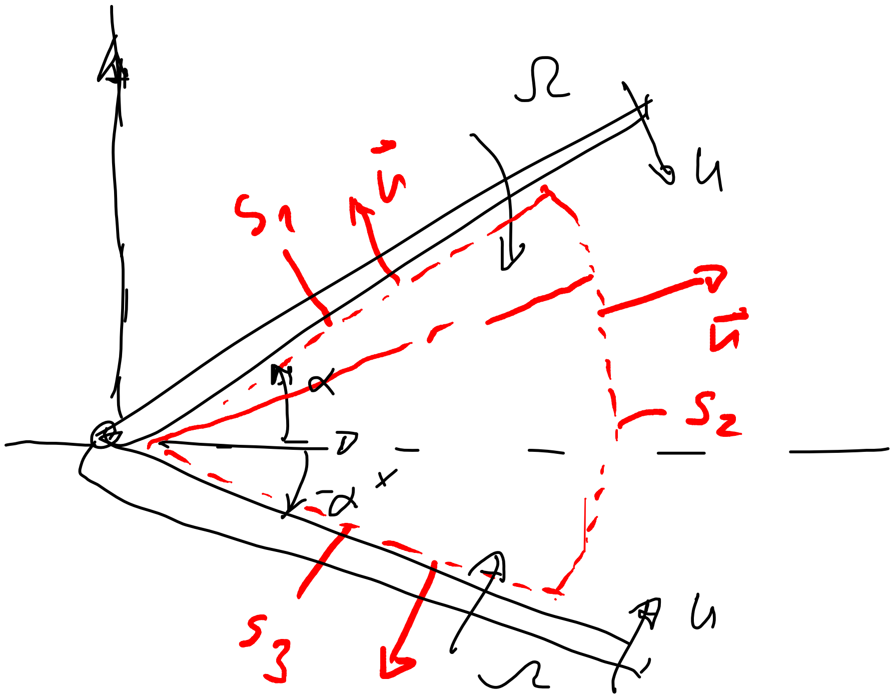
1. Kontinuitätsgleichung
2. Bernoulli

# Konti-Gleichung




$$u(r,\varphi) = u_r(r,\varphi)\vec{e}_r + u_\varphi(r,\varphi)\vec{e}_\varphi$$

mit $u_r(r,\varphi) = \underline{\underline{f(r)}}\cos\left(\frac{\pi}{2\alpha}\varphi\right)$

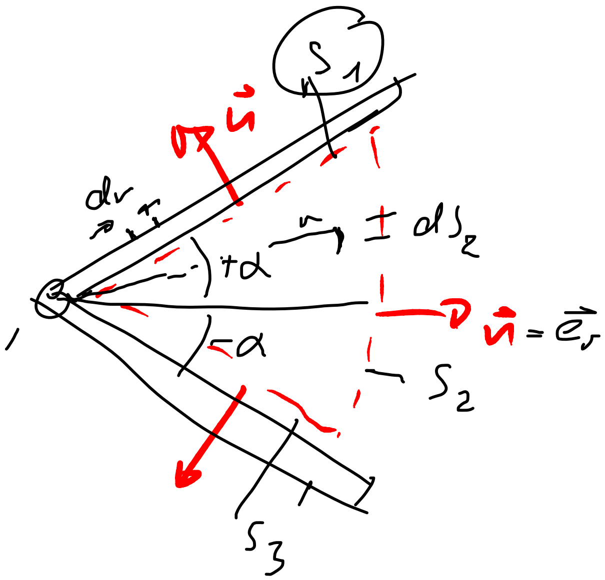
1) Wandgeschwindigkeit? $\vec{u}_w = ?$

2) $f(r)$ bestimmen

$$\boxed{\vec{u} = \vec{\Omega} \times \vec{x}}$$

$$\begin{pmatrix} u_r \\ u_\varphi \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \pm\Omega \end{pmatrix} \times \begin{pmatrix} r \\ 0 \\ 0 \end{pmatrix} = \pm \begin{pmatrix} 0 \\ \Omega r \\ 0 \end{pmatrix}$$

$$\Rightarrow u_\varphi = \pm \Omega r$$




Konti: $\oint_S \vec{u}\cdot\vec{n}\, dS = 0$

$$\Rightarrow \int_{S_1} \vec{u}\cdot\vec{n}\, dS + \int_{S_2} \vec{u}\cdot\vec{n}\, dS + \int_{S_3} \vec{u}\cdot n\, dS = 0$$

Flächenelement pro Tiefe

$S_1$: $\vec{u}\cdot\vec{n} = -\Omega r$ $\qquad dS_1 = dr$

$S_3$: $\vec{u}\cdot\vec{n} = -\Omega r$ $\qquad dS_3 = dr$

$S_2$: $\underset{\vec{e}_r}{\vec{u}\cdot\vec{n}} = u_r \vec{e}_r \cdot \vec{e}_r = u_r = f(r)\cos\left(\frac{\pi}{2\alpha}\varphi\right)$

$$dS_2 = r\, d\varphi$$

$=$ Einsetzen in Kondi:

$$\underbrace{\int_0^r -\Omega r \, dr}_{S} + \underbrace{\int_0^r -\Omega r \, dr}_{S_3} + \underbrace{\int_{-\alpha}^{\alpha} f(r) \cos\left(\frac{\pi}{2\alpha}\varphi\right) r \, d\varphi}_{S_2} = 0$$

$$\Rightarrow -\Omega r^2 + r f(r) \left( \sin( \ldots ) \right) = 0$$

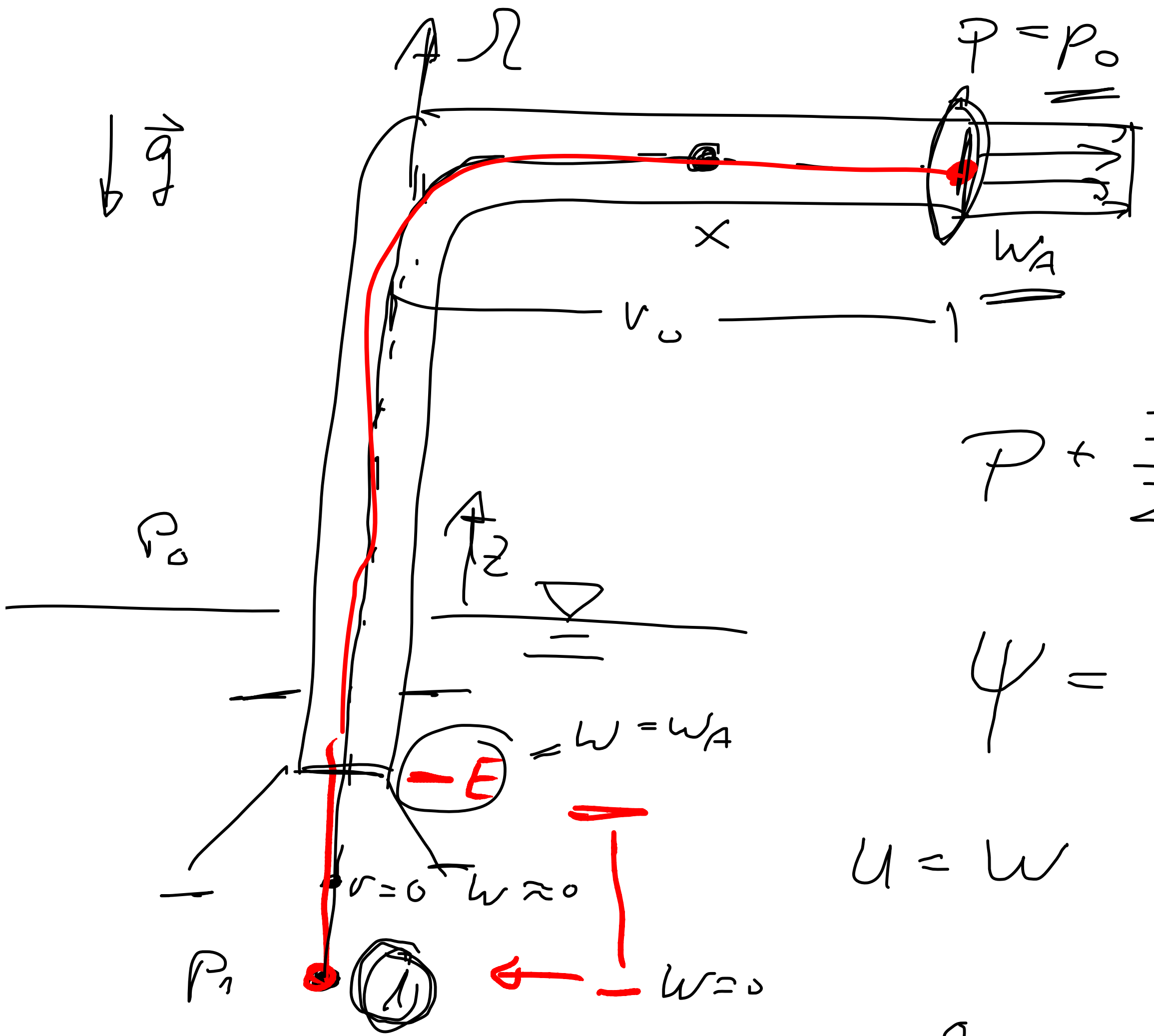
$$\Rightarrow \underline{\underline{f(r) = \frac{\Omega r \pi}{4\alpha}}}$$

