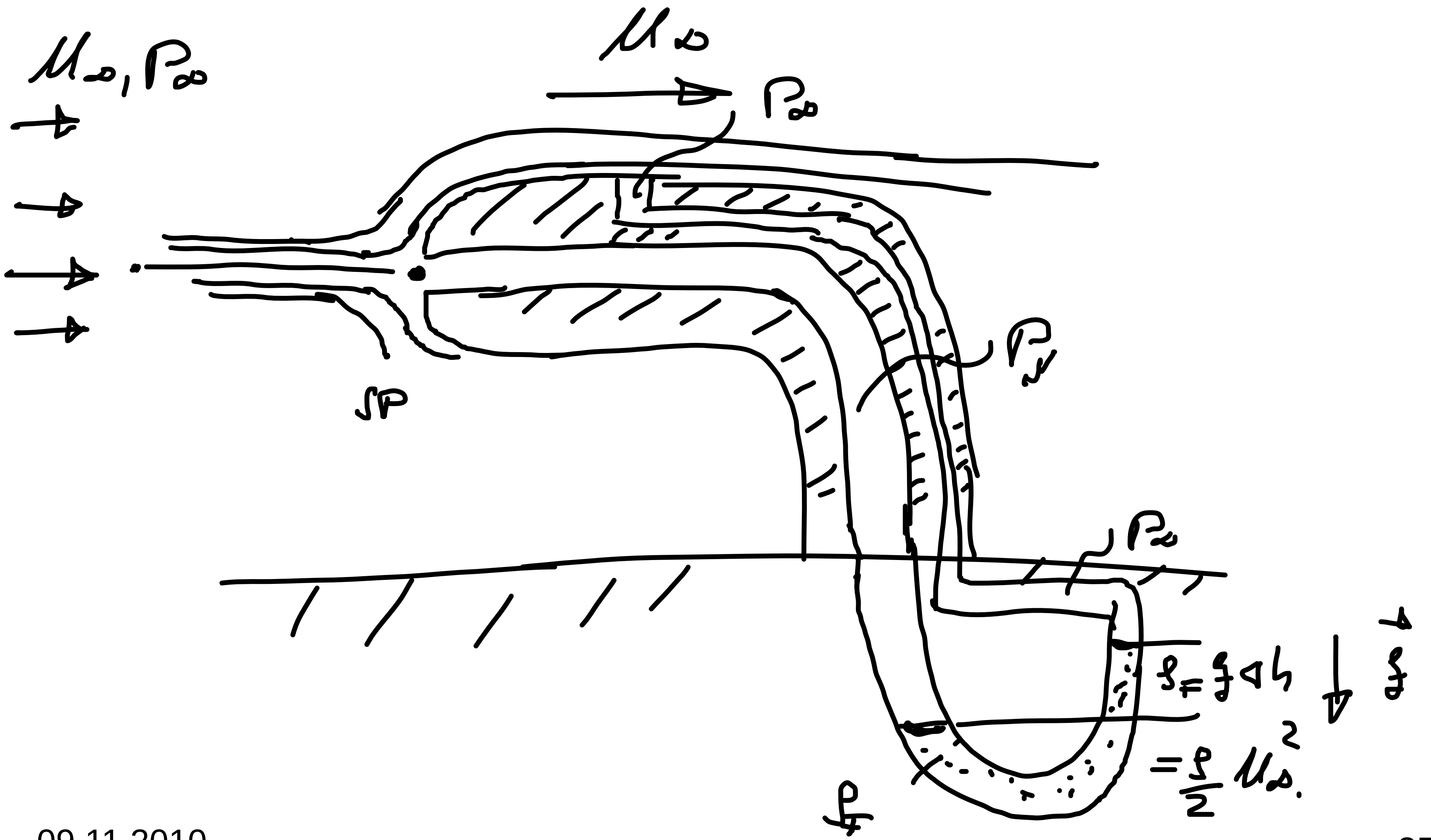
Beispiele für Staupunktströmung.

