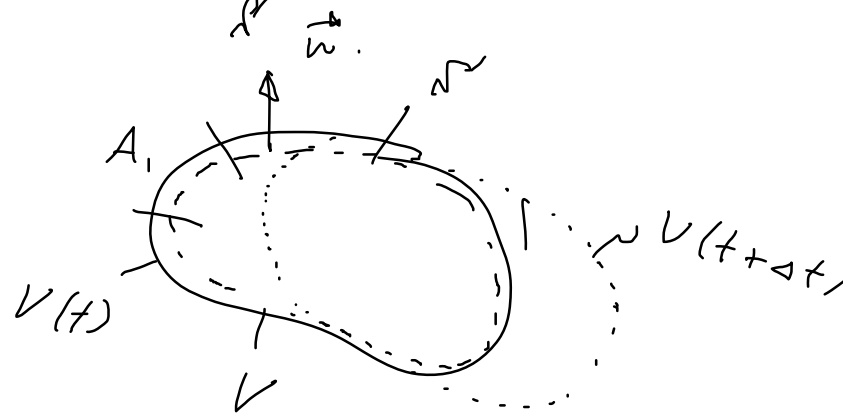


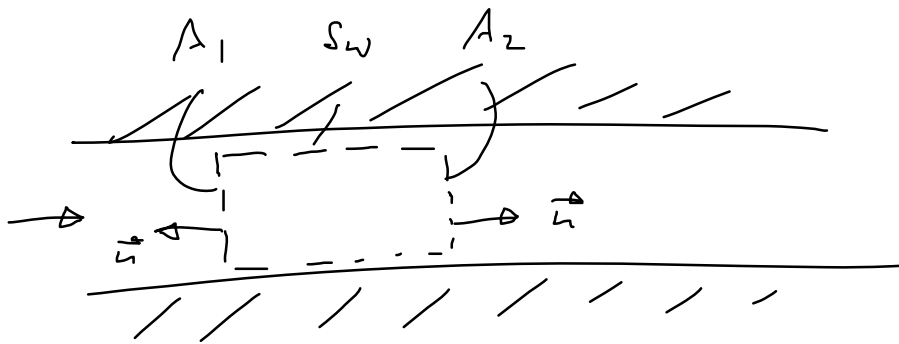
Kont. Gleichung

$$\frac{Dm}{Dt} = \frac{\partial}{\partial t} \int_V \rho dV + \oint \rho \vec{u} \cdot \vec{n} dS' = 0.$$



Navier-Stokes

$$\rho \vec{u}_1 = \left( \int_{A_1} \rho \vec{u} \cdot \vec{n} dS' \right)$$



$$\frac{Dm}{Dt} = 0.$$

stationäre Strömung  $\frac{\partial}{\partial t} \equiv 0$ .

$$0 = \underbrace{\int_{A_1} \rho \vec{v} \cdot \vec{n} dA}_{-m} + \underbrace{\int_{A_2} \rho \vec{v} \cdot \vec{n} dA}_{+m} + \int_{S_W} \rho \vec{v} \cdot \vec{n} dA \stackrel{\equiv 0}{=} 0 \text{ an der Wand.}$$

$$-m + m = 0 \quad \checkmark$$

## Bernoullische Gleichg.

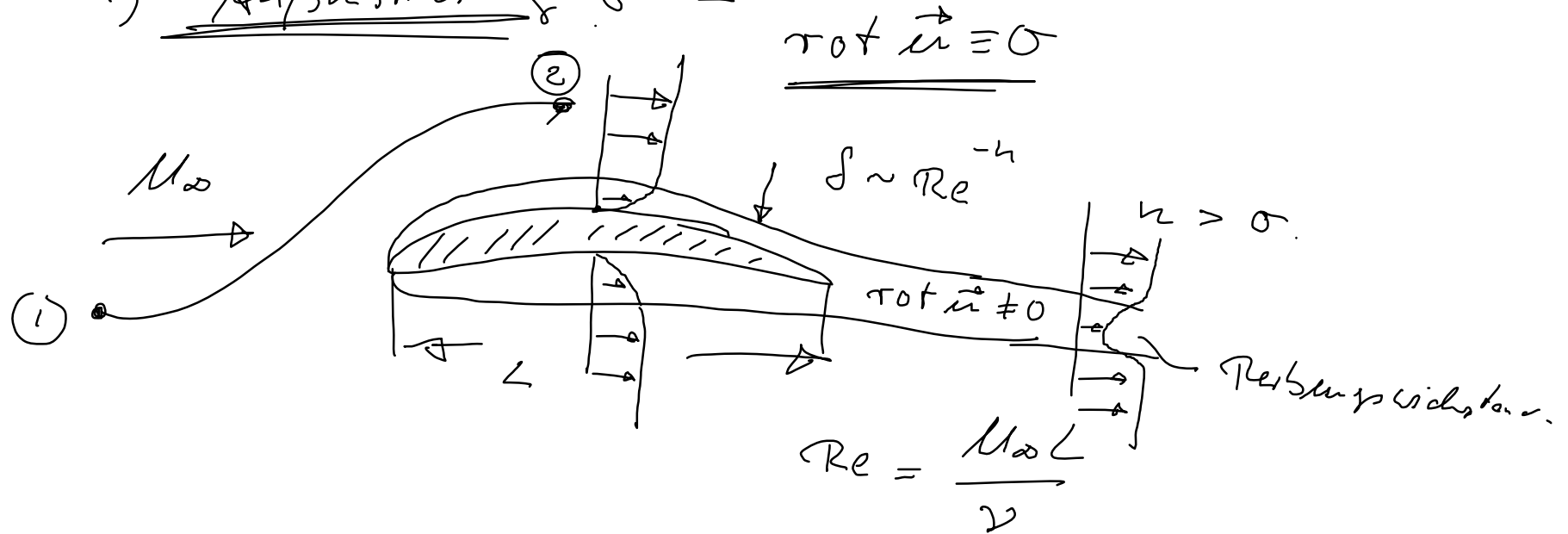
David Bernoulli \* 1700 in Groning  
† 1782 in Petersburg.

Fred

Leonard Euler \* 1702 in Basel.  
† 1783 " "

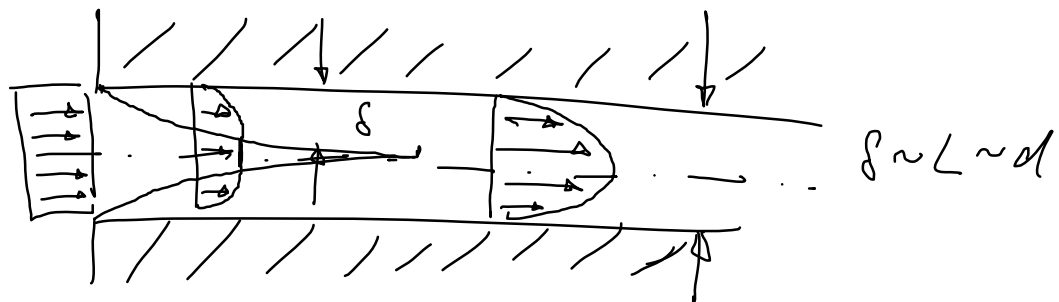
An welche Stellen in einer Lösung muss Reibung  
beachtet werden.

a) Außenströmung  $\delta \ll L$



$\leadsto$  Häufig ist es ausreichend  
Reibung nicht zu berücksichtigen

b) Innenströmung  
 $\delta \sim L$



# Bernoulli's Gleichung

mechanische Energie.  
↳ keine Verluste.

$$P + \frac{\rho}{2} u^2 + \psi = C$$

$P$  statische Druck

$C$  Bernoulli'sche Konstante.

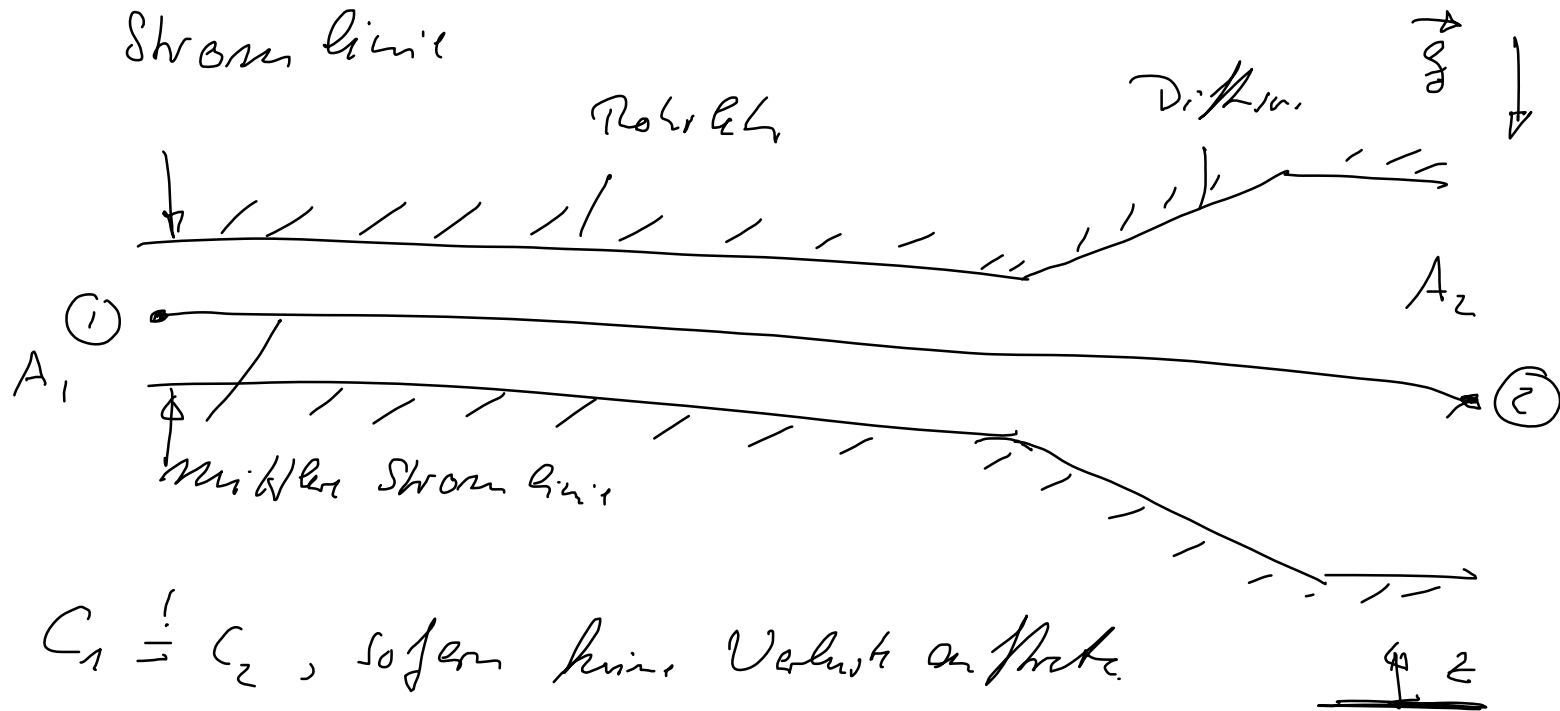
$\rho$  Dichte

$\frac{\rho}{2} u^2$  Staudruck

$\psi$  Potential der Volumenkräfte

$C'$  ändert sich nicht mit längs einer

Stromlinie



$C_1 \stackrel{!}{=} C_2$ , sofern keine Verluste an Probe.

$$P_1 + \frac{\rho}{2} u_1^2 + \psi_1 = P_2 + \frac{\rho}{2} u_2^2 + \psi_2$$

$$\psi_1 = \psi_2 \quad P_1 + \frac{\rho}{2} u_1^2 = P_2 + \frac{\rho}{2} u_2^2$$

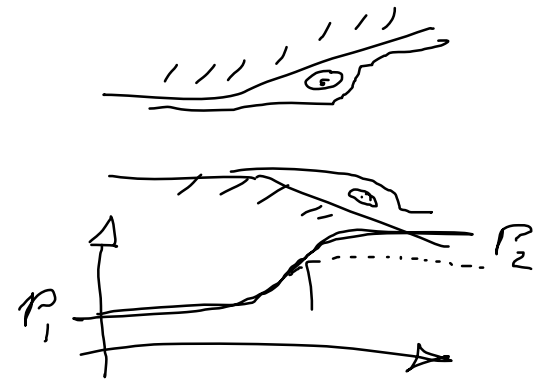
$$P_2 - P_1 = \frac{\rho}{2} \mu_1^2 \left( 1 - \left( \frac{\mu_2}{\mu_1} \right)^2 \right)$$

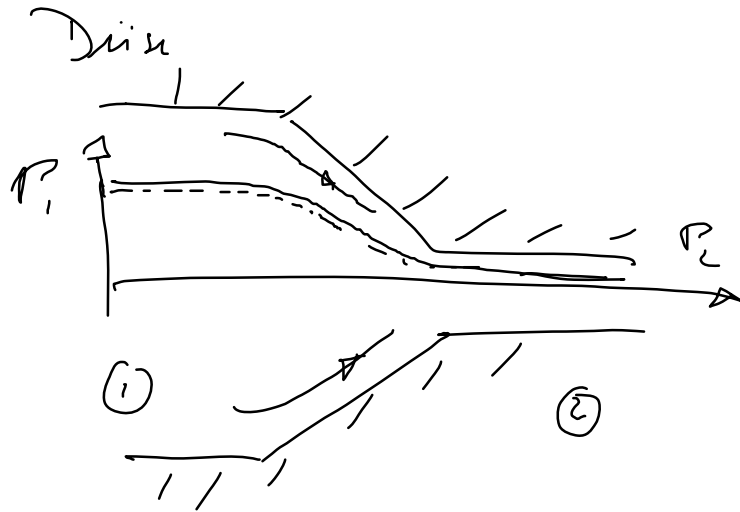
Kontin.  $v_1 = v_2$

$$\rho \mu_1 A_1 = \rho \mu_2 A_2 \quad \text{und} \quad \frac{\mu_2}{\mu_1} = \frac{A_1}{A_2}$$

$$P_2 - P_1 = \frac{\rho}{2} \mu_1^2 \left( 1 - \left( \frac{A_1}{A_2} \right)^2 \right) > 0$$

ideale Durchströmung im Diffuser.