## 2) Bernoulli

$\rho = const$

$\Omega_{max}$ für

$p(r,z) \neq p_D$




$$p + \frac{\rho}{2}u^2 + \psi = C$$

$$\psi = \rho g z - \frac{\rho}{2}\Omega^2 r^2$$

$$u = w$$

$$p_1 + \cancel{\frac{\rho}{2}w_1^2}^{\,0} + \rho g z_1 - \cancel{\frac{\rho}{2}\Omega^2 r_1^2}^{\,0} = p + \frac{\rho}{2}w^2 + \rho g z - \frac{\rho}{2}\Omega^2 r^2 \quad \text{I}$$

$$p_1 = p_0 - \rho g z_1 \quad \text{II}$$

$$p = p_0 - \frac{\rho}{2} w^2 - \rho g z + \frac{\rho}{2} \Omega^2 r^2 = p(z, r)$$

Kontinuitätsgleichung $w = w_A$

$p(r = r_0, z = h)$

$\Rightarrow$ Einsetzen

$$p = p_0 = p_0 - \frac{\rho}{2} w_A^2 - \rho g h + \frac{\rho}{2} \Omega^2 r_0^2$$

$$\Rightarrow \frac{\rho}{2} w_A^2 = \frac{\rho}{2} w^2 = \frac{\rho}{2} \Omega^2 r_0^2 - g \rho h$$

Einsetzen: $\boxed{p(r, z) = p_0 + \frac{\rho}{2} \Omega^2 (r^2 - r_0^2) + \rho g (h - z)}$

$r = 0$ $z = h$

$$p_{min}(z=h, r=0) = p_0 - \frac{\rho}{2}\Omega^2 r_0^2 \overset{!}{>} p_D$$

$$\Rightarrow \boxed{\Omega < \sqrt{\frac{2(p_0-p_D)}{\rho r_0^2}}}$$

Rohr verschlossen, zum Zeitpunkt $t=0$ geöffnet

Gesucht: Beschleunigung $b(t) = \frac{dw_A}{dt}$

$\Rightarrow$ Bernoulli instationär!

$$\rho\int_1^A \frac{\partial w}{\partial t}\,ds + \underline{p_A + \rho g h} - \frac{\rho}{2}\Omega^2 r_A^2 + \frac{\rho}{2}w_A^2 = \underline{\rho g z_1 - \frac{\rho}{2}\Omega^2 r_1^2 + \frac{\rho}{2}w_1^2}$$

$w = w_A = \text{const} \neq f(s)$ $\qquad p_1 = p_0 - \rho g z_1$

Einsetzen:

$$\rho \frac{\partial w_A}{\partial t} \int_E^A ds + \cancel{p_0} + \rho g h - \frac{\rho}{2} \Omega^2 r_0^2 + \frac{\rho}{2} w_A^2 = \cancel{p_0} \quad \Big| \frac{1}{\rho}$$

$$\boxed{\frac{\partial w_A}{\partial t} = b(t) = \frac{1}{2\ell}\Big(\underbrace{\Omega^2 r_0^2 - 2gh}_{C} - w_A^2\Big)} \Longrightarrow \text{lösen}$$

$t = 0: \quad w_A(t=0) = 0$

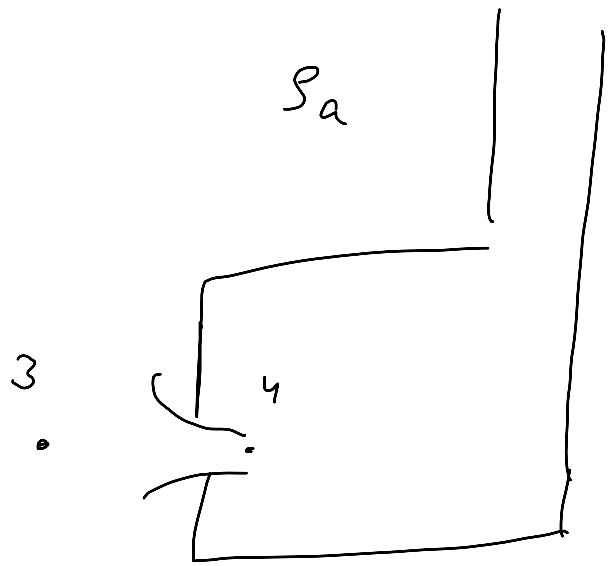
$$\Longrightarrow \quad \frac{\partial w_A}{\partial t} = b(t) = \frac{1}{2\ell}\left(\Omega^2 r_0^2 - 2gh\right)$$

$$\frac{\partial w_A}{\partial t} = \frac{1}{2\ell}\left(C - w_A^2\right) \xrightarrow{\text{TdV}} \underbrace{\int \frac{dw_A}{(C - w_A^2)}} = \underbrace{\int \frac{1}{2\ell}\, dt}_{\frac{t}{2\ell}}$$