Staudruck $p_s = p_\infty + \frac{\varrho}{2} u_\infty^2$

Prandtl-Sonde



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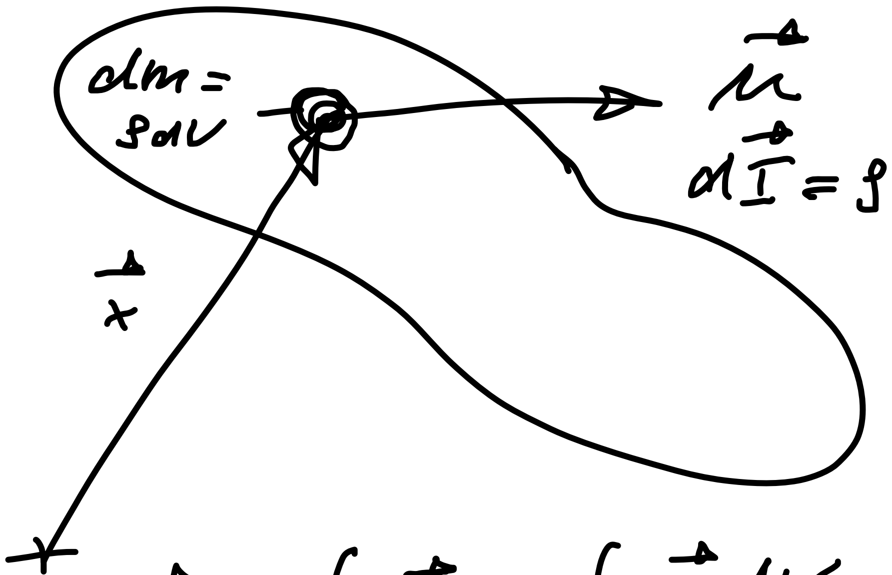
Impulsbilanz / Impulssatz.

LS:

RS:

Impulsänderung = Kraft



$d\vec{I} = \rho \vec{u} dV$

$$\vec{I} = \int d\vec{I} = \int_{V(t)} \rho \vec{u} dV$$

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Impulssätze

$$\frac{D\vec I}{Dt} = \frac{D}{Dt}\int_{V(t)} \rho\vec u\, dV = \underbrace{\frac{\partial}{\partial t}\int_V \rho\vec u\, dV}_{\text{lokale Impulsänderung}} + \underbrace{\oint_S \rho\vec u\; \vec u\cdot\vec n\, dS}_{\text{Impulsflüsse}}$$

RS

$$\text{Kraft} = \int_S \vec t\, dS + \int_V \rho\vec k\, dV$$

Spannungsvektor $\vec t := \lim_{\Delta S \to 0} \frac{\Delta \vec F}{\Delta S}$ <u>Oberflächenkraft</u>

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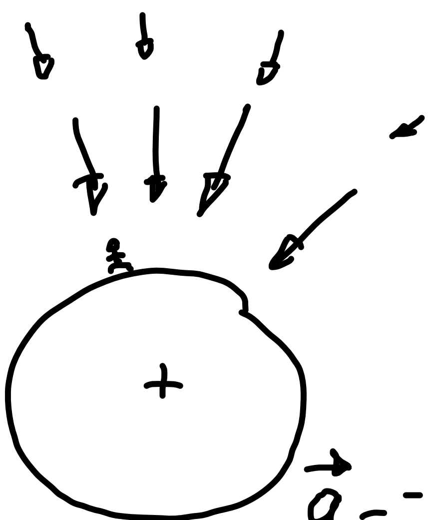
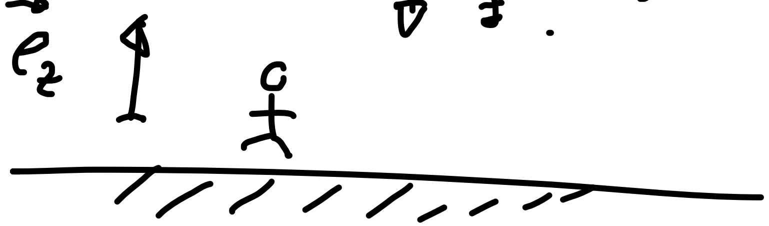
Volumenkraft $\vec{f} = \rho \vec{k} := \lim\limits_{\Delta V \to 0} \frac{\Delta \vec{F}}{\Delta V}$