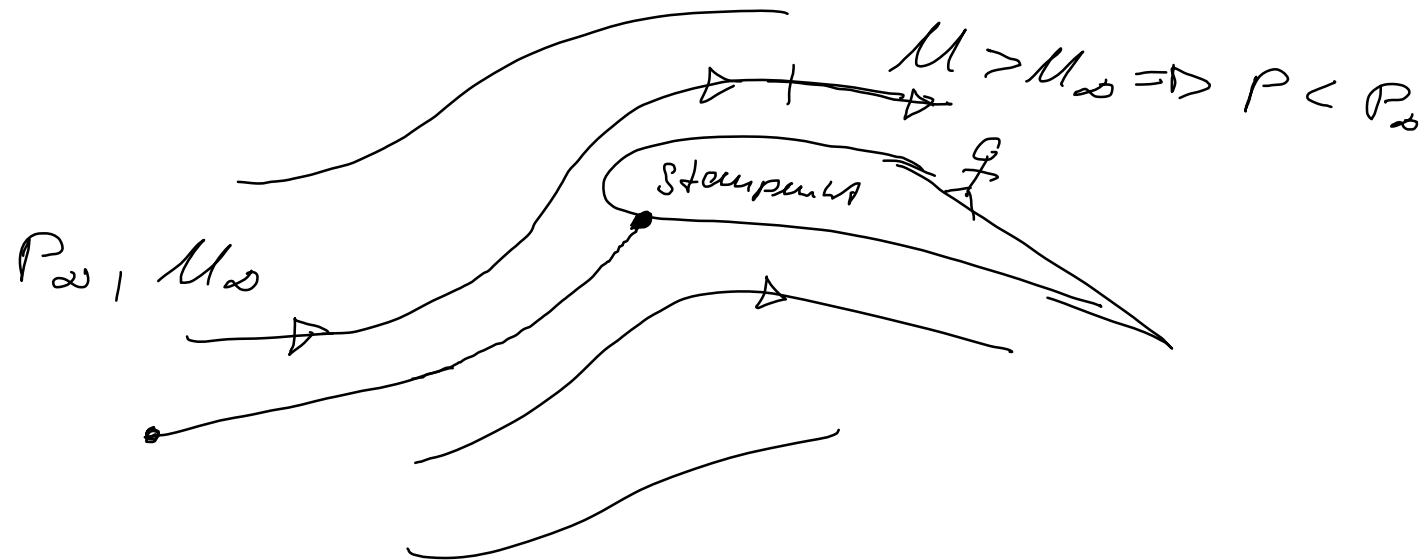
$$P_1 > P_2$$

$$P_1 - P_2 = \frac{\rho}{2} u_1^2 \left( 1 - \left( \frac{A_2}{A_1} \right)^2 \right)$$



# Definitione und Begriff.

Gesamtdruck  $P_g = P + \frac{\rho}{2} u^2$



Druck im Staupunkt  $P_\infty + \frac{\rho}{2} u_\infty^2$

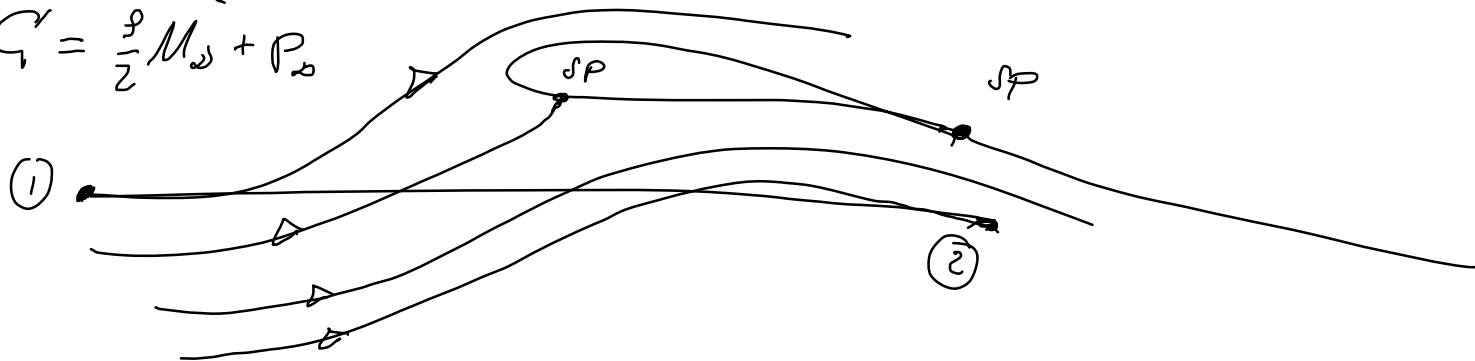
piezometrischer Druck  $p^* = p + \psi$

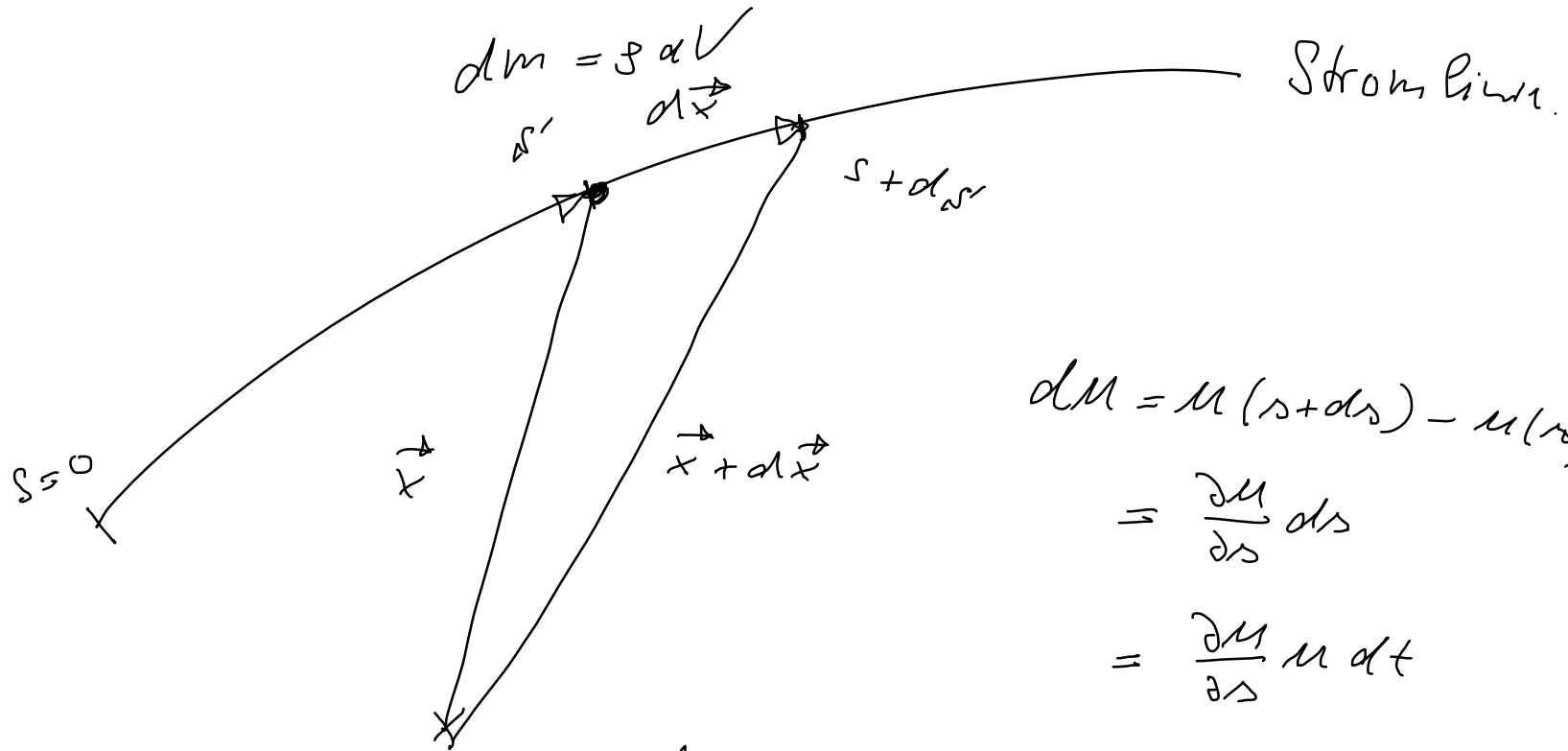
---

Bernoulli gilt längs einer Stromlinie.

$\text{rot } \vec{u} \equiv 0$   $\leadsto$  Bernoulli gilt auch  
für zu der Stromlinie.

$$C = \frac{\rho}{2} M^2 + P_2$$





$$dM = M(s+ds) - M(s)$$

$$= \frac{\partial M}{\partial s} ds$$

$$= \frac{\partial M}{\partial s} u dt$$

$$a_s = \frac{\partial M}{\partial s} u = \frac{dM}{dt}$$

Bestl.

Länge der  
Stromlinie

Masse  $\times$  Beschleunigung = Kraft.

Newton.

$$\rho dV \frac{\partial M}{\partial s} u = - \frac{\partial P}{\partial s} dV + \int_{\partial V} dV$$

Einheit.

$$\rho \frac{\partial M}{\partial s} u = - \frac{\partial P}{\partial s} + \int_{\partial V}$$

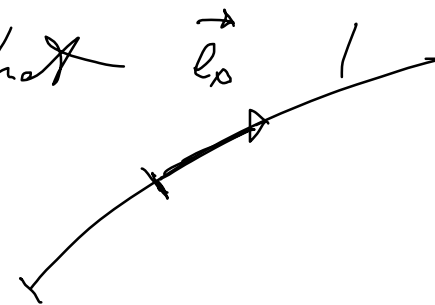
$$\frac{\partial \mathcal{L}}{\partial \dot{\alpha}} \dot{\alpha} = \frac{1}{2} \frac{\partial}{\partial \dot{\alpha}} (\dot{\alpha}^2) \quad \text{Identität (Leibnizregel)}$$

$$\frac{\partial}{\partial \dot{\alpha}} \left( \frac{\rho}{2} \dot{\alpha}^2 \right) = - \frac{\partial \mathcal{P}}{\partial \dot{\alpha}} + \mathcal{J}_{\dot{\alpha}}$$

Strom  $\vec{e}_s$ :

Sofern die Volumenleiste ein Potential hat  $\vec{e}_s$

$$\vec{\mathcal{J}} = -\nabla \Psi$$

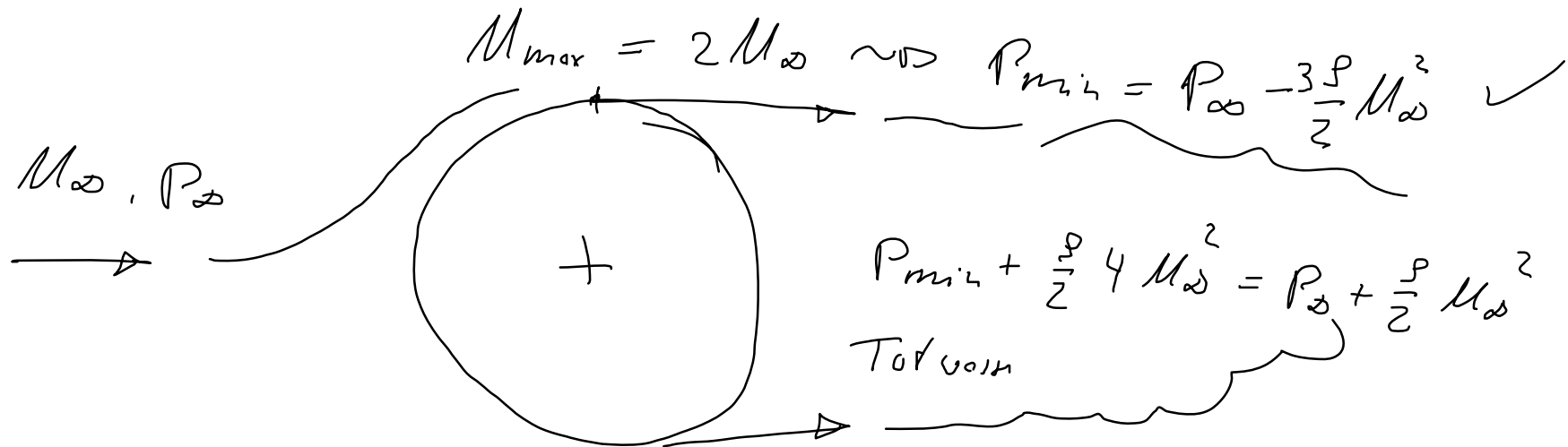


$$\mathcal{J}_{\dot{\alpha}} = \vec{\mathcal{J}} \cdot \vec{e}_s = -\nabla \Psi \cdot \vec{e}_s = - \frac{\partial \Psi}{\partial \dot{\alpha}}$$

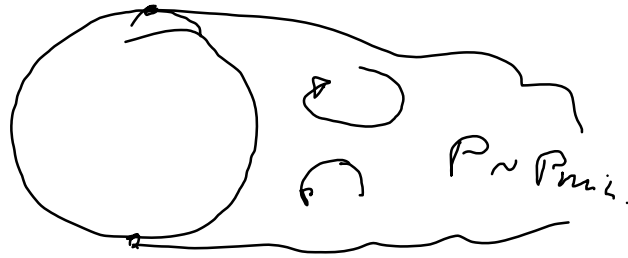
$$\frac{\partial}{\partial \dot{\alpha}} \left( \frac{\rho}{2} \dot{\alpha}^2 + \mathcal{P} + \Psi \right) = 0.$$

Integrationskonstante  $\zeta'$

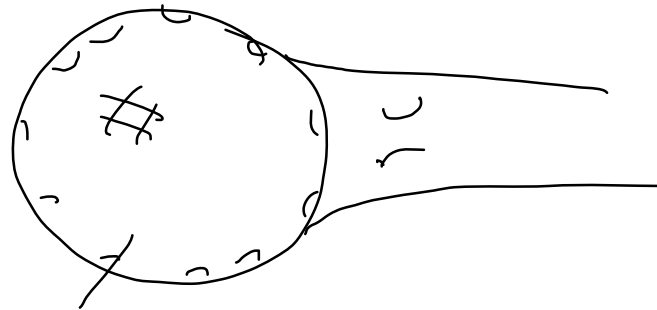
$$\frac{\rho}{2} u^2 + p + \psi = \zeta'$$



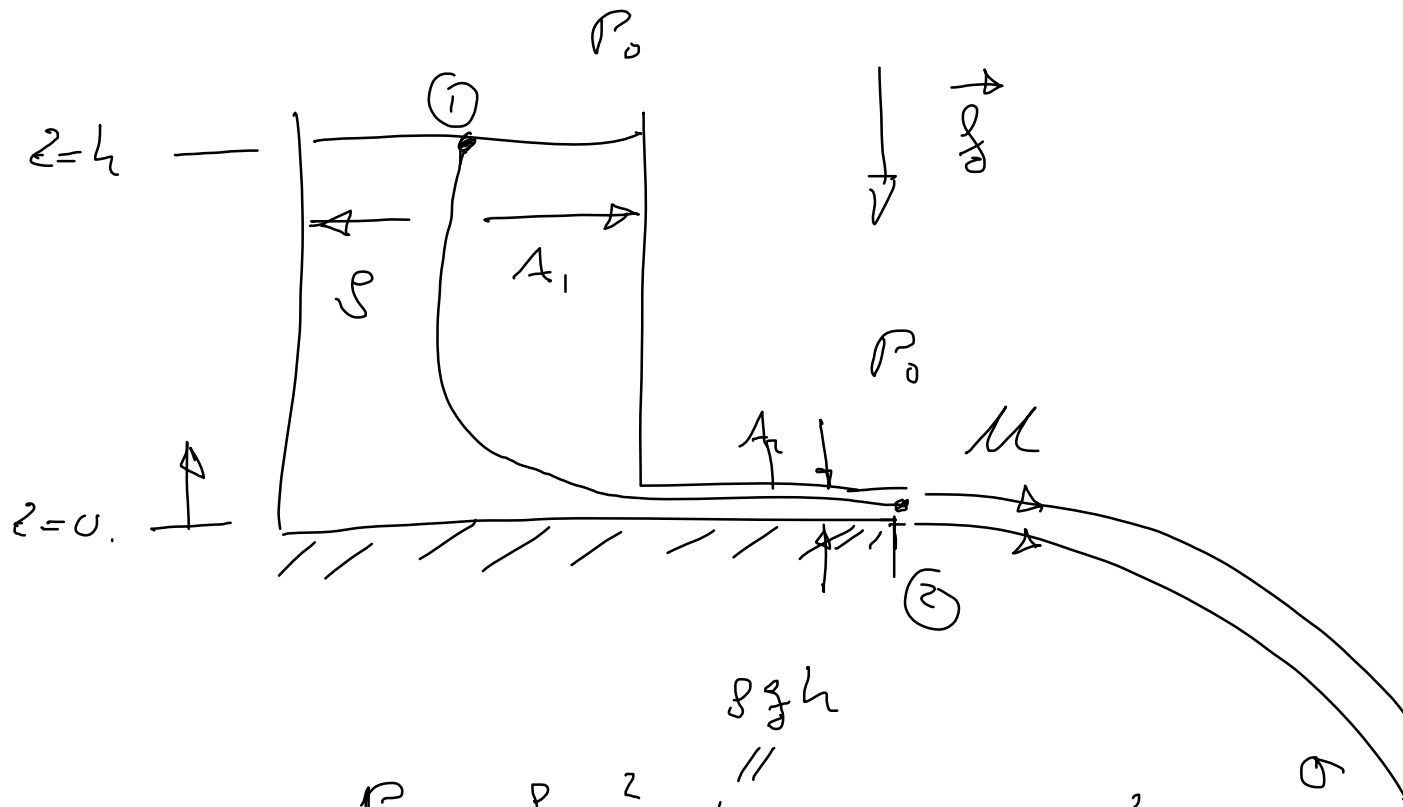
$U_2$   
→



→



Golgi body.