$$\boxed{w_A(t) = \sqrt{c}\,\tanh\left(\frac{\sqrt{c}}{2\ell}t\right)}$$

# 2. VRÜ

Korrektur 3.2




Gl: 37 $\quad p_0 = p_1 + \frac{\rho_a}{2} u_1^2$

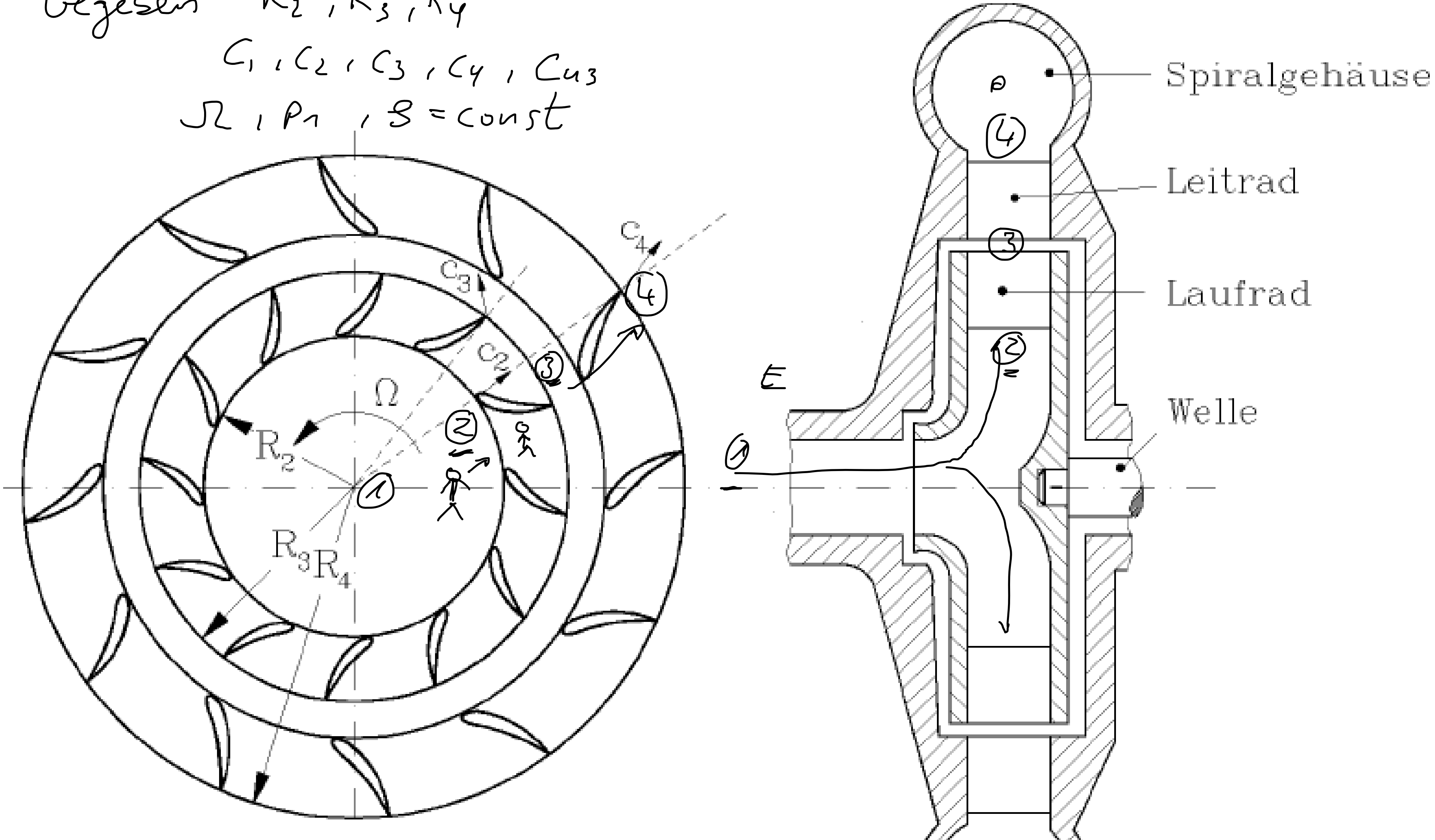
3.3 Rohrpumpe

Gl 51 : rechte Seite $+ p_1$

Gegeben $R_2, R_3, R_4$

$c_1, c_2, c_3, c_4, c_{u3}$

$\Omega, p_1, \rho = const$




gesucht $p_2, p_3, p_4$

Bernoulli ① → ②

$$p_1 + \frac{\rho}{2} c_1^2 = p_2 + \frac{\rho}{2} c_2^2 \Longrightarrow p_2 = p_1 + \frac{\rho}{2}\left(c_1^2 - c_2\right)$$

Bernoulli ② → ③ bewegtes Koordinatensystem

$$p_2 + \frac{\rho}{2} w_2^2 - \underline{\frac{\rho}{2} \Omega^2 R_2^2} = p_3 + \frac{\rho}{2} w_3^2 - \underline{\frac{\rho}{2} \Omega^2 R_3^2}$$

$$p_3 = p_2 + \frac{\rho}{2}\left(\underline{w_2}^2 - \underline{w_3}^2\right) + \frac{\rho}{2} \Omega^2 \left(R_3^2 - R_2^2\right)$$

$$w^2 = ? \qquad \vec{c} = \vec{w} + \vec{u} \Rightarrow \underline{\vec{w}} = \underline{\vec{c}} - \underline{\vec{u}}$$

$$\vec{u} = \text{Umfangsgeschw. } \underline{\vec{\Omega} \times \vec{x}} = \Omega R\, \vec{e}_u$$

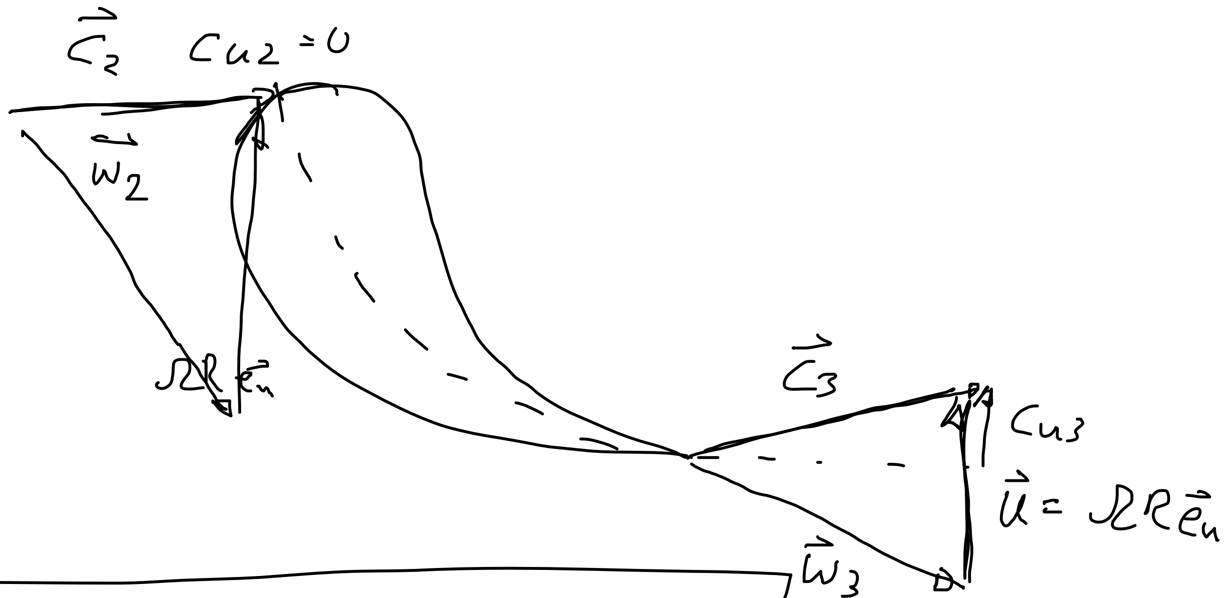
Vektorschreibweise
$$\begin{pmatrix} w_r \\ w_u \end{pmatrix} = \begin{pmatrix} c_r \\ c_u \end{pmatrix} - \begin{pmatrix} 0 \\ \Omega R \end{pmatrix}$$

$$w^2 = |\vec{w}|^2 = w_r^2 + w_u^2$$
$$= \underbrace{c_r^2 + c_u^2}_{} - 2 c_u \Omega R + (\Omega R)^2$$

$$\vec{c} = \begin{pmatrix} c_r \\ c_u \end{pmatrix} \qquad |\vec{c}|^2 = c_r^2 + c_u^2 \Rightarrow c^2$$

$$w^2 = \underline{c}^2 - 2 c_u \Omega \underline{R} + (\Omega R)^2$$

$$\vec{c} = \vec{w} + \vec{u}$$




$$\boxed{w^2 = c^2 - 2 c_u \Omega R + \Omega^2 R^2} \leftarrow$$

$$\rightarrow \quad w_2^2 = c_2^2 - 2 \overset{0}{\cancel{c_{u2}}} \Omega R_2 + \Omega^2 R_2^2$$