Massenkraft $\vec{k}$

Beispiel für Volumenkräfte

Schwerefeld. $\rho \vec{g} = -\rho g \vec{e}_z$





$\vec{g} = -\frac{G}{r^2} \vec{e}_r$

$\vec{e}_z$ $\quad \vec{g} = \text{const.}$

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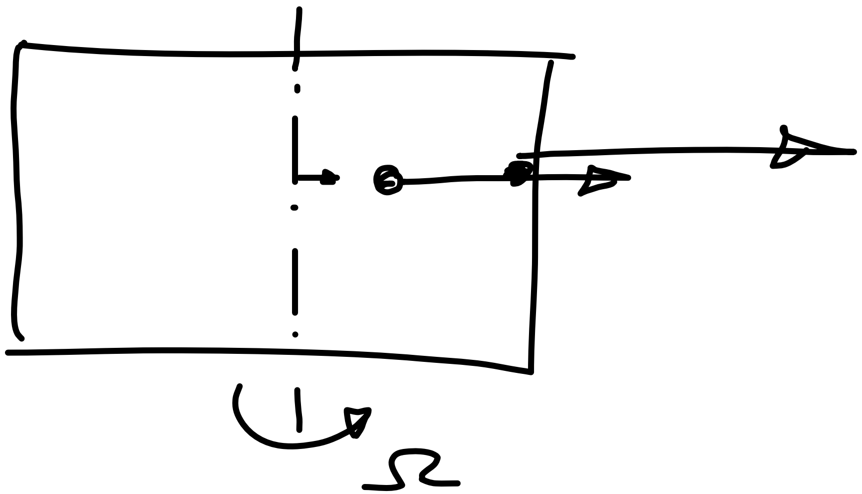
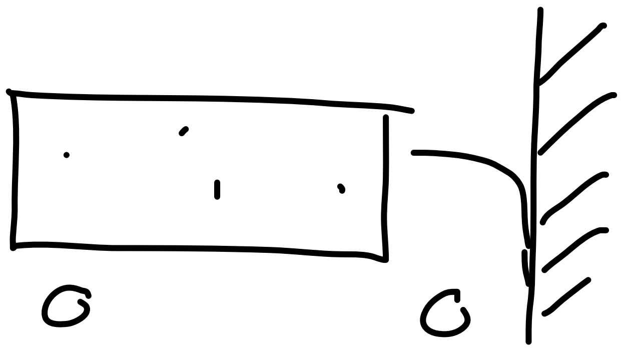
Zentrifugalfeld

$$\rho \vec{k} = \rho r \Omega^2 \vec{e}_r$$





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Impulssatz in integraler Form.

$$\boxed{\frac{\partial}{\partial t}\int_V \rho \vec{u}\, dV + \oint_S \rho \vec{u}\, (\vec{u}\cdot\vec{n})\, dS = \oint_S \vec{t}\, dS + \int_V \rho \vec{k}\, dV}$$

Herleitung des Impulssatzes in differentieller Form.

$$\frac{D}{Dt}\int_{V(t)} \vec{u}\, \underbrace{\rho\, dV}_{dm} = \ldots\ldots$$

$$\frac{D}{Dt}\int_m \vec{u}\, dm = \int \frac{D\vec{u}}{Dt}\rho\, dV = \int_V \rho \vec{k}\, dV + \int_S \vec{n}\cdot\underset{\sim}{T}\, dS$$

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Satz vom Gauß

$$\vec{n}\cdot(\;)\,dS = \nabla\cdot(\;)\,dV$$

$$\int_V \rho \frac{D\vec{u}}{Dt}\,dV = \int_V \rho \vec{k}\,dV + \int_V \nabla\cdot \underset{\sim}{T}\,dV$$

$$\int_V \underbrace{\rho \frac{D\vec{u}}{Dt} - \rho\vec{k} - \nabla\cdot \underset{\sim}{T}}_{\equiv 0}\,dV = 0.$$

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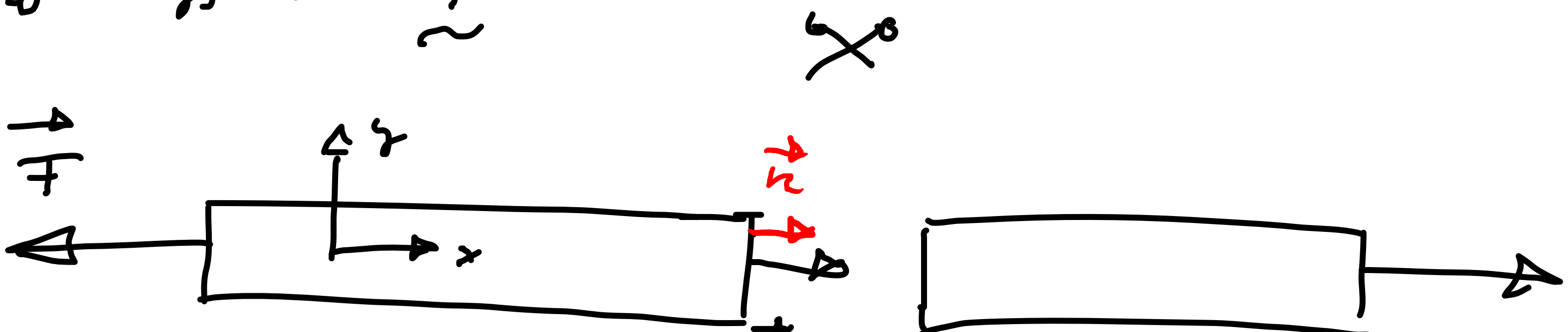
$$\rho \frac{D\vec{u}}{Dt} = \underbrace{\rho \vec{k} + \nabla \cdot \underset{\sim}{T}}$$ Cauchy-Gleichung.