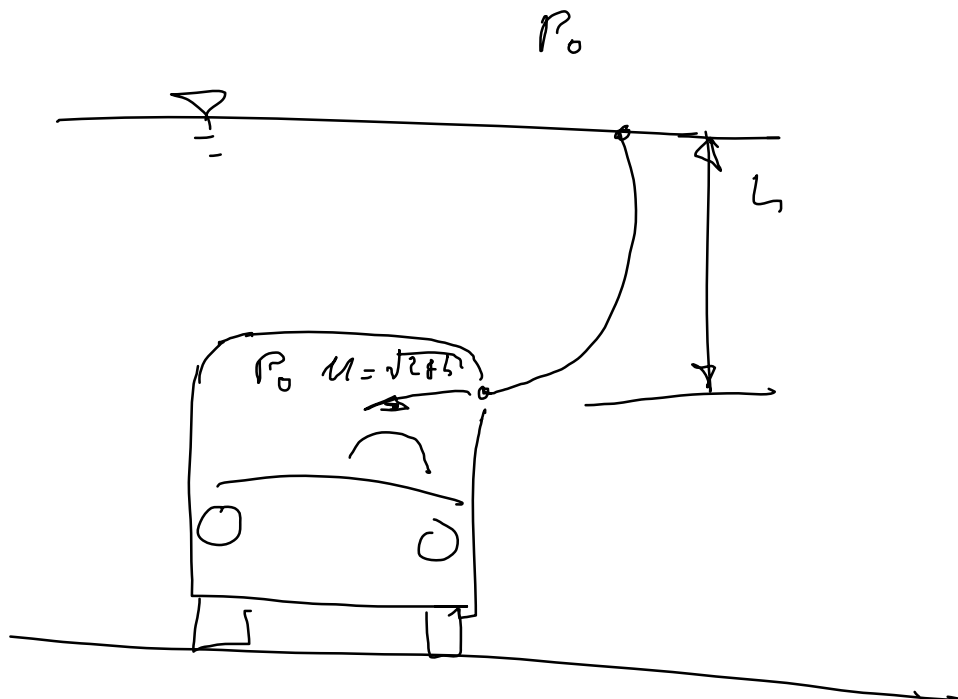
$$P_1 + \frac{\rho}{2} v_1^2 + \psi_1 = P_2 + \frac{\rho}{2} u_2^2 + \psi_2$$

$\parallel$   $\approx 0$   $\parallel$   
 $P_0$   $\approx 0$   $P_0$   
 wenn  
 $A_1 \gg A_2$

Druck im Strahl = Umgebungsdruck.  
 sofern  $u \ll a$

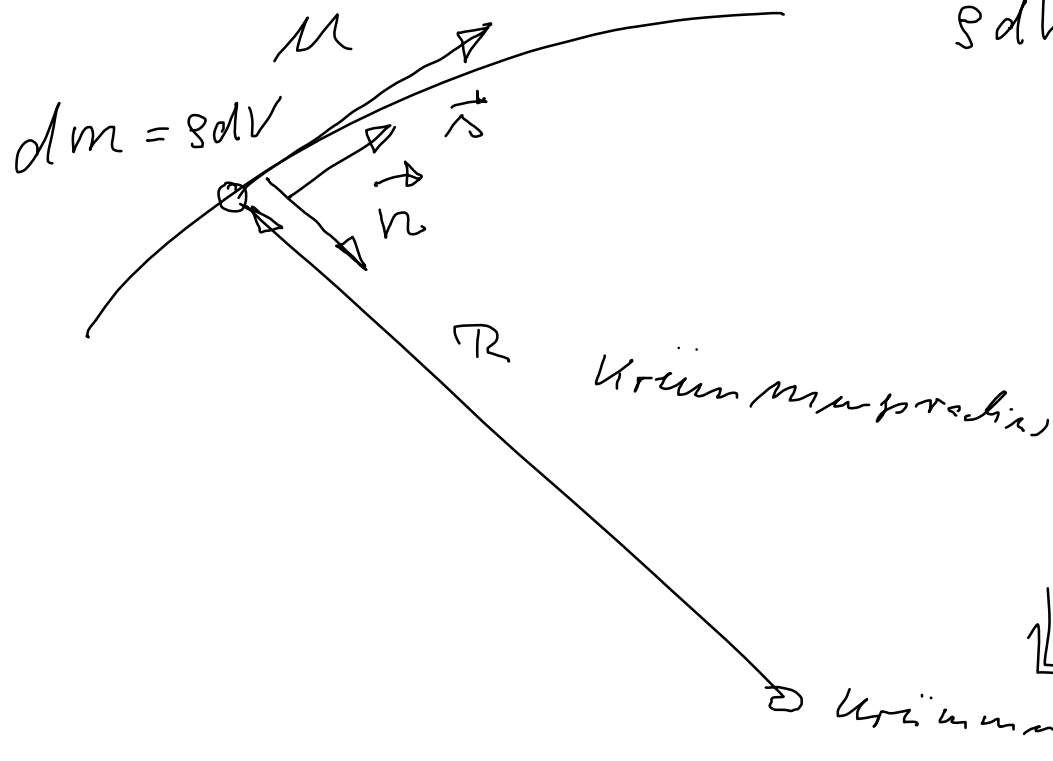
$$\leadsto u = \sqrt{2gh}$$

Torricellische  
Ausflußgeschw.







Durchänderung Sekunde zu Stromlinie



$$\begin{aligned} \rho dV \frac{v^2}{R} &= - \frac{\partial p}{\partial n} dV + \int \rho g h dV \\ &= - \frac{\partial}{\partial n} (p + \rho \psi) dV \\ &= - \frac{\partial P^*}{\partial n} dV \end{aligned}$$

$$\frac{\partial P^*}{\partial n} = - \rho \frac{v^2}{R}$$

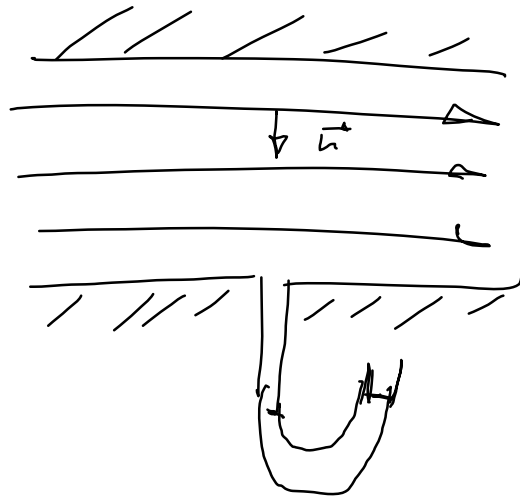
$\Rightarrow$  Im Freistrah (  ) und im Kapillarrohr (  )

sind die Stromlinien in der Düse nicht  
gleichmäßig ( $\Omega \rightarrow \partial\Omega$ )

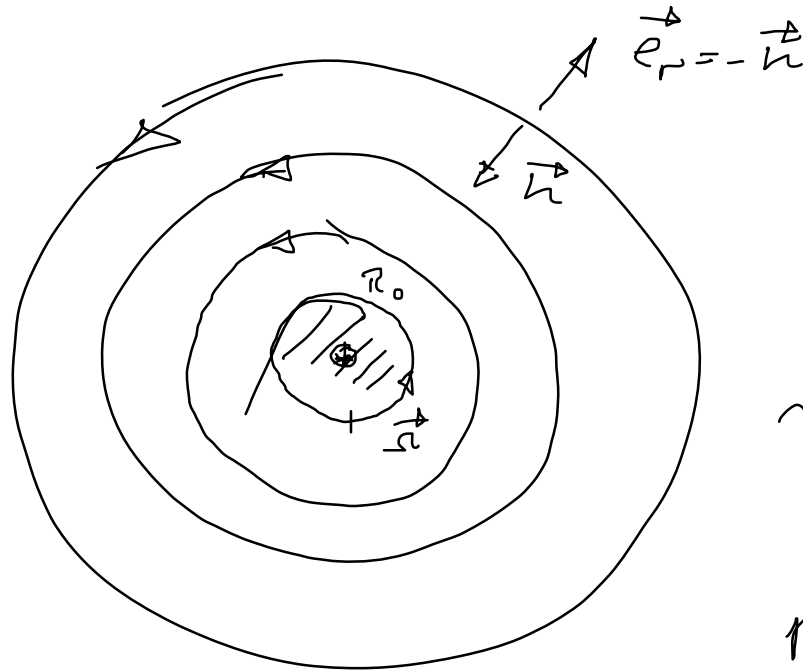
$$\leadsto \frac{\partial p}{\partial n} \equiv 0$$

$\leadsto$  homogener Druck über der Drehzylinder.  
( $\mu \ll a$ ).

---



$\frac{\partial p}{\partial n} \equiv 0$ .  $\leadsto$  homogener Druck  
über dem  
Rotationszylinder.



$$\frac{dP}{dr} = - \frac{dP}{dr} = -\rho \frac{u^2}{r}$$

$$\leadsto \frac{dP}{dr} = \rho \frac{u(r)^2}{r}$$

$$p(r) = p(R_0) - \rho \frac{\Omega^2 R_0^2}{2} \left( 1 - \left( \frac{R_0}{r} \right)^2 \right)$$

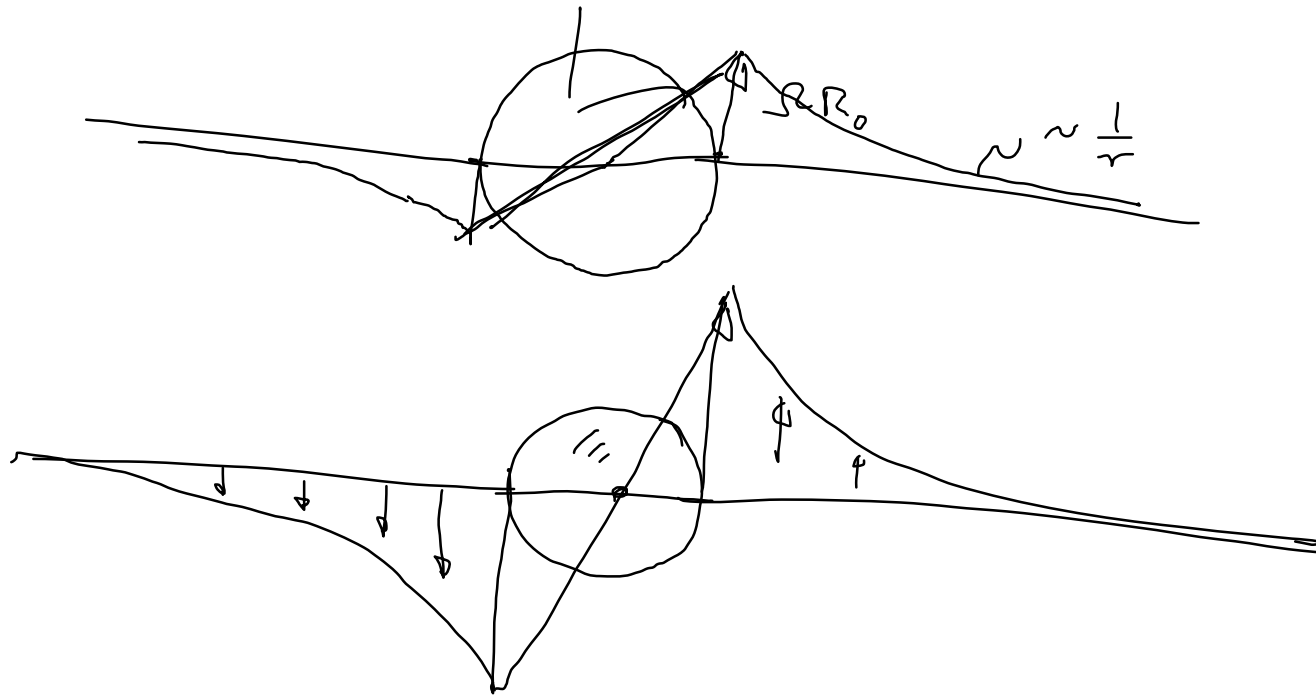
$\leadsto$  Druckverteilung im inneren  
Potentialwirbel.

$$u(r) = \Omega R_0 \frac{R_0}{r}$$

$$\text{rot } \vec{u} \equiv 0.$$

Geschwindigkeit  $u$  ist pot  
 lines Potential wirbels.