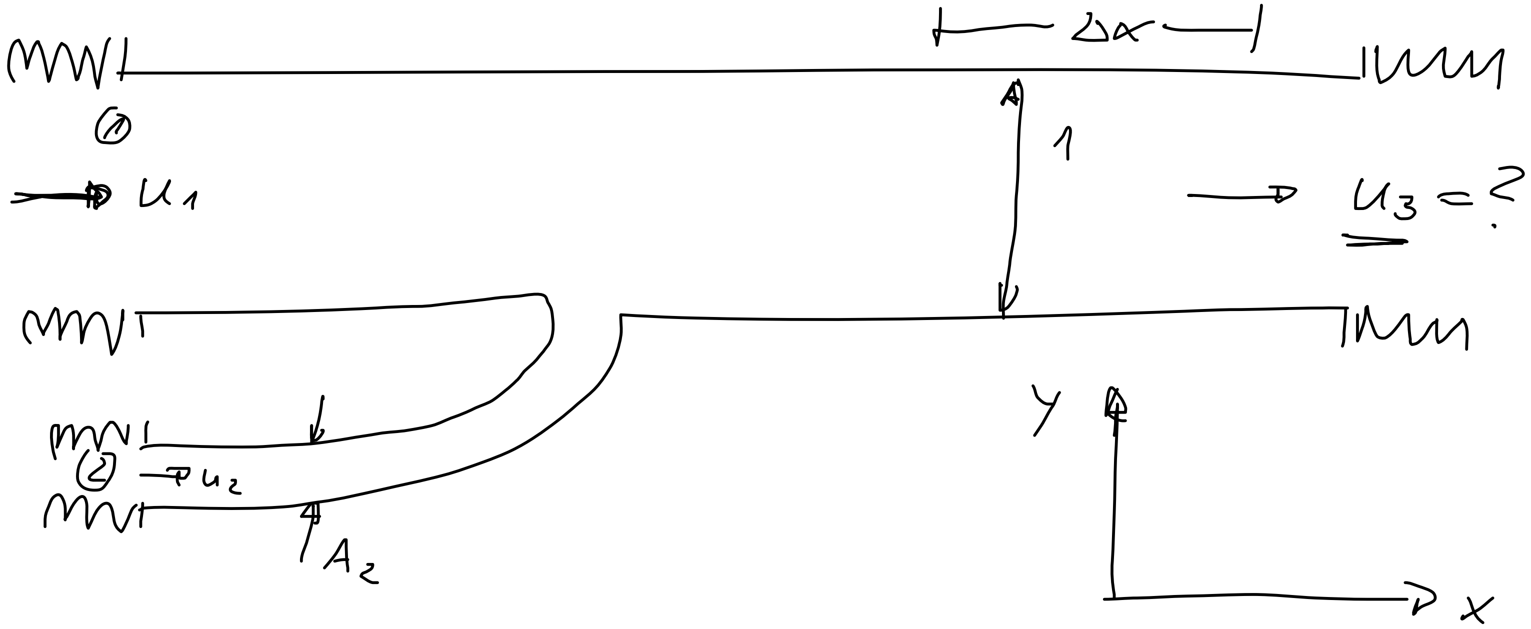
$$\rightarrow \quad w_3^2 = c_3^2 - 2 c_{u3} \Omega R_3 + (\Omega R_3)^2$$

$$\Rightarrow \quad p_3 = p_1 + \frac{\rho}{2} \left( c_1^2 - c_3^2 + 2 \Omega R_3 c_{u3} \right)$$

3 $\rightarrow$ 4

$$p_3 + \frac{\rho}{2} c_3^2 = p_4 + \frac{\rho}{2} c_4^2 \Longrightarrow p_4 = \ldots$$

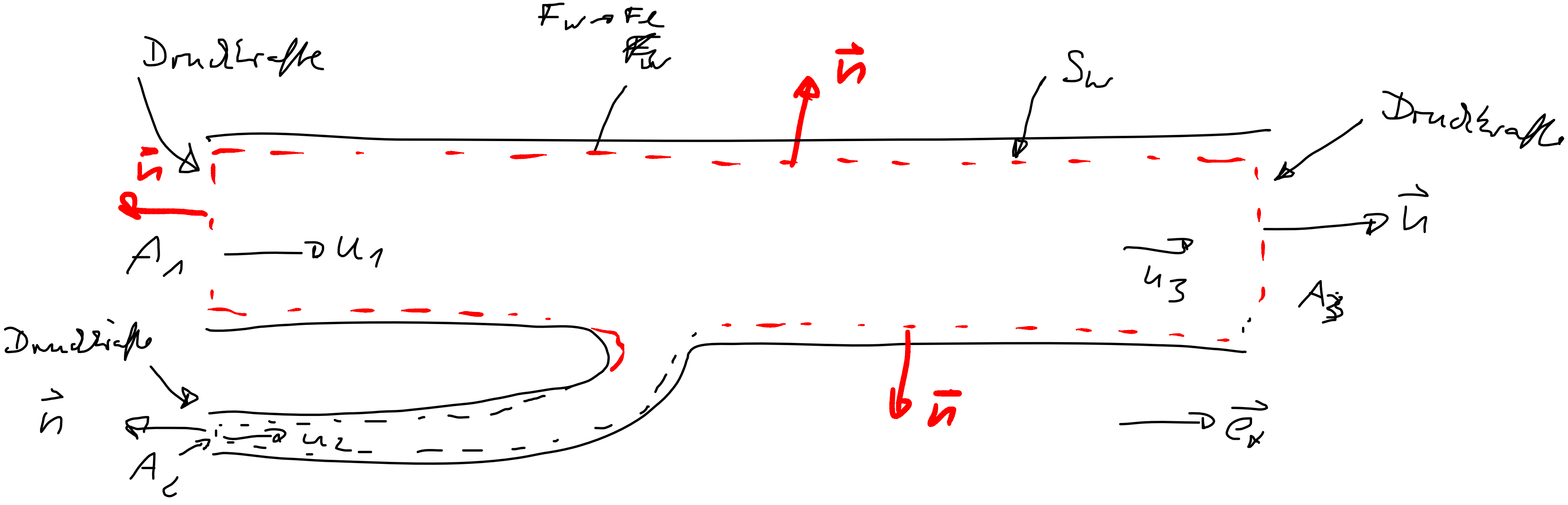
# Impulssatz




gegeben $p_1, p_2, p_3$, $u_1, u_2$, $A_1, A_2$ $\rho = const$

$c_{ges}$

⟹ Impulssatz → Kraft gesucht




Kontigleichung $\boxed{\cancel{\varrho}\oint_S \cancel{\varrho}\,\vec{u}\cdot\vec{n}\,dS = 0}$

$S = A_1 + A_2 + A_3 + S_w$

$$\Longrightarrow \int_{A_1} \vec{u}\cdot\vec{n}\,dS + \int_{A_2} \ldots \int_{S_w} \vec{u}\cdot\vec{n}\,dS$$