$\frac{\text{Masse}}{\text{Volumen}}$ — Beschleunigung. — Kraft pro Volumeneinheit auf ein Flüssigkeitsteilchen.

Zusammenhang Spannungsvektor $\vec{t}$ und Spannungstensor $\underset{\sim}{T}$



$\vec{t} = \sigma_{xx} \vec{n}$

$\vec{n} = \vec{e}_x$.

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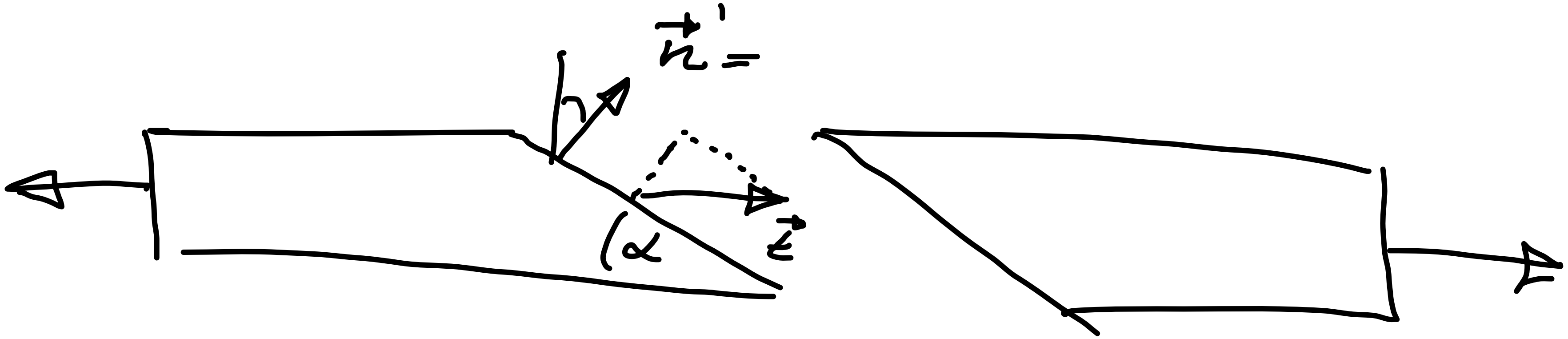
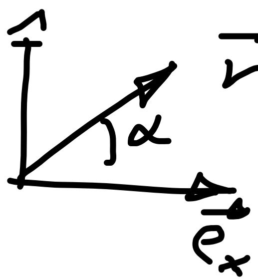


$$\vec{t} = \vec{n}' \cdot \underset{\sim}{T}$$

$$\underset{\sim}{\tau} = \underline{\tau_{xx}}\,\underline{\vec{e}_x\vec{e}_x} + \tau_{xy}\,\vec{e}_x\vec{e}_y + \tau_{xy}\,\vec{e}_y\vec{e}_x + \tau_{yy}\,\vec{e}_y\vec{e}_y .$$

Komponent   Basis der Tens.

Spannungstensor in Ebene.



$$\vec{n} = \cos\alpha\,\vec{e}_x + \sin\alpha\,\vec{e}_y$$

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Skalarprodukt

$$\vec{t} = \vec{n} \cdot \underset{\sim}{T} = \underset{\sim}{T} \cdot \vec{n}$$

$$= \left(\tau_{xx}\,\vec{e}_x\vec{e}_x + \tau_{xy}\,\vec{e}_x\vec{e}_y + \dots\right)\cdot\left(\cos\alpha\,\vec{e}_x + \sin\alpha\,\vec{e}_y\right)$$

$$= \tau_{xx}\cos\alpha\,\vec{e}_x + \tau_{yx}\cos\alpha\,\vec{e}_y + \tau_{xy}\sin\alpha\,\vec{e}_x + \tau_{yy}\sin\alpha\,\vec{e}_y .$$

$$\underline{\vec{t} = \left(\tau_{xx}\cos\alpha + \tau_{xy}\sin\alpha\right)\vec{e}_x + \left(\tau_{yx}\cos\alpha + \tau_{yy}\sin\alpha\right)\vec{e}_y}$$

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