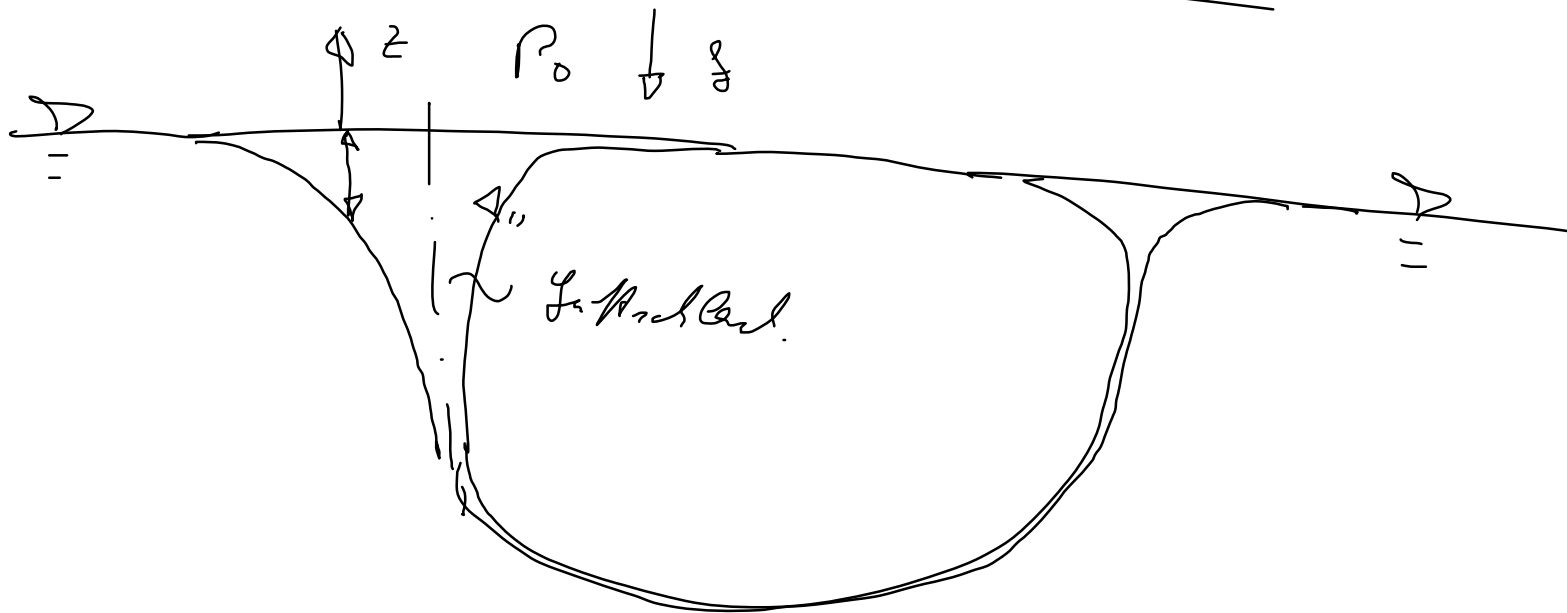
Wirbelkern



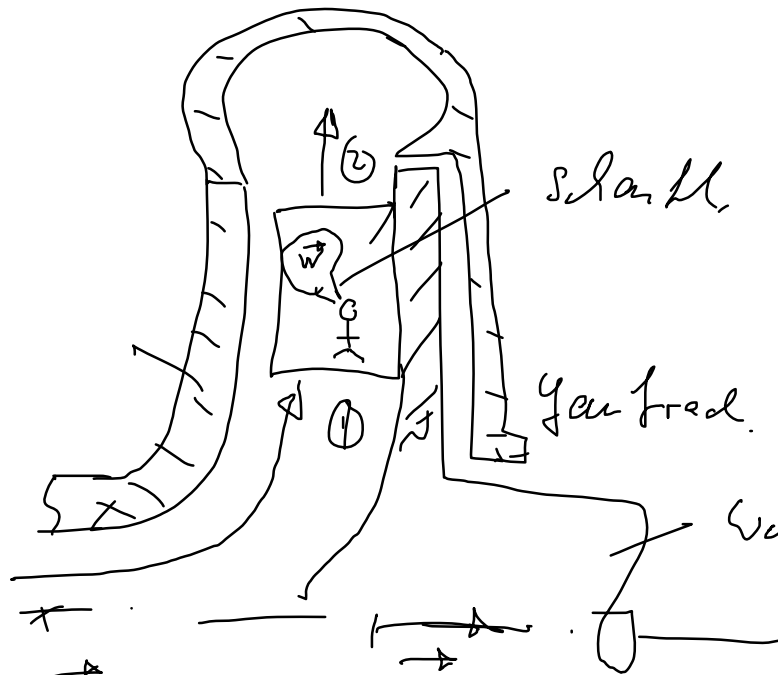
$$p(r_0) + \frac{\rho}{2} (\Omega r_0)^2 = p(r) + \frac{\rho}{2} \omega^2 r^2 \quad r \geq r_0$$

$$= p(r) + \frac{\rho}{2} (\Omega r_0)^2 \left(\frac{r_0}{r}\right)^2$$

$$p(r) = p(r_0) + \frac{\rho}{2} (\Omega r_0)^2 \left[ 1 - \left(\frac{r_0}{r}\right)^2 \right]$$



# Bernoulli im rotierenden System



im rotierende System. / (Gasfreed.)

→ stationäre Strömung

Well. → Trägheitskräfte

- Zentrifugalkraft
- Corioliskraft

$\vec{u}$  Führungsgeschw.  
 $\vec{w}$  relative Geschwindigkeit

$\vec{c}$  absolute Geschwindigkeit

$\vec{u}$  Umlaufgeschw.  $\vec{u} = \vec{\Omega} \times \vec{x}$

$$\vec{c} = \vec{w} + \vec{u} + \vec{u}$$

Bei Turbomaschine  $\vec{u} \equiv 0$ .

$$\vec{c} = \vec{w} + \vec{u}$$

$$\frac{\rho}{2} w^2 + p + \psi = c$$

$$\psi = \underline{\underline{\rho \frac{1}{2} (\Omega r)^2}} \left( + \rho g z + \dots \right)$$

$$\vec{f} = \rho \vec{\Omega} \times (\vec{\Omega} \times \vec{x})$$

$$\psi = -\frac{\rho}{2} |\vec{\Omega} \times \vec{x}|^2$$

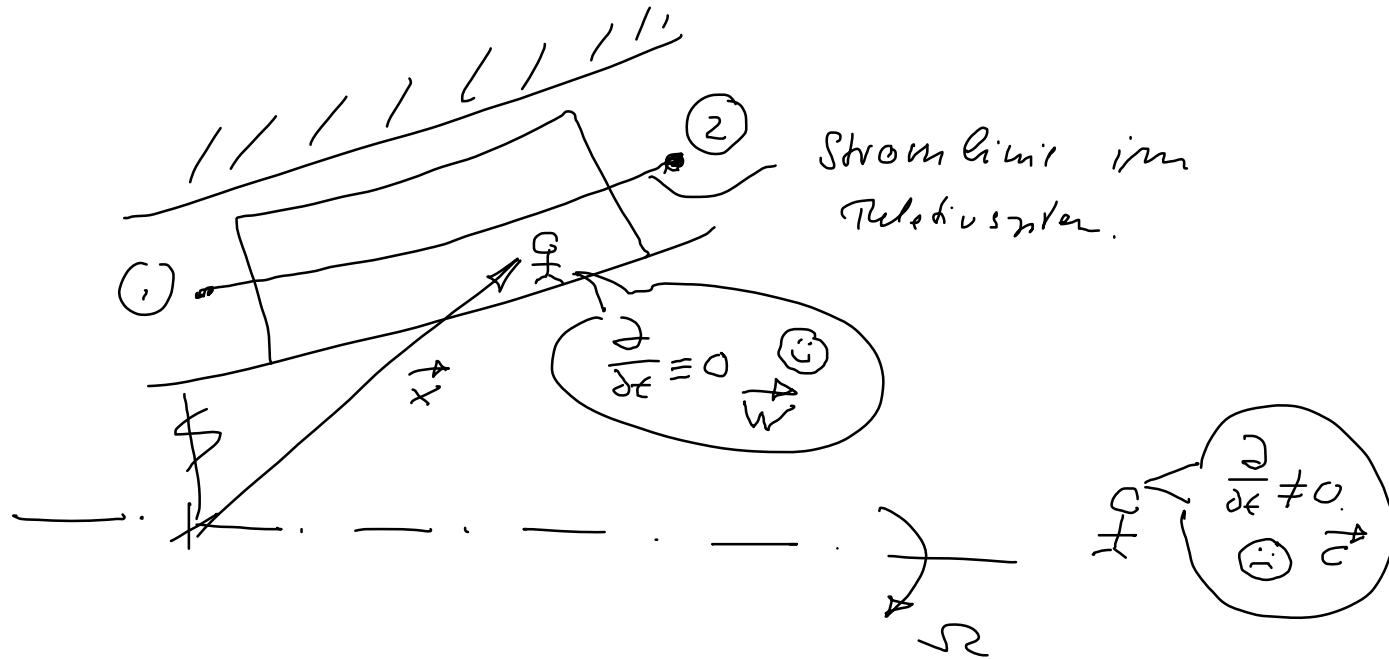
$$\nabla \psi = -\vec{f}$$

$$\frac{\rho}{2} \omega_1^2 + P_1 = \frac{\rho}{2} (\Omega r)_1^2 = \frac{\rho}{2} \omega_2^2 + P_2 = \frac{\rho}{2} (\Omega r)_2^2$$

Bemerk: im rotierenden System.



# Bernoullische Gleichung im rotierenden System



$$\vec{c} = \vec{w} + \vec{\Omega} \times \vec{r}$$

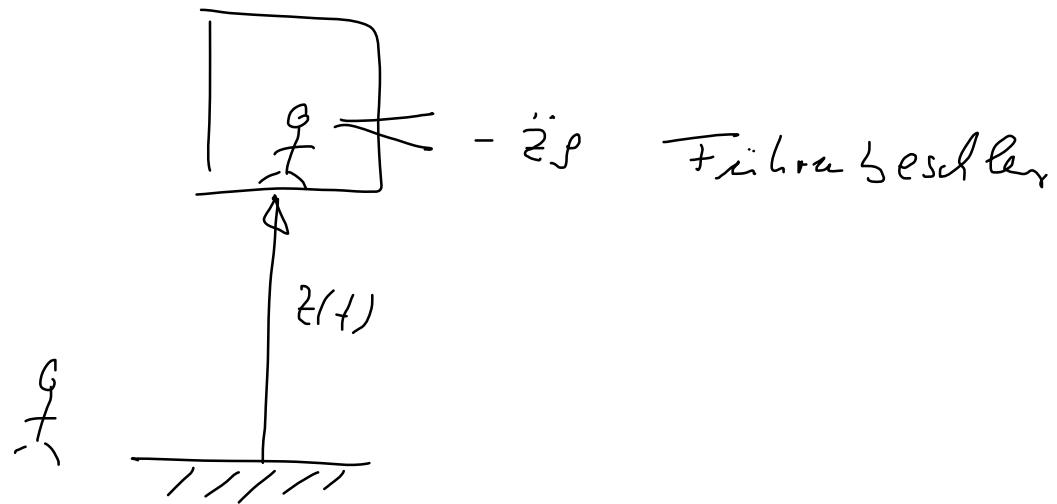
$$= \vec{w} + \vec{u}$$

$$P_1 + \frac{\rho}{2} w_1^2 - \frac{\rho}{2} (\Omega r_1)^2 =$$

$$= P_2 + \frac{\rho}{2} w_2^2 - \frac{\rho}{2} (\Omega r_2)^2$$

$$- \frac{\rho}{2} (\Omega r)^2 = \psi \text{ Potential der Zentrifugalkraft.}$$

- $\vec{c}$  Absolutgeschw.
- $\vec{w}$  Relativgeschw.
- $\vec{u}$  Mitführungsgeschw.



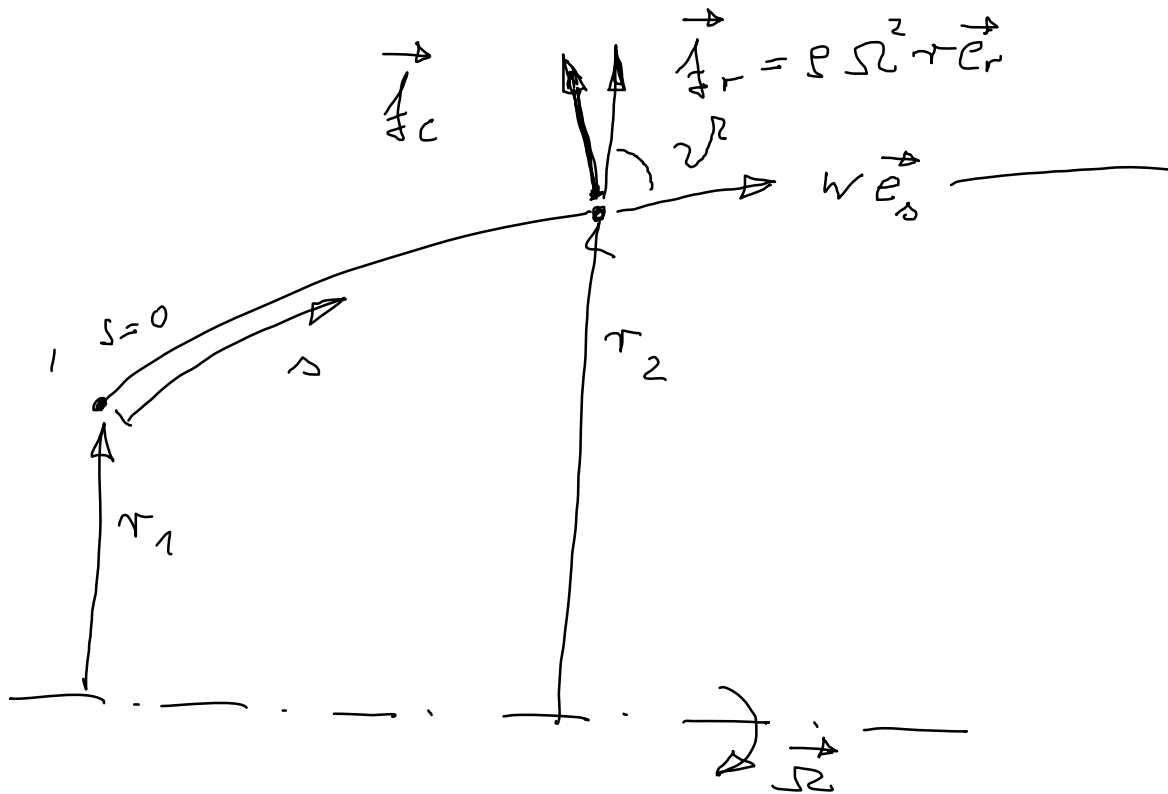
$$\vec{f}_r = \rho \Omega^2 r \vec{e}_r \quad \text{Volumenkraft der Zentrifugalkraft.$$

$$\vec{f}_c = 2\rho \vec{\Omega} \times \vec{w} \vec{e}_r$$

$\vec{w} = w \vec{e}_r$

Corioliskraft.

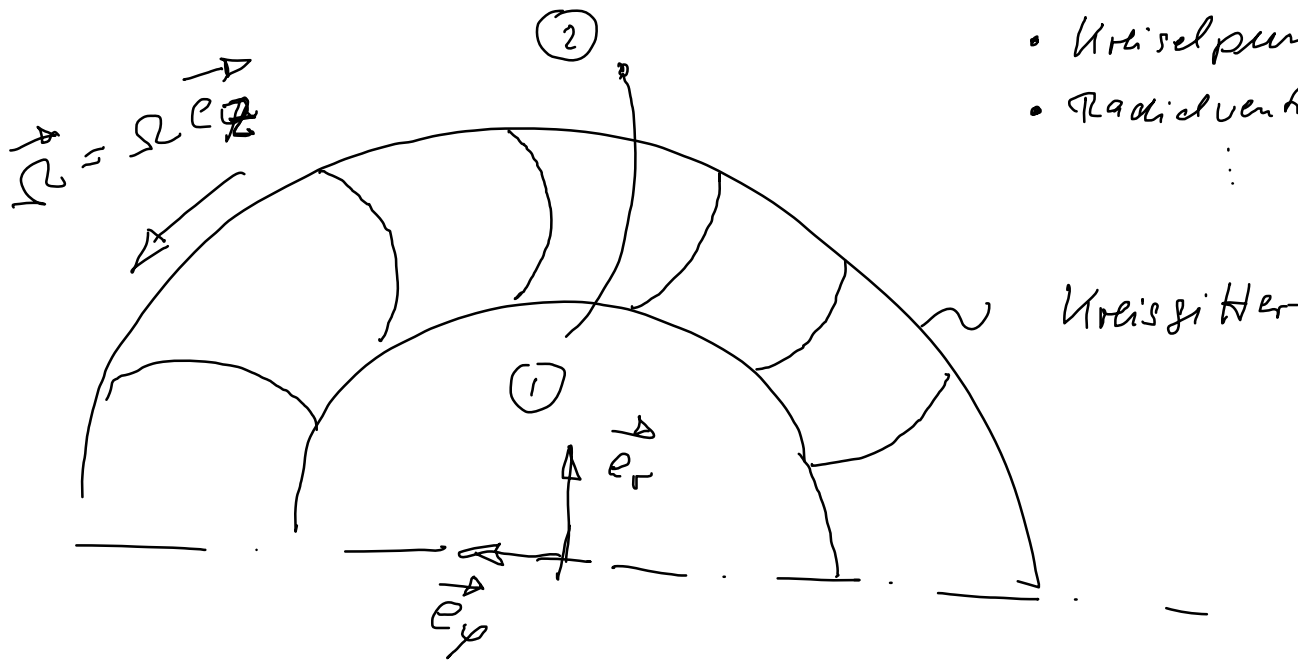




$$\rho w \frac{\partial w}{\partial s} = - \underbrace{\frac{\partial p}{\partial s} + \rho \Omega^2 r \cos \vartheta}_{\text{Kraft}}$$

Mass \* Beschl. = Kraft

$$\left[ \frac{\rho}{2} w^2 + p \right]_1^2 = \int_1^2 \underbrace{\rho \Omega^2 r \cos \vartheta}_{\frac{dr}{ds}} ds = \frac{\rho \Omega^2}{2} (r_2^2 - r_1^2)$$

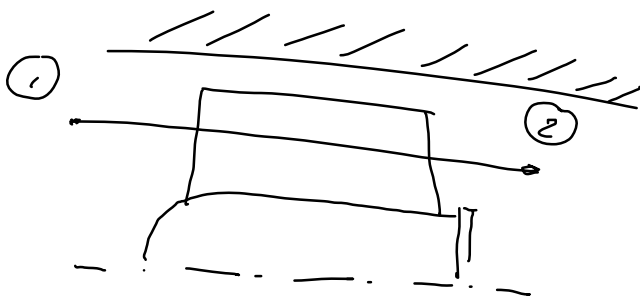


- Kreiselpumpe (Arbeitsmaschine)
- Radialventil. ( " )

Kreisgitter

Radialmaschine: radial Durchströmung der Maschine.