$A_1$: $\vec{u}\cdot\vec{n} = u_1 \vec{e}_x \cdot (-\vec{e}_x)$
$= -u_1$

$A_2$: $\vec{u}\cdot\vec{n} = -u_2$

$A_3$: $\vec{u}\cdot\vec{n} = u_3 \cdot \vec{e}_x \cdot \vec{e}_x = u_3$

$dS = dA_1$ , $dS = dA_2$

$S_w$: $\vec{u}\cdot\vec{n} = 0$

Einsetzen $\Rightarrow$ $\underline{\underline{u_3}} = u_1 + u_2 \frac{A_2}{A_1}$

$$\oint_S \rho \vec{u} \cdot (\underline{\underline{\vec{u} \cdot \vec{n}}}) dS = \vec{F}_{\to Fl} \qquad S = A_1 + A_2 + A_3 + S_w$$

$$\int_{A_1} \rho \vec{u} \cdot (\vec{u} \cdot \vec{n}) dS + \int_{A_2} \ldots \ldots \int_{S_w} \rho \vec{u} \cdot (\vec{u} \cdot \vec{n}) dS = \underline{\underline{\vec{F} \big|_{\cdot \vec{e}_x}}}$$

$$\Rightarrow \int_{A_1} \rho \underline{\underline{\underbrace{\vec{u} \cdot \vec{e}_x}}} (\underline{\underline{\underbrace{\vec{u} \cdot \vec{n}}}}) dS \ldots \ldots = \vec{F} \cdot \vec{e}_x$$

$A_1$: $\vec{u} \cdot \vec{e}_x = u_1 \vec{e}_x \cdot \vec{e}_x = u_1$ $\qquad$ $A_3$: $\vec{u}_3 \cdot \vec{e}_x = u_3$

$A_2$: $\vec{u} \cdot \vec{e}_x = u_2$

$$\oint_S \rho \vec{u} \cdot \vec{e}_x (\vec{u} \cdot \vec{n})\, dS = \underline{\underline{-\rho u_1^2 A_1 - \rho u_2^2 A_2 + \rho u_3^2 A_1}}$$

$$\oint_S$$

rechte Seite $\underline{\underline{\vec{F}_{\to Fl} \cdot \vec{e}_x}}$

$A_1$ : $\vec{F}_{p_1} = p_1 \cdot A_1\, \vec{e}_x \leftarrow$

$A_2$ : $\vec{F}_{p_2} = p_2 \cdot A_2 \cdot \vec{e}_x$

$A_3$ : $\vec{F}_{p3} = p_3\, A_1\, (-e_x) = -p_3\, A_1\, \vec{e}_x$

$S_w$ : $\underline{\vec{F}_{W \to Fl}} = - \underline{\vec{F}_{Fl - W}}$

$$\left.\begin{matrix} \\ \\ \\ \end{matrix}\right\} \vec{F}$$

$$\underline{\underline{\vec{F}_{\to Fl} \cdot \vec{e}_x}} = p_1 A_1 \underbrace{\vec{e}_x \cdot \vec{e}_x}_{1} + p_2 A_2 \underbrace{\vec{e}_x \cdot \vec{e}_x}_{1} - p_3 A_1 \underbrace{\vec{e}_x \cdot \vec{e}_x}_{1} - \vec{F}_x$$

$$\underline{\underline{F_x}} = \left(p_1 A_1 + p_2 A_2 \; \ldots\ldots \right)$$

$$\boxed{\underline{\Delta x} = \frac{F}{c_{ges}} = \frac{1}{c_{ges}} \left(p_1 A_1 + p_2 A_2 \ldots\ldots \right)}$$

Aufgabe GL 13

$$\int_{A_1} \ldots dS = \varrho u_1 \vec{e}_x \cdot \vec{e}_x (u_1 e_x \cdot (-\vec{e}_x) A_1 = \ldots$$