Axialmaschine: axial Durchstr. " "



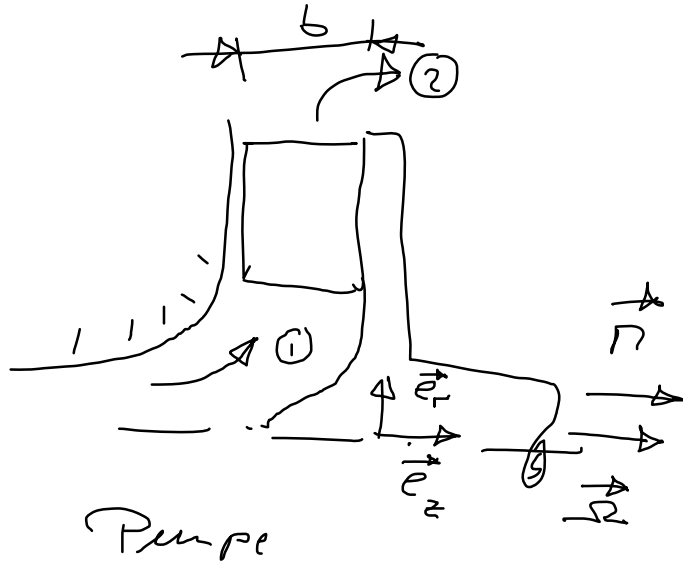
- Kaplan turbine (Kraftmaschine)
- Ventilatoren (Arbeitsm.)
- Schraubpropeller (Arbeitsm.)
- Windturbinen (Kraftm.)

$$P = \frac{dW}{dt} > 0$$

Arbeitsmaschine

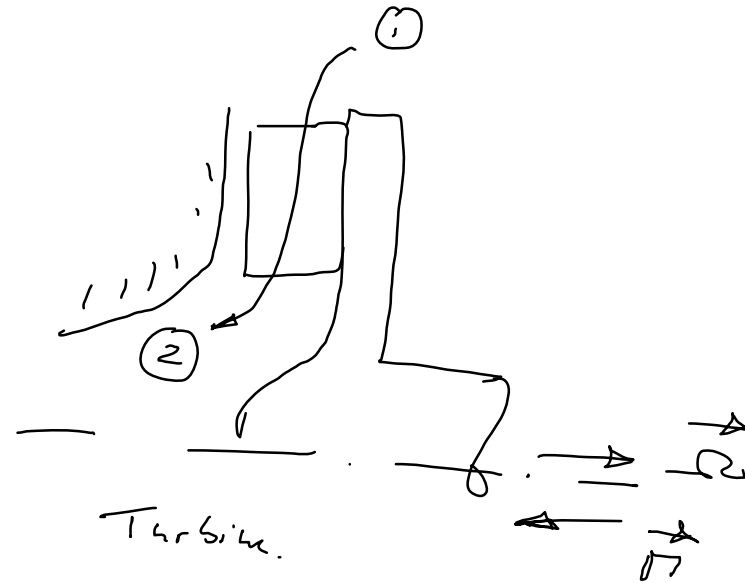
$$P = \frac{dW}{dt} < 0$$

Kraftmaschine.



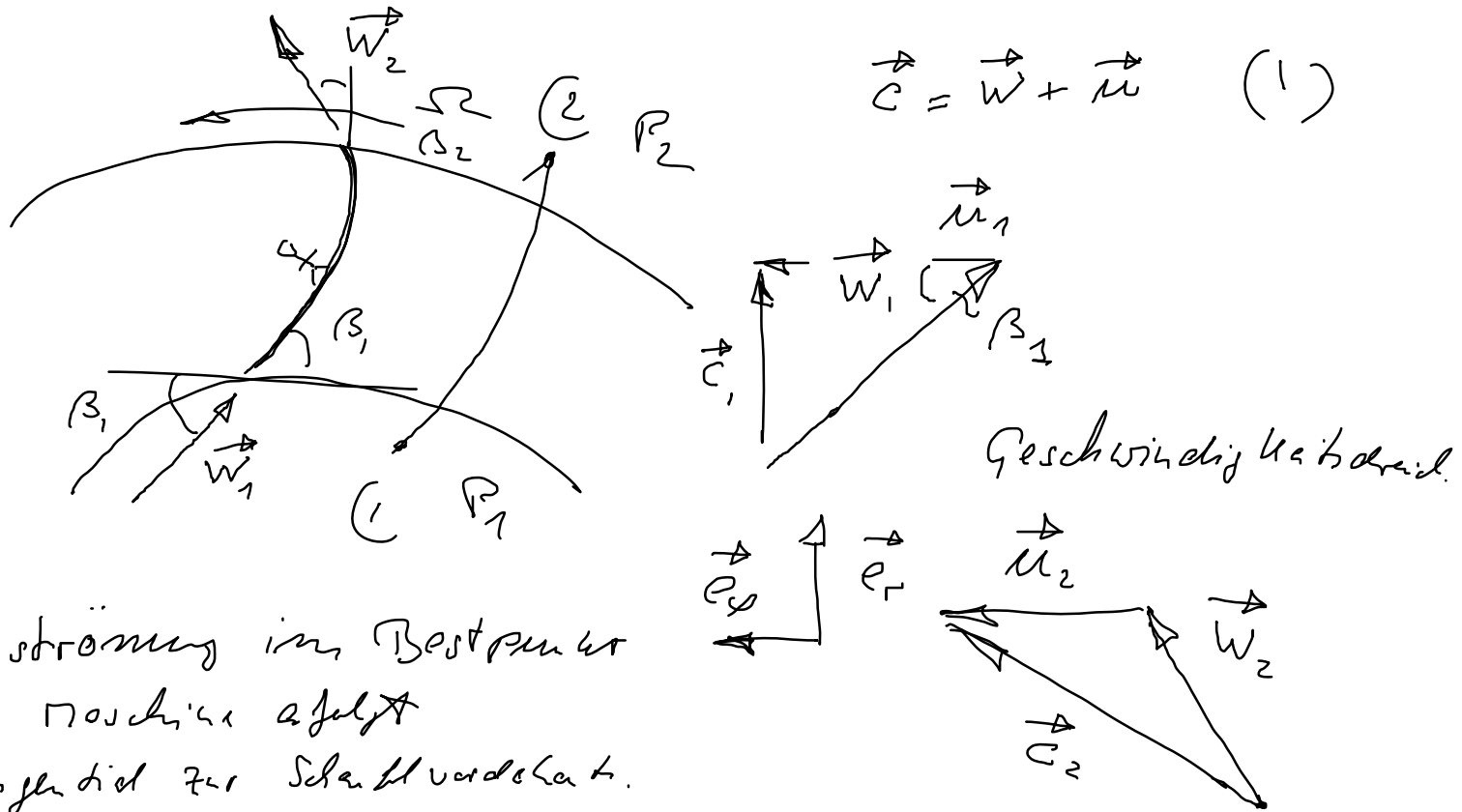
Pumpe

$$P = T \cdot \Omega > 0$$



Turbine.

$$P = T \cdot \Omega < 0$$



Anströmung im Bestpunkt der Maschine erfolgt tangential zur Schaufeloberfläche.

Sofern  $C_{1\varphi} C_{\varphi} \equiv 0$  spricht man von einer dreifach freien Strömung.

Drall  $C_{\varphi r}$  eine Flüssigkeitkegel = Umfangswgp. \* Radius.

## Schaufelkonstrukt Abströmung:

- Strömungsrichtung stimmt mit der Schaufelrichtung überein.
- Voraussetzung: Hohe Schaufelanzahl.

### 1. Bernoulli

$$P_1 + \frac{\rho}{2} w_1^2 - \frac{\rho}{2} (\Omega r_1)^2 = P_2 + \frac{\rho}{2} w_2^2 - \frac{\rho}{2} (\Omega r_2)^2$$

$$\frac{P_2 - P_1}{\rho/2} = w_1^2 - w_2^2 + \left( \frac{\Omega d_2}{2} \right)^2 \left[ 1 - \left( \frac{d_1}{d_2} \right)^2 \right]$$

### 2. Konti

$$\dot{V} = \underbrace{\pi d_1 b}_{\text{Ringfläch.}} \underbrace{w_1 \sin \beta_1}_{\text{Tangentialgeschw.-Komponente}} = \pi d_2 b w_2 \sin \beta_2$$

$$\frac{P_2 - P_1}{\left(\frac{\Omega d_2}{2}\right)^2 \frac{8}{2}} = \underbrace{\left(\frac{V/\pi d_1 b \sin \beta_1}{\Omega d_2 / 2}\right)^2}_{\psi} \left(1 - \left(\frac{d_1 \sin \beta_1}{d_2 \sin \beta_2}\right)^2\right) + 1 - \left(\frac{d_1}{d_2}\right)^2$$

$\psi$  Durchfluss.

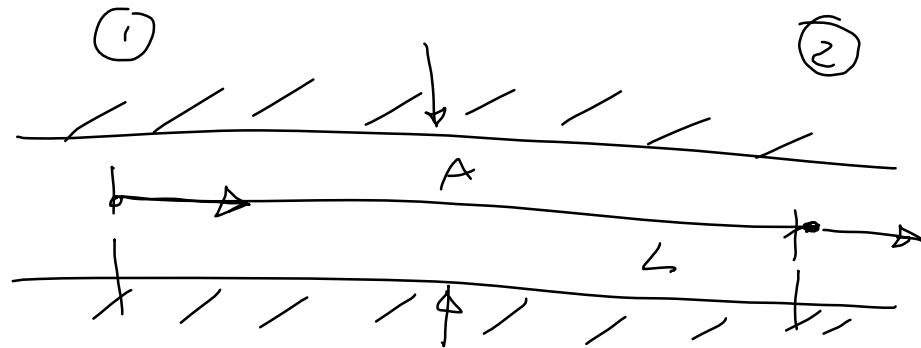
$\psi$  Durchflusszahl.

$$\psi = \psi(\psi, d_1/d_2, \beta_1, \beta_2)$$





Bernoullische Gleichung für instationäre Strömung.



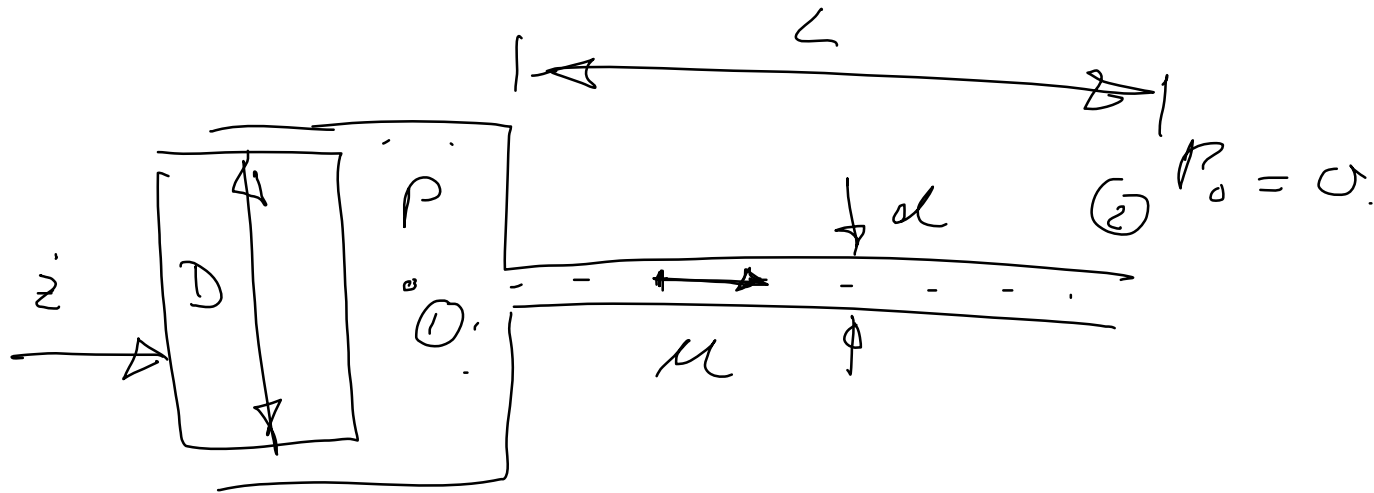
Beschleunigung  $a = \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial s}$  (siehe Taylorrech.)  
 Allg.

Im Beispiel  $\frac{\partial u}{\partial s} \equiv 0$ , sofern die Strömung inkompressibel ist.

Newton'sche Beuhl. =  $\rho a$

$$A L \rho \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial s} \right) = (P_1 - P_2) A$$

$$P_1 - P_2 = \rho \mu L$$



$$\underbrace{\frac{\rho}{2} \dot{z}^2}_{\approx 0} + P_1 = \frac{\rho}{2} \mu^2 + P_2 + \rho \mu L$$

$$\mu_1 = \dot{z} \ll \mu, \text{ da } \dot{z} \frac{\pi}{4} D^2 = \mu \frac{\pi}{4} d^2$$

$$\mu = \dot{z} \left( \frac{D}{d} \right)^2 \text{ hydraulisch \u00fcbersetz.}$$

$$P_1 - P_2 = \frac{\rho}{2} u_2^2 + \rho L \dot{u}$$

$$= \frac{\rho}{2} \dot{z}^2 \left( \frac{D}{d} \right)^4 + \rho L \ddot{z} \left( \frac{D}{d} \right)^2$$

$$F = \frac{\pi}{4} D^2 (P_1 - P_2) = \left[ \frac{\rho}{2} \dot{z}^2 \left( \frac{D}{d} \right)^4 + \rho L \ddot{z} \left( \frac{D}{d} \right)^2 \right] \frac{\pi}{4} D^2$$

$$= \frac{\rho}{2} \dot{z}^2 \left( \frac{D}{d} \right)^4 \frac{\pi}{4} D^2 + m' \ddot{z}$$

$m'$  virtuelle Masse

$$m' = \rho L \frac{\pi}{4} D^2 \left( \frac{D}{d} \right)^4 \gg m = \rho L \frac{\pi}{4} d^2$$

Anwendung:

- Hydraulische  
Gewe. ☺
- Einspritzstrahl  
☹



Manue Hydrodynamics

Neumann

MIT-Prin

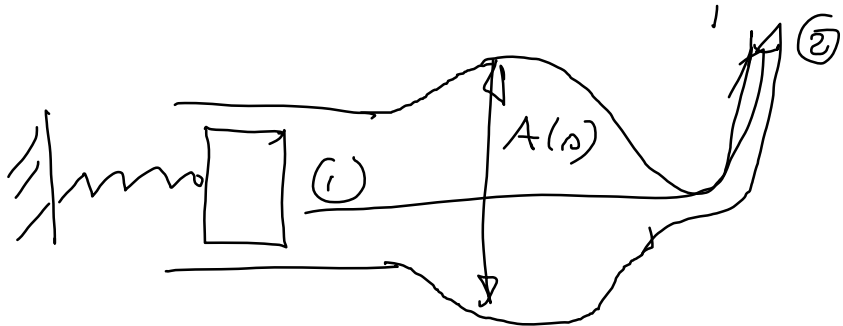
Standard work

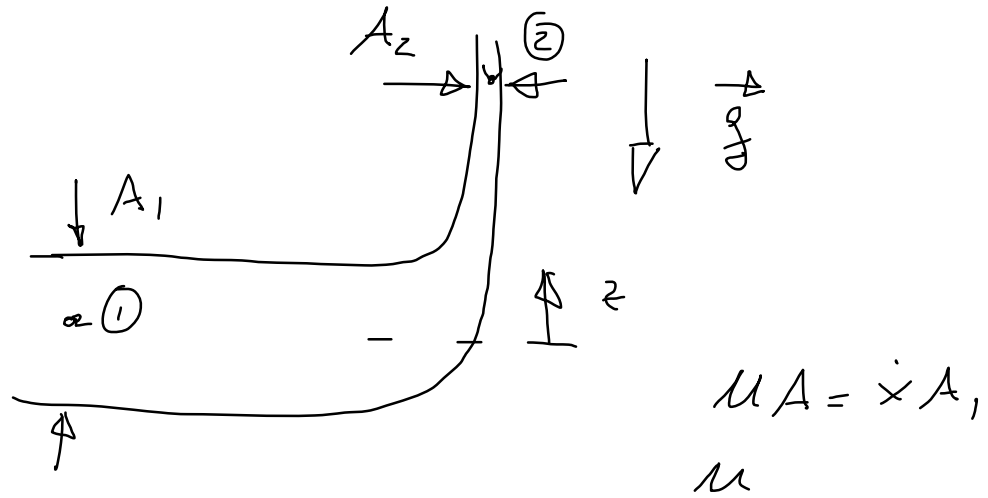
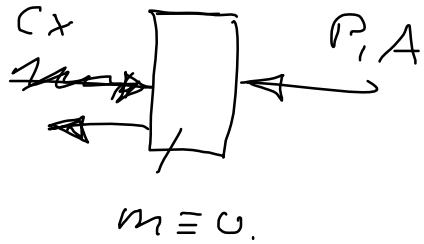
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$$P_1 + \frac{\rho}{2} U_1^2 + \psi_1 = P_2 + \frac{\rho}{2} U_2^2 + \psi_2 + \int_1^2 \rho \frac{\partial \psi}{\partial t} ds$$

$$\int_1^2 \rho \frac{\partial \psi}{\partial t} ds = \rho u L \text{ slope}$$

$$A = \text{const int.}$$





$$\frac{\rho}{2} \dot{x}^2 + P_1 = \frac{\rho}{2} u_2^2 + P_0 + \rho g z_2 + \int_1^2 \rho u \, ds$$

$$A_2 u_2 = \dot{x} A_1$$

$$\leadsto u_2 = \dot{x} \frac{A_1}{A_2}$$

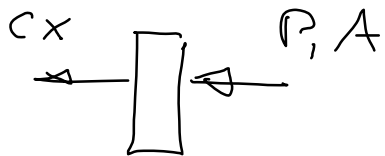
$$\rho g z_2 = \rho g x \frac{A_1}{A_2}$$

$$\int_1^2 \rho \left( \frac{A(s)}{A_1} \right)^{-1} \dot{x} \, ds =$$

$$\rho \dot{x} \underbrace{\int_1^2 \left( \frac{A(s)}{A_1} \right)^{-1} ds}_{= L}$$

$$\frac{\rho}{2} \dot{x}^2 + P_1 = \frac{\rho}{2} \dot{x}^2 \left( \frac{A_1}{A_2} \right)^2 + \rho g x \frac{A_1}{A_2} + \rho \ddot{x} \int \frac{A_1}{A(s)} ds$$

$$P_1 = \frac{\rho}{2} \left[ \left( \frac{A_1}{A_2} \right)^2 - 1 \right] \dot{x}^2 + \rho g \frac{A_1}{A_2} x + \rho L \ddot{x}$$



$$\rho L \ddot{x} + \left( C + \rho g \frac{A_1}{A_2} A_1 \right) x + \frac{\rho}{2} \left[ \left( \frac{A_1}{A_2} \right)^2 - 1 \right] \dot{x} |\dot{x}| = 0$$

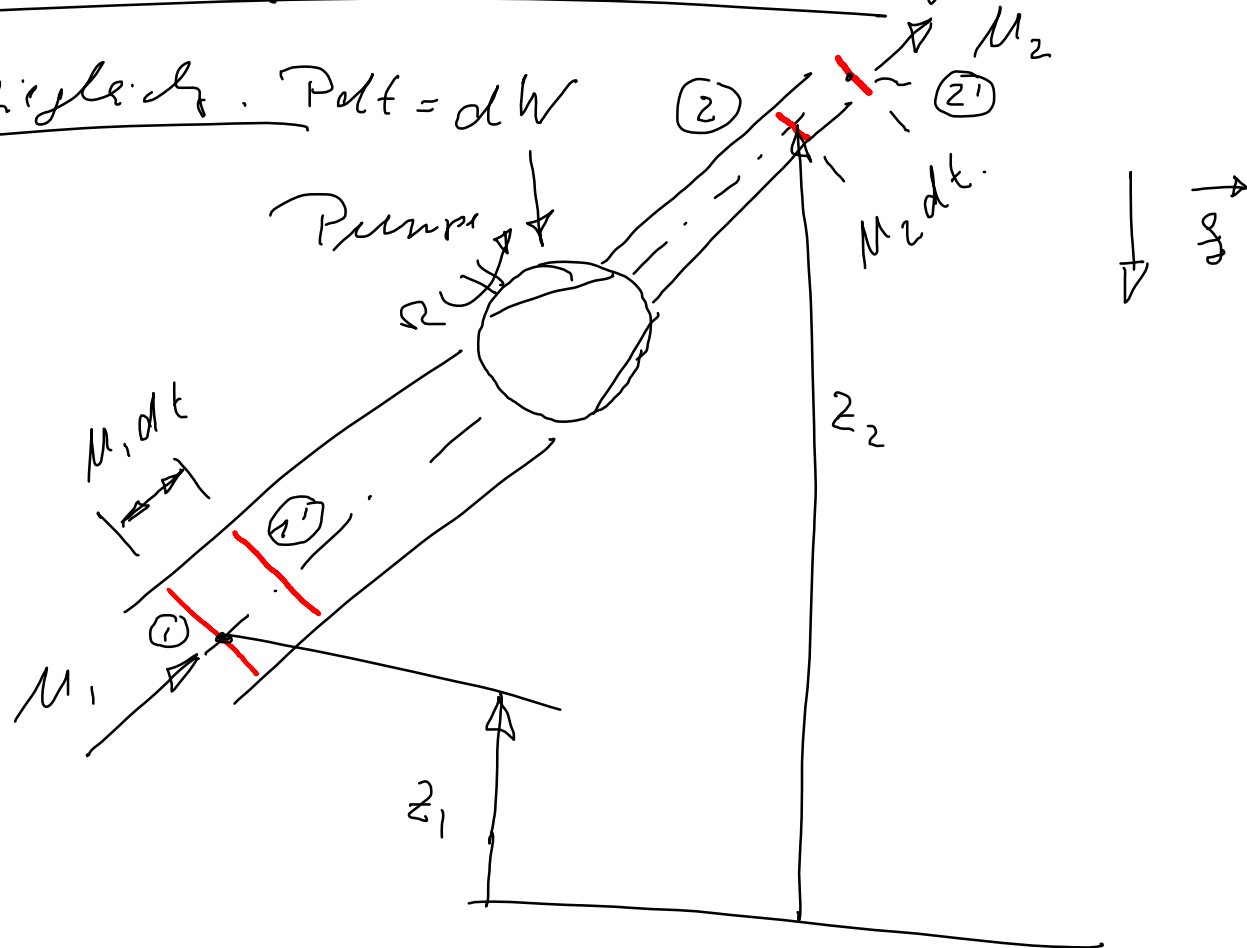
$$\omega^2 = \frac{C}{\rho L A_1}$$

$$\frac{C + \rho g \frac{A_1}{A_2} A_1}{\rho L A_1}$$

Eigenfrequenz

# Zusammenhang Bernoulli'sche Gleichung

Energiegleichung:  $P \, dt = dW$



## Mechanische Energie

Die Änderung von kinetischer Energie und potentieller Energie im Zeitintervall  $dt =$  Leistung die an die Flüssigkeit verrichtet wird



$$dE + dK = dW_{ges.}$$

$$dW_{ges.} = dW + P_1 A_1 u_1 dt - P_2 A_2 u_2 dt$$

Gewinn an potentieller Energie + kinetischer Energie

$$\int P_2 A_2 u_2 dt z_2 + \int P_2 A_2 u_2 dt \frac{u_2^2}{2}$$

Verlust an Energie

$$\int P_1 A_1 u_1 dt z_1 + \int P_1 A_1 u_1 dt \frac{u_1^2}{2}$$

mit  $\dot{V} = u_1 A_1 = u_2 A_2$

$$\frac{dW}{dt} = \dot{P} = \dot{V} \left[ \overbrace{\left( p_2 + \rho g z_2 + \rho \frac{u_2^2}{2} \right)}^{P_2 \text{ Gesamtdruck}} + \left( p_1 + \rho g z_1 + \rho \frac{u_1^2}{2} \right) \right]$$

Wenn  $\dot{P} = 0$ .

$\leadsto$  Bernoulli'sche Gleichung.

$$\boxed{\dot{P} = \dot{V} (P_{E2} - P_{E1})} \quad !$$

$\dot{P} \neq 0$

# Energiegleichung

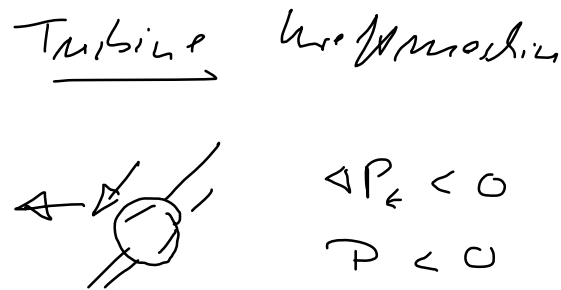
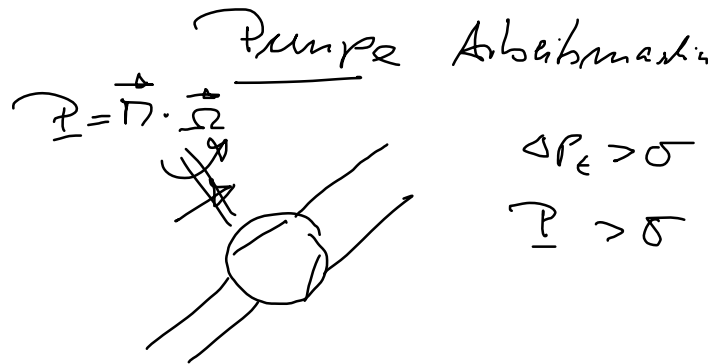
$$P = Q \Delta P_t$$

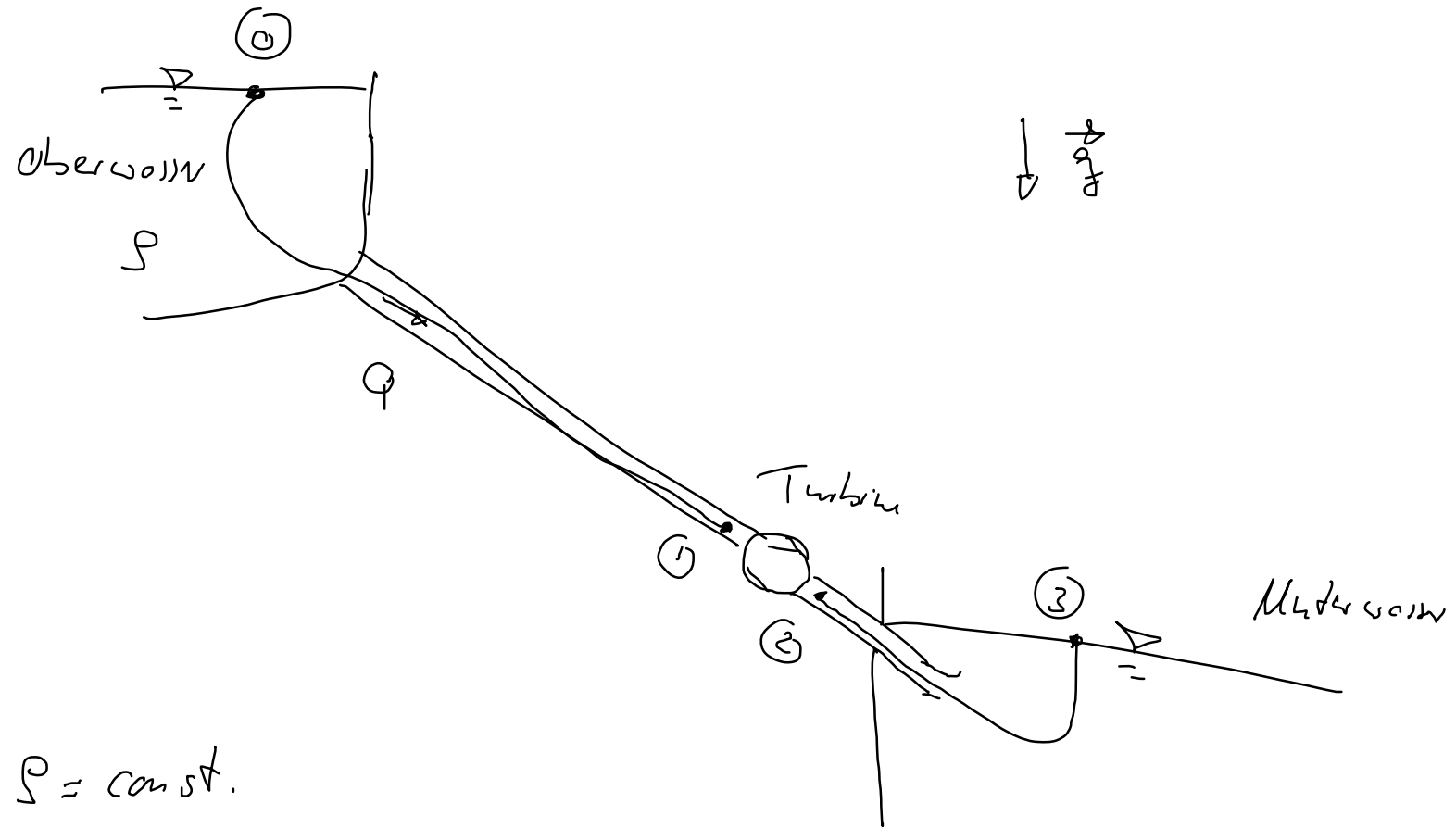
$\dot{V}$ ,  $Q$  Volumenstrom

$\Delta P_t$  Totaldruckdifferenz

Spezialfall  $P = 0$

$$\underbrace{P_1 + \frac{\rho}{2} u_1^2 + \rho g z_1}_{P_t} = P_2 + \frac{\rho}{2} u_2^2 + \rho g z_2 = C$$





# 4. Impulsnotz

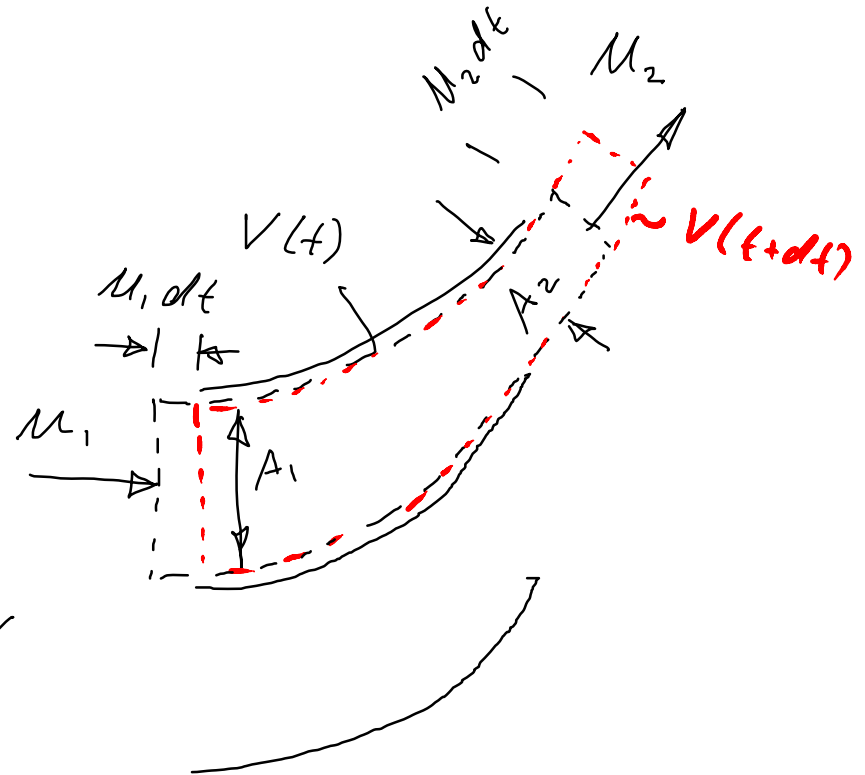
masse \* Besch. = Kraft. zu Spezial!