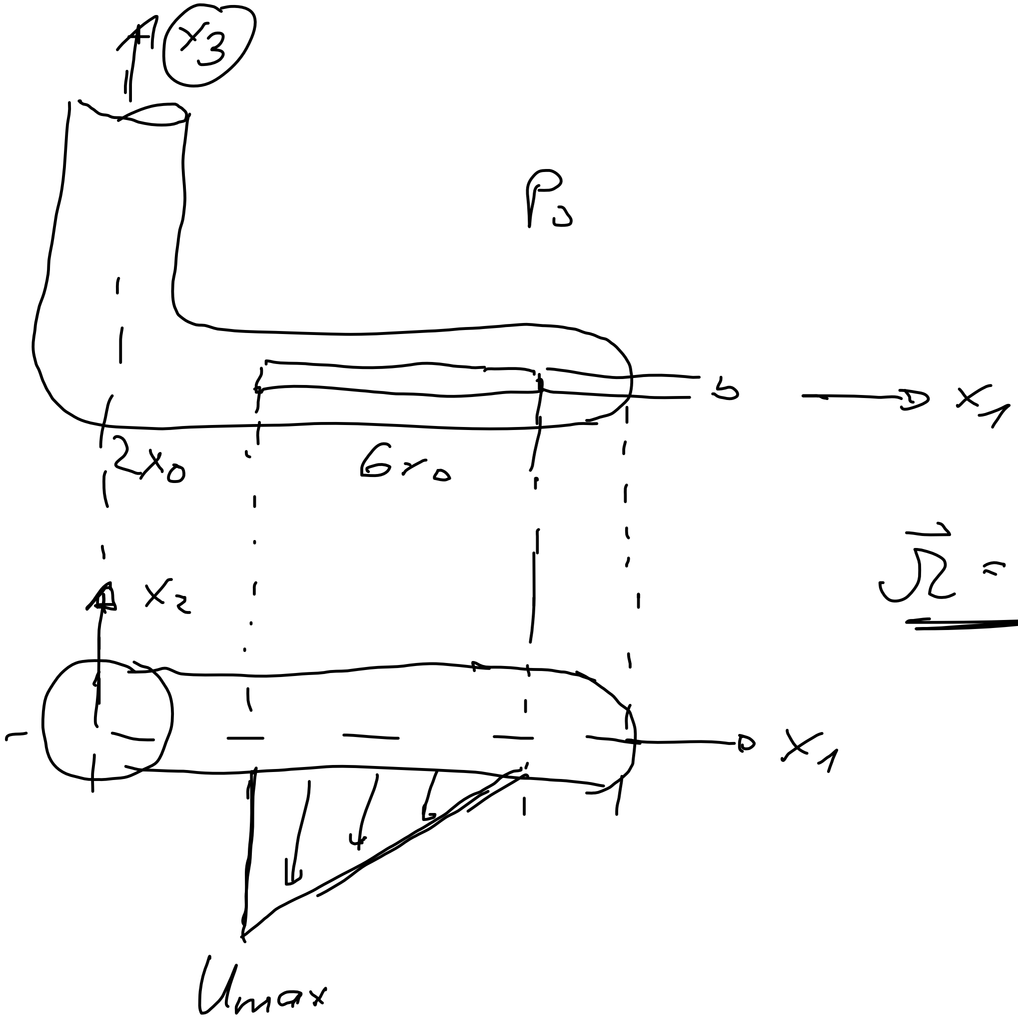
$$\int_{A_2} - - - - - A_2$$

$$\int_{A_3} \qquad A_3$$

# Drallsatz

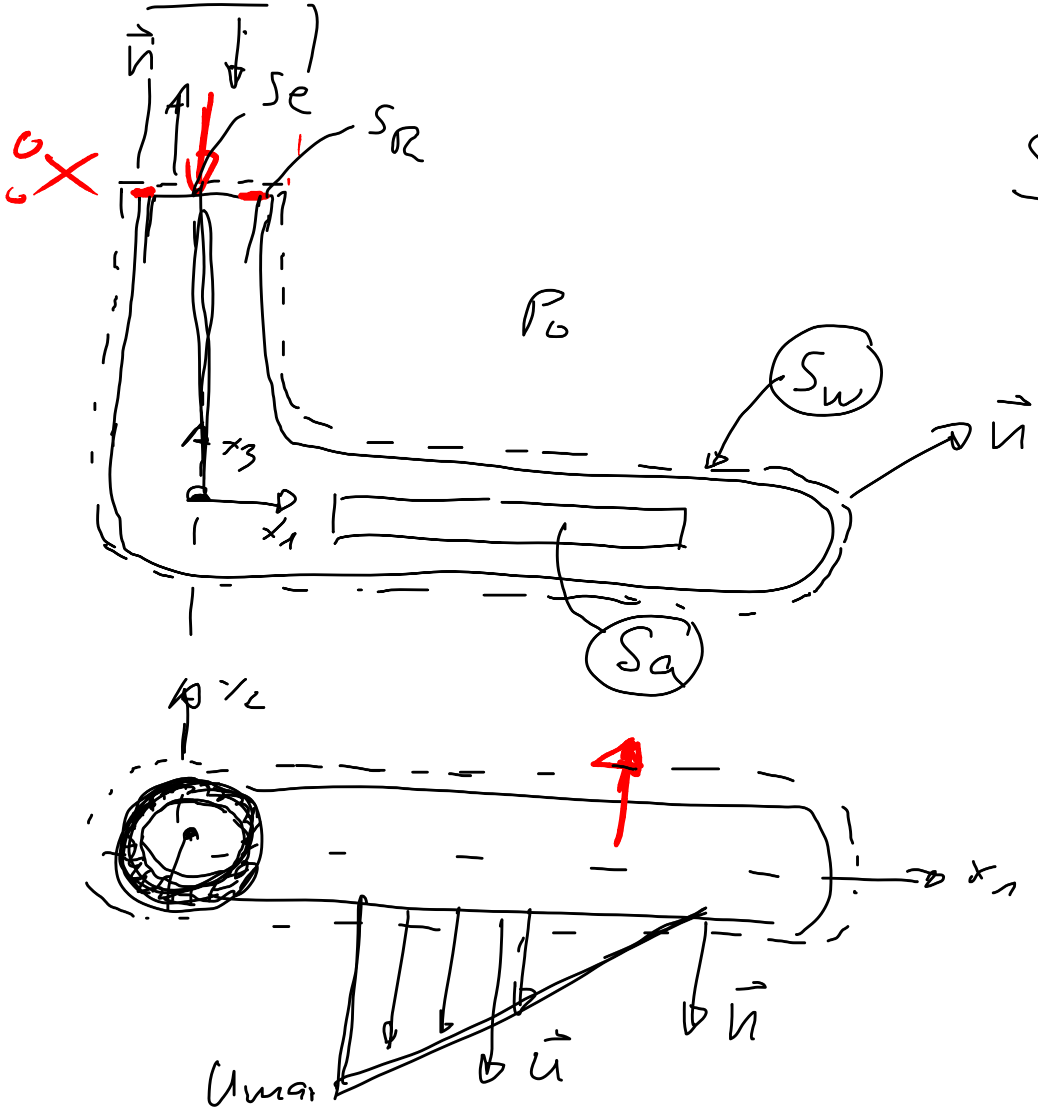
gesucht 1) $M_3 \, (U_{max})$

2) $U_{max} \, (Q)$

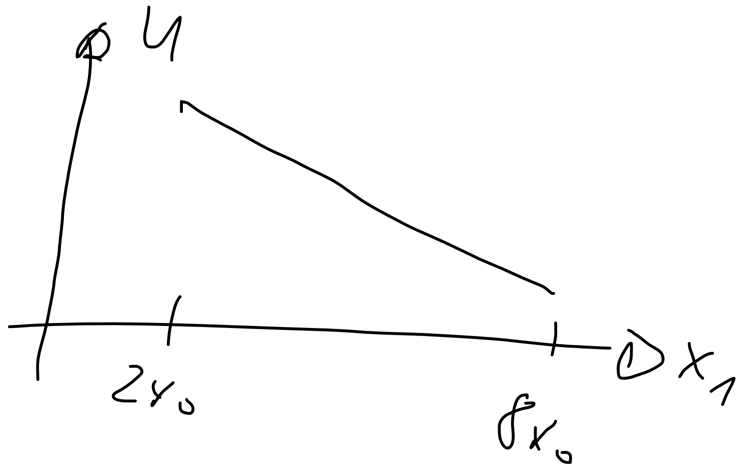



$\vec{\Omega} = \vec{0}$

gegeben $b, x_0, Q, \rho, p_0$




$$S = S_e + S_a + S_w + S_R$$




$$u = m x + b$$

$$u_2(x_1) = \frac{u_{max}}{6x_0}(8x_0 - x_1)$$

Drallsatz: $\oint \rho\, \underline{\underline{\vec e_3}} \cdot \underline{(\vec x \times \vec u)}\, \underline{(\vec u \cdot \vec n)}\, dS = \vec M_{Fe} \cdot \vec e_3$

linke Seite

für $S_W$ und $S_R$ : $\vec{u} \cdot \vec{n} = 0$

$$\int_{S_e} \rho \vec{e}_3 \, \underline{(\vec{x} \times \vec{u})} \, \underline{(\vec{u} \cdot \vec{n})} \, dS + \int_{S_a} \rho \vec{e}_3 \, \underline{(\vec{x} \times \vec{u})} \, \underline{(\vec{u} \cdot \vec{n})} \, dS = \vec{M}_{\to F\ell} \, \vec{e}_3$$

$S_e$ $\quad \vec{e}_3 \cdot (\vec{x} \times \vec{u})$

$$\vec{x} = x_1 \vec{e}_1 + x_2 \vec{e}_2 + x_3 \vec{e}_3$$
$$\vec{u} = -u_3 \vec{e}_3$$

$$\Rightarrow (\vec{x} \times \vec{u}) = x_2 u_3 \underline{\vec{e}_1} - x_1 u_3 \underline{\vec{e}_2} \quad \Big| \, \vec{e}_3$$

$$\Rightarrow \vec{e}_3 \, (\vec{x} \times \vec{u}) = 0$$

5a :

$$\vec{x} = x_1 \vec{e}_1 + x_2 \vec{e}_2 + x_3 \vec{e}_3$$

$$\vec{u} = u_2 (-\vec{e}_2)$$

$$(\vec{x} \times \vec{u}) = u_2 x_3 \vec{e}_1 - \underline{u_2 x_1 \vec{e}_3}$$

$$\vec{e}_3 \cdot (\vec{x} \times \vec{u}) = \underline{-u_2 x_1}$$

$\vec{u} \cdot \vec{n}$

$$\vec{n} = -\vec{e}_2$$

$$\Rightarrow \vec{u} \cdot \vec{n} = -u_2 \vec{e}_2 \cdot (-\vec{e}_2) = u_2$$

$$\int_{S_a} \rho \vec{e}_3 \cdot (\cancel{\vec{x}} \times \vec{u}) (\vec{u} \cdot \vec{n}) \, dS = -\int_{S_a} u_2^2 \rho x_1 \, dS$$

$u_2$ ?

$$\int_{S_a} \rho \vec{e}_3 \cdot (\vec{x} \times \vec{u})(\vec{u} \cdot \vec{n}) \, dS = -\rho \int_{-b/2}^{+b/2} \int_{2x_0}^{8x_0} x_1 \left[\frac{u_{max}}{6x_0}(8x_0 - x_1)\right]^2 dx_1 \, dx_2$$

$$\Rightarrow \int_{S_a} \ldots \, dS = \boxed{-7 \rho b \, u_{max}^2 \, x_0^2}$$