Impulsänder = Kraft

$$\frac{D \vec{I}}{Dt} = \vec{F}$$

$$d\vec{I} = \vec{u} \rho dV$$

$$\vec{I} = \int_V d\vec{I} = \int_V \vec{u} \rho dV$$



Impulsänder  $D\vec{I} = \vec{I}(t+dt) - \vec{I}(t)$

$$= \left( \vec{I}_0 + \rho A_2 u_2 dt \vec{u}_2 \right) - \left( \vec{I}_0 + \rho A_1 u_1 dt \vec{u}_1 \right)$$

$$\frac{D \vec{I}}{Dt} = m (\vec{u}_2 - \vec{u}_1) \frac{D}{Dt}$$

$$\frac{D \vec{I}}{Dt} = m (\vec{u}_2 - \vec{u}_1)$$

$m$  in  $M_2$ , Impulsfluss an  
der Stelle 1, 2

$$m = M_1 A_1 \rho = M_2 A_2 \rho$$

bei stationärem  
Strömung und  
inkompressibel.

Alternativ Herleitung  $\leadsto \frac{Dm}{Dt} = 0$ ;  $m = \int_{V(t)} \rho dV$

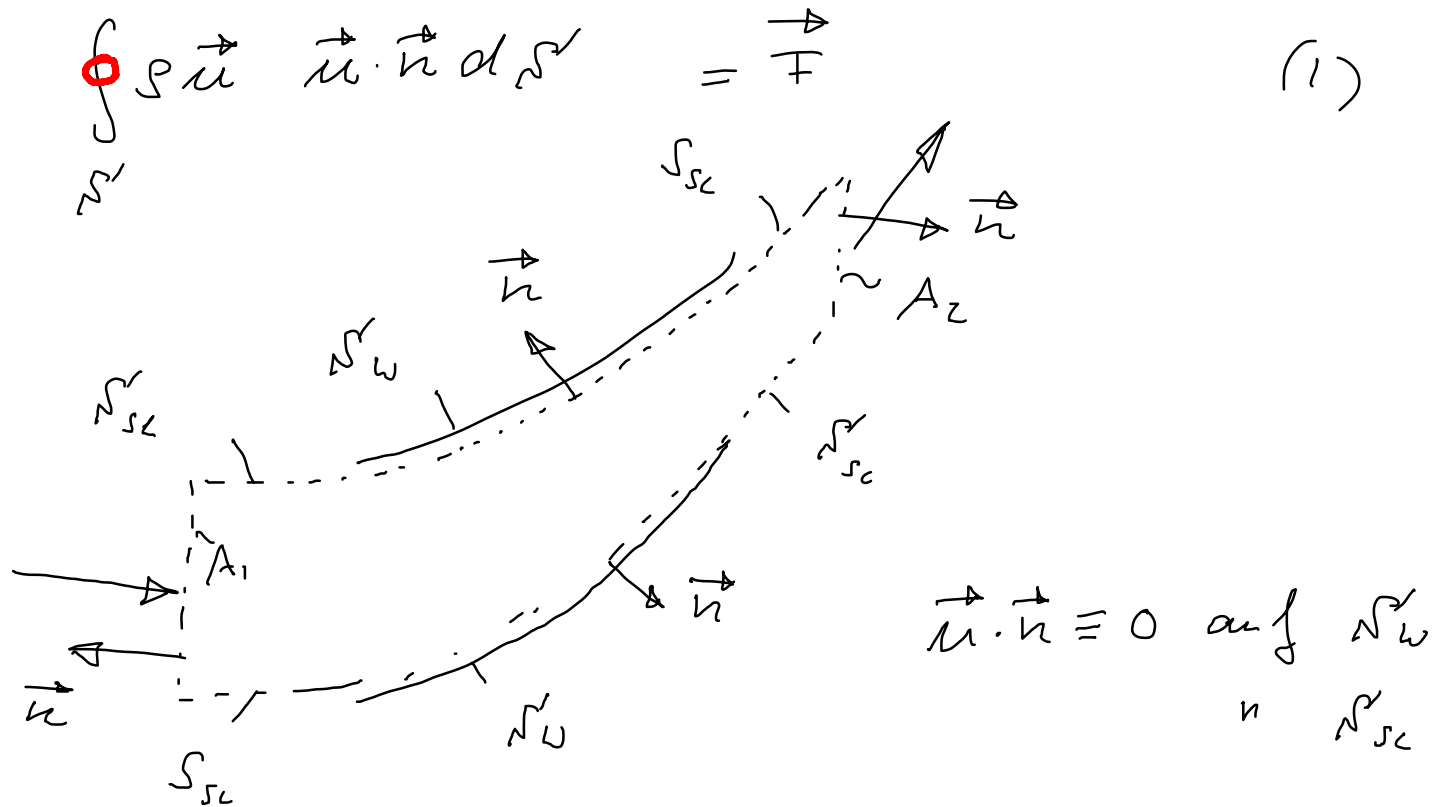
$$\frac{D \vec{I}}{Dt} = \vec{F}$$

$$\frac{\partial}{\partial t} \int_V \rho dV + \underbrace{\oint_{\partial V} \rho \vec{u} \cdot \vec{n} d\Omega}_{\text{Fluss durch Rand}} = 0$$

$$\frac{\partial}{\partial t} \int_V \rho \vec{u} dV + \oint_{\partial V} \rho \vec{u} \vec{u} \cdot \vec{n} d\Omega = \vec{F}$$

$$-M_1 \rho A_1 + M_2 \rho A_2 = 0.$$

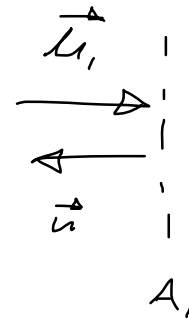
Bei stationärem Ström, verschwindet  $\frac{\partial}{\partial t} \equiv 0$ .



$$\vec{S} = A_1 + A_2 + S_w + S_{sc}$$

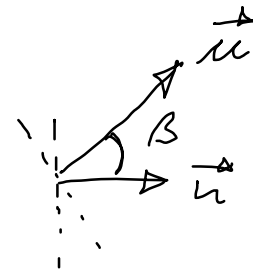
$$\int_{A_1 + A_2} \vec{u} \cdot \vec{n} \, dS = \frac{\rho}{\epsilon_0} F$$

Auf  $A_1$   $\vec{u} = \vec{u}_1 = \text{const}$   
 $\vec{u} \cdot \vec{n} = -u_1$



Auf  $A_2$   $\vec{u} = \vec{u}_2 = \text{const}$

$$\vec{u} \cdot \vec{n} = u_2 \cos \beta$$



$$-u_1 A_1 + u_2 A_2 = \frac{\rho}{\epsilon_0} F$$

$$u_2 A_2 - u_1 A_1 = \frac{\rho}{\epsilon_0} F$$



$$\frac{D\Phi}{Dt} = \frac{\partial}{\partial t} \int_V \varphi dV + \oint_{\mathcal{R}'} \varphi \vec{u} \cdot \vec{n} d\mathcal{R}'$$

$$\Phi = \int_{V(t)} \varphi dV$$

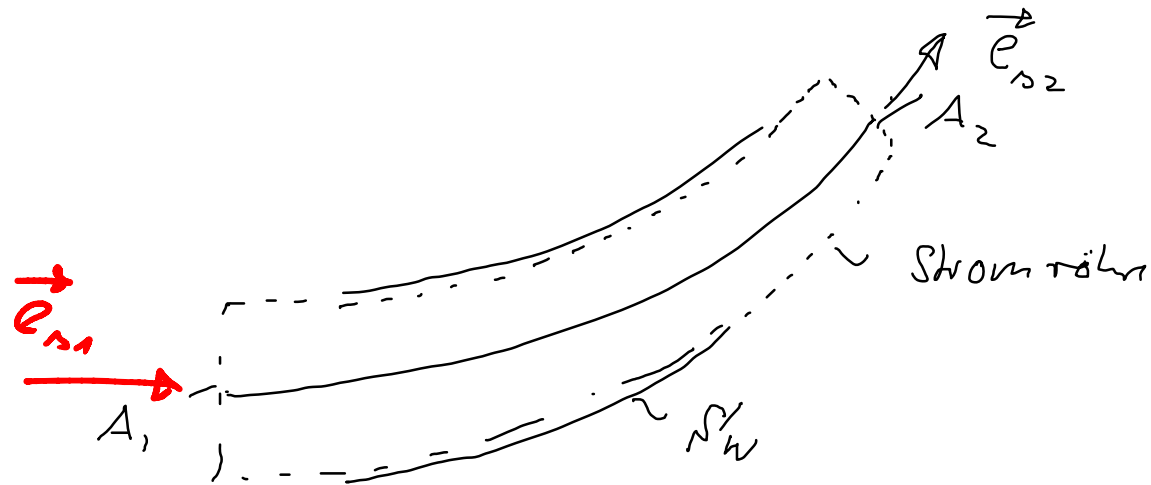
} Reynolds  
Transportthea

Anwendung auf die Kont.

- ↳ ↳ ↳ Impulskont.
- ↳ ↳ Drehmom.
- ↳ ↳ Energiekont.
- ↳ ↳ ZW Kont.

$$\frac{DN}{Dt} = \sigma \Rightarrow$$

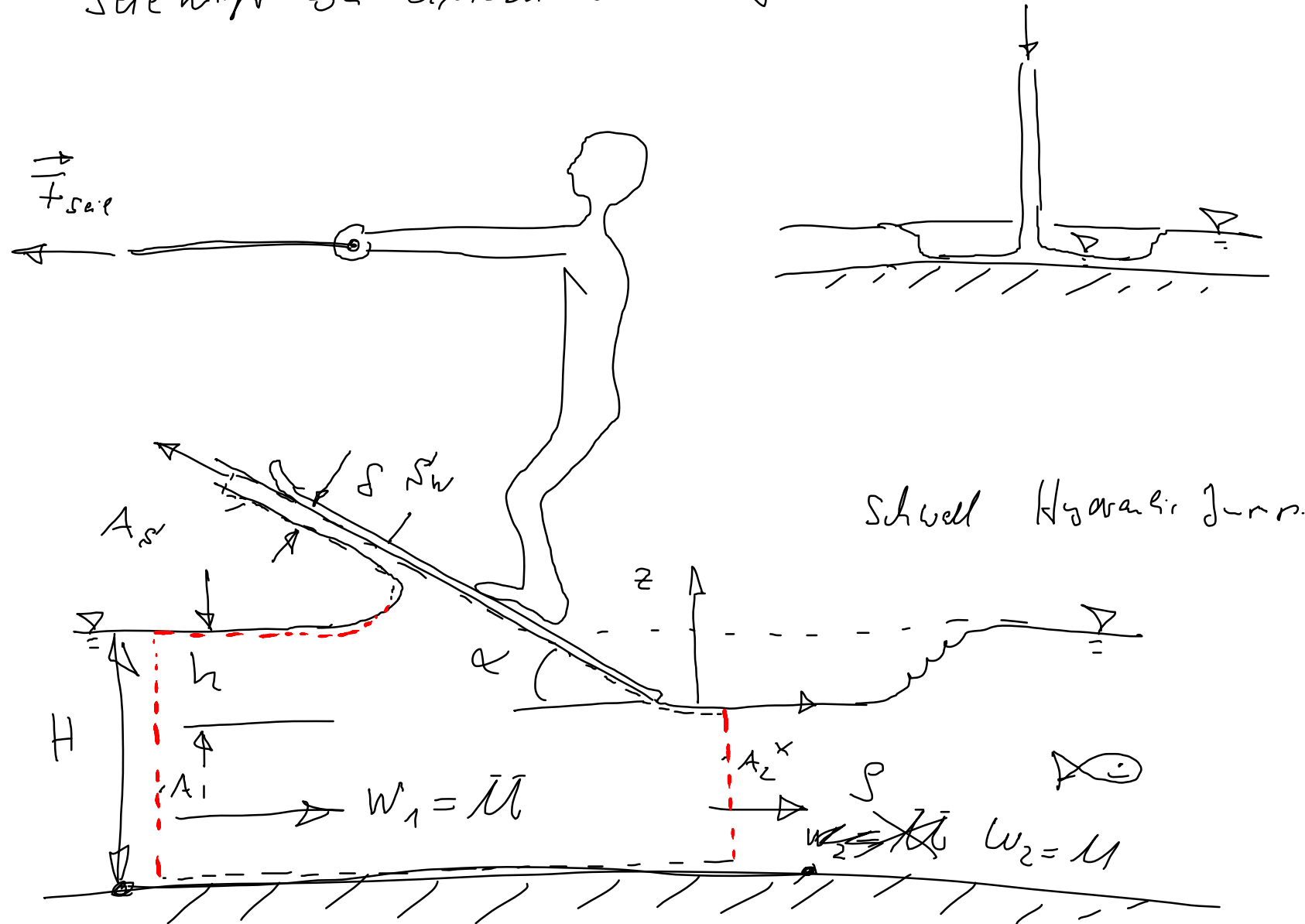
$$N = \int_{V(t)} c dV$$



$$\frac{D\vec{I}}{Dt} = m(\vec{u}_2 - \vec{u}_1) = \underbrace{\vec{F}_{W \rightarrow Fe}}_{\vec{F}_{Fe \rightarrow W} = -\vec{F}} + P_1 A_1 \vec{e}_{s1} - P_2 A_2 \vec{e}_{s2}$$

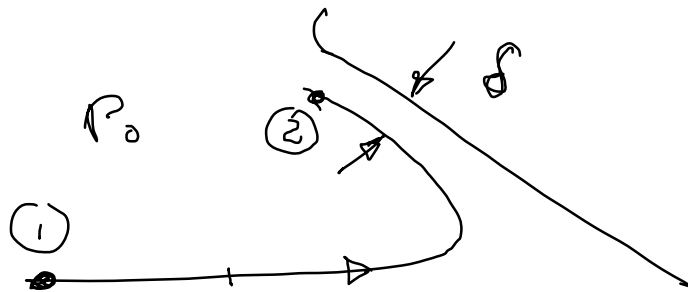
$$\vec{F} = P_1 A_1 \vec{e}_{s1} - P_2 A_2 \vec{e}_{s2} + m(\vec{u}_1 - \vec{u}_2) \quad \text{!}$$

# Seilhaft bei einem Gasrohr fallen



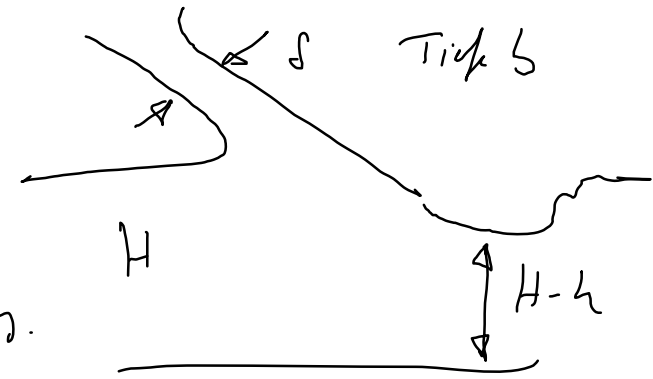
$$w_1 = U$$

Wasser Geschwindigkeit im den Strahl.



$$\left. \begin{array}{l} P_1 + \frac{\rho}{2} w_1^2 = P_2 + \frac{\rho}{2} w_2^2 \\ \parallel \qquad \parallel \qquad \parallel \\ P_0 \qquad U \qquad P_0 \end{array} \right\}$$

kontinuität.



$$= \rho M H b + \rho M (H-h) b + \rho M \delta b = 0$$

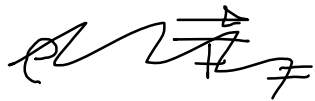
$$\leadsto \delta = h$$

$$\rightarrow w_2 = U$$

$\leadsto$  Geschwindigkeit im Strahl ist gleich der  
Fahrtgeschw.