= linke Seite vom Drallsatz

rechte Seite : $\vec{M}_{\to Fe} \cdot \vec{e}_3$

$$= (\vec{F} \times \vec{x}) \vec{e}_3$$

Auf $S_W$ ~~und~~ + $S_a$ $\qquad \vec{M} = \vec{0}$

$S_e$ : $\underline{\vec{F}} = -\underline{\rho_e}\, \underline{S_e}\, \underline{\vec{e}_3} \qquad \vec{x} = x_3 \underline{\vec{e}_3}$

$$\Rightarrow \vec{F} \times \vec{x} = 0$$

$S_R$ : $\vec{M}_{\to Fe} \cdot e_3 = \boxed{-\vec{M}_{\to Rohr} \cdot \vec{e}_3}$

$$\boxed{-\vec{M}_{\to Rohr} \cdot \vec{e}_3 = -786\, u_{max}^2 x_0^2}$$

$$U_{max} = f(Q)$$

$$Q = \oint_S \vec{u} \cdot \vec{n} \, dS = \int_{S_a} \underline{\vec{u}} \cdot \vec{n} \, dS$$

$$\Rightarrow Q = \int_{-b/2}^{+b/2} \int_{2x_0}^{8x_0} \frac{U_{max}}{6x_0} (8x_0 - x_1) \, dx_1 \, dx_2 = \underline{3b\, U_{max}} \, x_0$$

$$\Rightarrow U_{max} = \frac{Q}{3b\, x_0}$$

2.1 Gl 14 $\left.\frac{D\cancel{\vec{b}}^{\vec{J}}}{Dt}\right|_J = \left.\frac{D\cancel{\vec{b}}^{\vec{J}}}{Dt}\right|_B$

gegeben : $m_{ges}$, $\rho_a$, $w_a$, $A_a$, $B$

1.1 $\cancel{\vec{M} = \vec{F} \times \vec{x}}$ $\vec{M} = \vec{x} \times \vec{F}$

20, 21, 22 : Klausurtagung

24 : Urlaub

28 : Klausur

# Impulssatz im bewegten Koordinatensystem

$$\frac{D\vec{J}}{Dt} = \vec{F} \quad , \quad \vec{J} = \int_{V(t)} \rho \vec{c} \, dV$$

$$\left.\frac{D\vec{J}}{Dt}\right|_{J} = \left.\frac{D\vec{J}}{Dt}\right|_{B} + \vec{\Omega} \times \vec{J}$$

$$\Rightarrow \underbrace{\frac{D}{Dt}\left[\int_{V(t)} \rho \vec{c} \, dV\right]_{B}}_{RTT} + \vec{\Omega} \times \int_{V} \rho \vec{c} \, dV = \vec{F}$$

$$RTT: \quad \frac{D}{Dt}\int_{V(t)} \varphi \, dV = \int_{V} \frac{\partial \varphi}{\partial t} dV + \int_{S} \varphi (\vec{u} \cdot \vec{n}) \, dS$$

$$\boxed{\left[\frac{\partial}{\partial t}\int_V \rho \vec{\underline{c}}\, dV\right]_B + \int_S \rho \vec{\underline{c}}\, \underbrace{(\vec{\underline{w}}\cdot\vec{n})}_{\text{Impulsfluss}}\, dS + \ldots + \vec{\Omega}\times\int_V \rho\vec{c}\, dV = \vec{F}}$$

$$\vec{\underline{c}} = \vec{\underline{w}} + \vec{\underline{v}} + \vec{u}$$

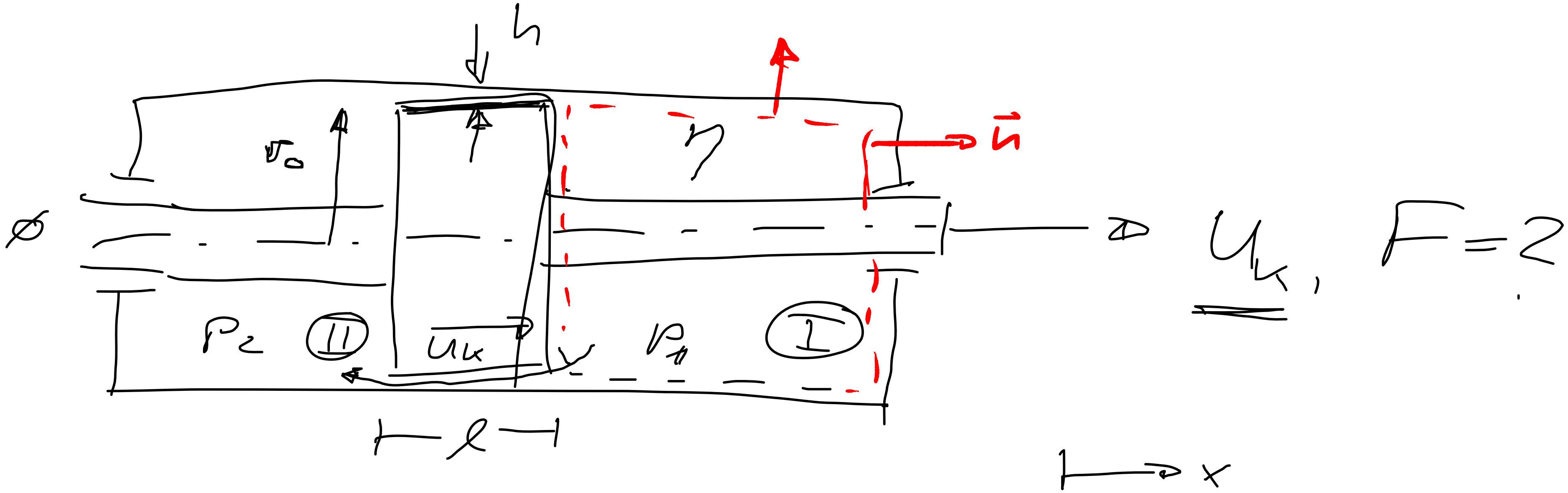
# Drallsatz im bewegten Koordinatensys.

$$\frac{D\vec{D}}{Dt} = \vec{M} \qquad \vec{D} = \int_{V(t)} \vec{x} \times (\rho \underline{\vec{c}})\, dV$$

$$\overset{RTT}{\Longrightarrow}$$

$$\boxed{\left[\frac{\partial}{\partial t} \int_V \vec{x} \times (\rho \vec{c})\, dV\right]_B + \int_S \vec{x} \times \rho \vec{c}\, (\underline{\vec{w}} \cdot \vec{n})\, dS + \vec{\Omega} \times \int_V \vec{x} \times (\rho \vec{c})\, dV = \vec{M}}$$

$$\boxed{\frac{D\varphi}{Dt} = \underline{\frac{\partial \varphi}{\partial t}} + \underline{\vec{u}\cdot\nabla\varphi}}$$




Durchmesser Kolbenstange vernachlässigbar

$$A_{Kolben}\, u_K = -A_{Spalt} \cdot \bar{u}$$

$$\Rightarrow \bar{u} = \frac{A_{Kolben}}{A_{Spalt}}\, u_K$$

$$A_{Kolben} = \pi r_0^2 \qquad A_{Spalt} = \underbrace{2\pi r_0 h}_{U \cdot h}$$

I $\quad \bar{u} = -\frac{r_0}{2h} u_K$

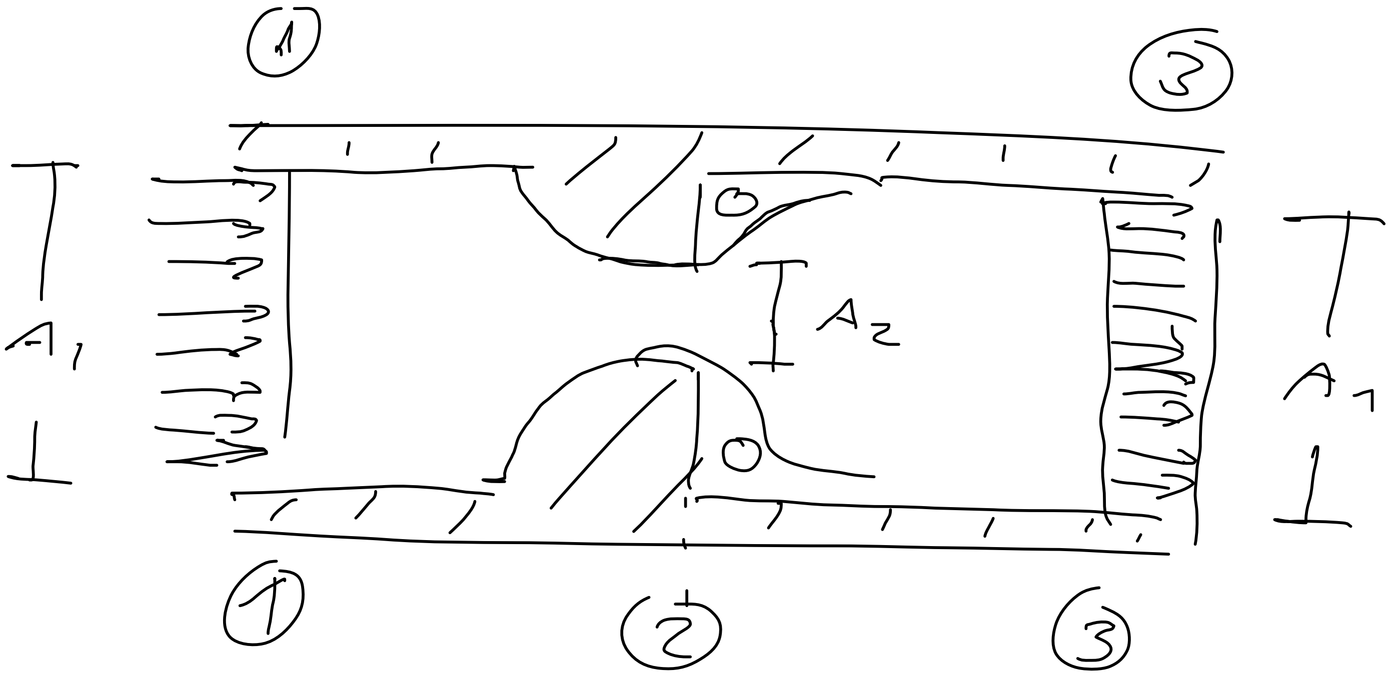
II $\quad \underline{\bar{u} = \frac{1}{6} \frac{\tau_w h}{\eta}}$ Spalt der Höhe $h$

III $\quad \underline{\tau_w = \frac{(p_2 - p_1)}{2l} h}$

I, II, III $\rightarrow$ $\underline{p_2 - p_1 = 6 \frac{r_0}{h^3} l \eta u_K}$

$$\Rightarrow \boxed{F = \Delta p \cdot A_K = 6\pi l \eta \left(\frac{r_0}{h}\right)^3 u_K}$$

# Bernoulli mit Verlusten




gegeben

$\rho = \text{const}$

$p_1, u_1, A_1, A_2$

gesucht:

1) Druckverlust $\Delta p_V$

2) Druck $p_3$

3) $\vec{F}_{Fl \to Strecke}$

1) $\Delta p_{VC} = \frac{\rho}{2}\left(\underline{u_2} - \underline{u_3}\right)^2$

Konti : $u_1 A_1 = u_3 A_1 = u_2 A_2$

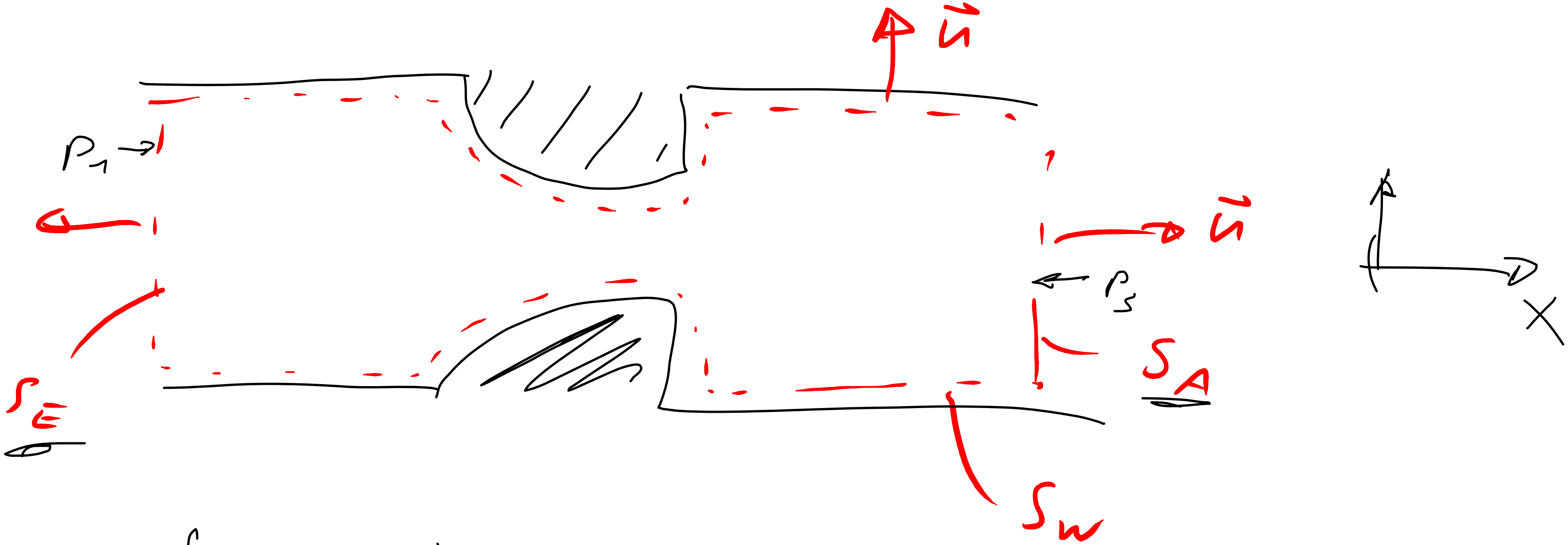
$\Rightarrow \Delta p_{VC} = \frac{\rho}{2} u_1^2 \left(\frac{A_1}{A_2} - 1\right)^2$

2) Bernoulli 1 $\rightarrow$ 3

$p_1 + \cancel{\frac{\rho}{2} u_1^2} + \cancel{\underline{\psi_1}} = p_3 + \cancel{\frac{\rho}{2} u_3^2} + \cancel{\underline{\psi_3}} + \Delta p_{VC}$

$\Rightarrow p_3 = p_1 - \Delta p_{VC} = p_1 - \frac{\rho}{2} u_1^2 \left(\frac{A_1}{A_2} - 1\right)^2$

# Kraft: Impulssatz




$$\int_S \rho \vec{u} \cdot \vec{e}_x (\vec{u} \cdot \vec{n})\, dS = \vec{F} \cdot \vec{e}_x$$

$$S_W : \quad \vec{u} \cdot \vec{n} = 0$$

$$S_E: \quad \vec{u} \cdot \vec{e}_x = u_1 \qquad \vec{u} \cdot \vec{n} = -u_1$$

$$S_A: \quad \vec{u} \cdot e_x = u_3 \qquad \vec{u} \cdot \vec{n} = u_3$$

$$\underline{-\rho u_1^2 A_1 + \rho u_3^2 A_1} = \underline{\vec{F}_{\to Fl} \cdot \vec{e}_x}$$

$$\underline{\vec{F} \cdot \vec{e}_x} = p_1 A_1 - p_3 A_1 + \underbrace{\vec{F}_{w \to Fl} \cdot \vec{e}_x}_{-\vec{F}_{Fl \to w} \cdot \vec{e}_x}$$

$$\Rightarrow \vec{F}_{Fl \to w} \cdot \vec{e}_x = \frac{\rho}{2} u_1^2 A_1 \left( \frac{A_1}{A_2} - 1 \right)^2$$

$$= \Delta p_{vc} A_1$$