Impuls der horizontalen Komponente.

$$\int \rho \vec{u} \cdot \vec{n} \cdot \vec{t} \, dS' = \vec{F} \cdot \vec{e}_x$$



$$\underbrace{-\rho M M H b}_{A_1} + \underbrace{\rho M^2 (H-L) b}_{A_2} - \underbrace{\rho M^2 b \int \cos \alpha}_{A_3} =$$

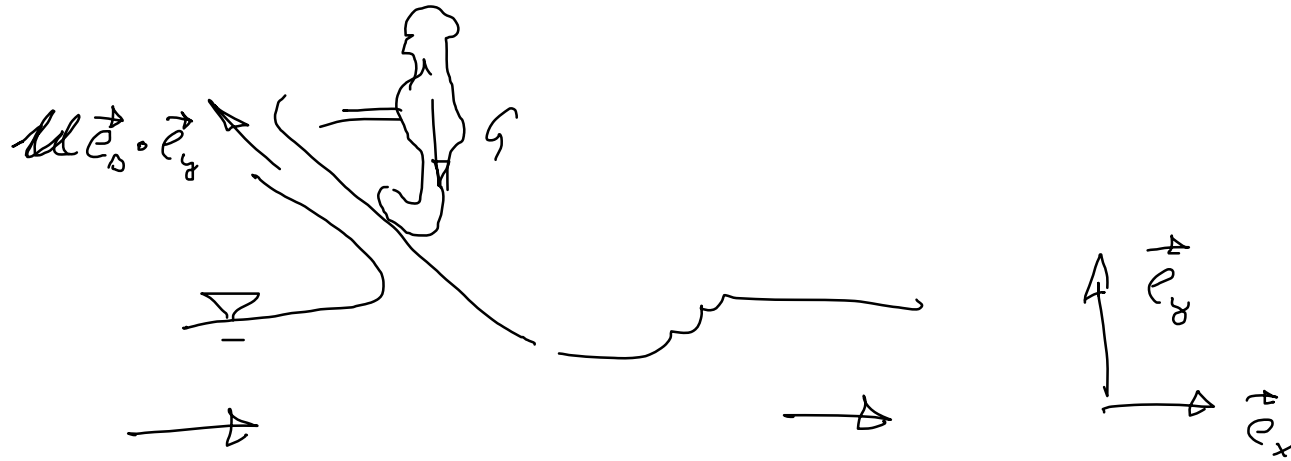
$$\int_{A_1} \rho \vec{u} \cdot \vec{e}_x \cdot \vec{n} \cdot \vec{t} \, dS' \quad \int_{A_2} \rho \vec{u} \cdot \vec{e}_x \cdot \vec{n} \cdot \vec{t} \, dS' \quad A_3$$

$$= F_x \text{ ski} \rightarrow F_e.$$



$$F_x \text{ ski} \rightarrow F_e = \rho M^2 h b (1 - \cos \alpha)$$

$\gamma$ -Komponente & der Impuls



$$+ \int_{A_D} \rho \vec{u} \cdot \vec{e}_y \vec{u} \cdot \vec{n} dA = F_{y, \text{Sch} \rightarrow \text{Fe}}$$

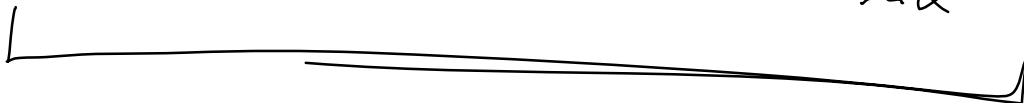
$$\leadsto F_{y, \text{Fe} \rightarrow \text{Sch}} = G = \rho M^2 b h \sin \alpha$$

$$\leadsto h = \frac{G}{\rho M^2} \frac{1}{\sin \alpha}$$

y-Komp. in die x-Komp. eingetrag.

$$F_{\text{seil}} = G \frac{1 + \cos \alpha}{\sin \alpha}$$

Freisprung

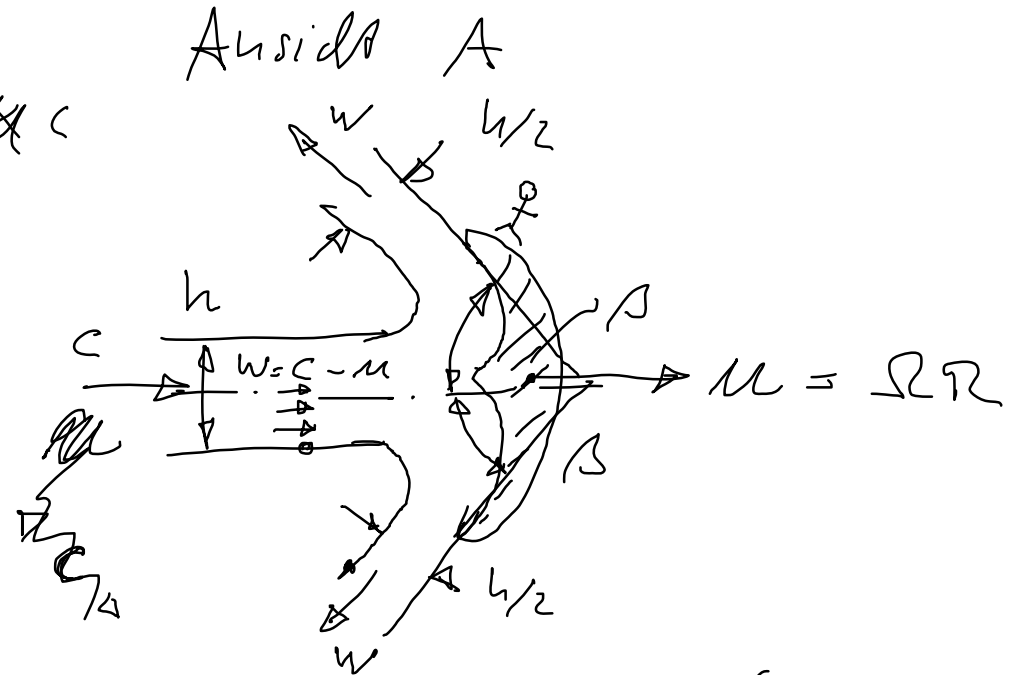
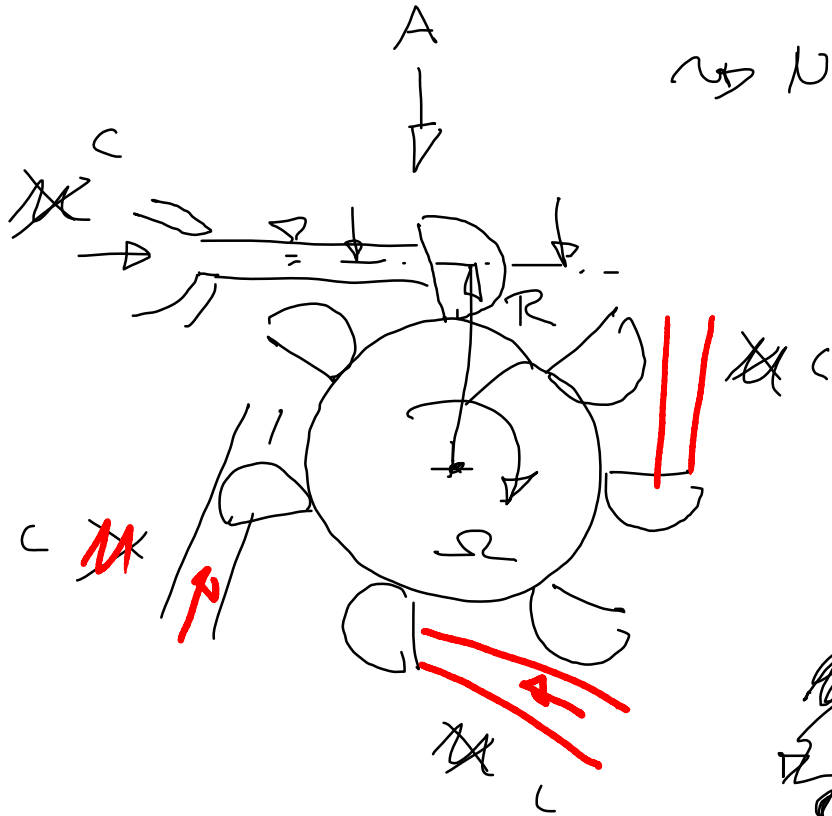
$$P = F_{\text{seil}} M = G M \frac{1 + \cos \alpha}{\sin \alpha}$$


# Zweites Beispiel

Pelton-Turbine.

Turbine für große Fallhöhe.

→ Norwegen



Impulsdruck: 
$$F = \rho w^2 h - \rho w^2 \frac{h}{2} \cos \beta * 2 = \rho (c-u)^2 h (1 - \cos \beta)$$

14.07.2009

Kraft pro Tiefe.



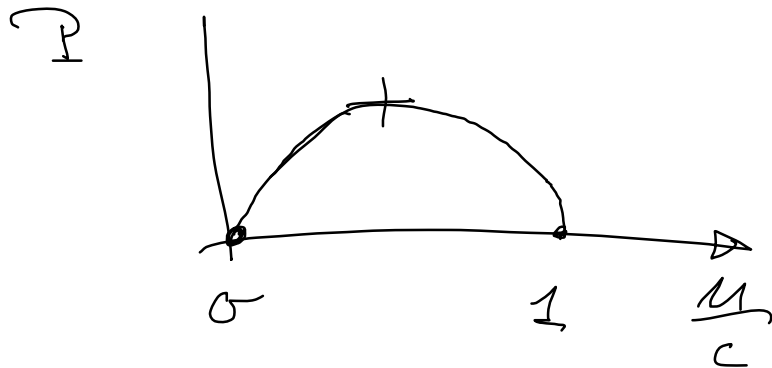
Leistung  $P = 2 F M$

$$= \rho c^3 h (1 - \cos \beta) \frac{M}{c} \left(1 - \frac{M}{c}\right) z$$

$M = \Omega R$  Mithrasgeschwindigkeit.

$z$  Schallhöhe.

$\frac{M}{c}$  Schallzahl.



Leistungszahl.

$\lambda, \zeta_p$

Coefficient of performance.

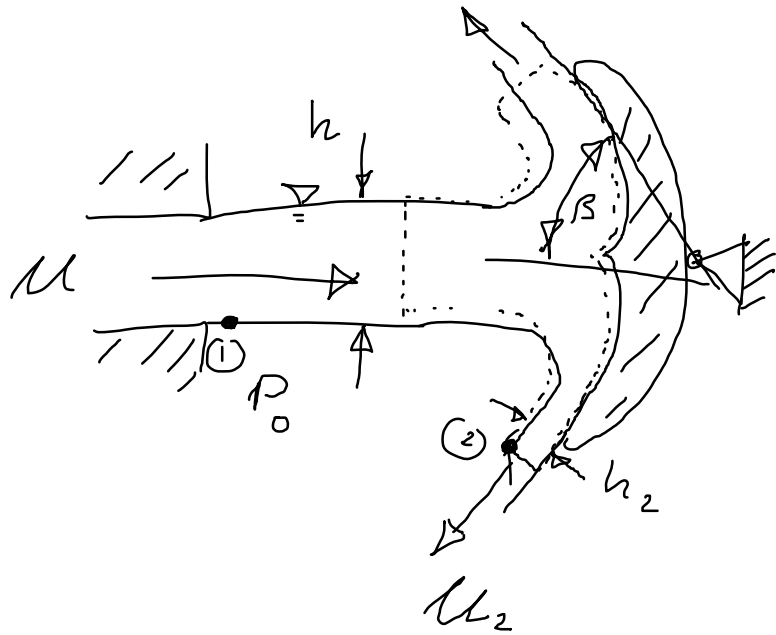
$$P_{\text{Maximal}} = \frac{\rho}{2} c^3 h z$$

$$\lambda = \frac{P}{P_{\text{Maximal}}} = 2(1 - \cos \beta) *$$

$$* \frac{M}{c} \left(1 - \frac{M}{c}\right)$$

$$\underline{\underline{\lambda_{\text{max}} = \frac{1}{2} (1 + \cos \beta)}}$$

Anwendung für die Impulsnetz Peltonmaschine.

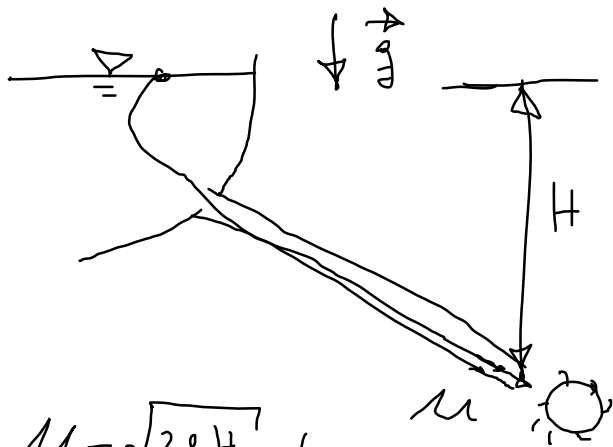


• Bernoulli langs der Stromlinie

① → ②

$$P_0 + \frac{\rho}{2} U^2 = P_0 + \frac{\rho}{2} U_2^2$$

$$\Rightarrow U_2 = U$$



• Kontinuität. -  $\rho M h + 2 \rho M h_2 = 0 \Rightarrow h_2 = h/2$

• Impuls.

$$-\rho M h + \rho M h_2 (-U \cos \beta) = F$$

Schaukt  $\rightarrow F$   
-F

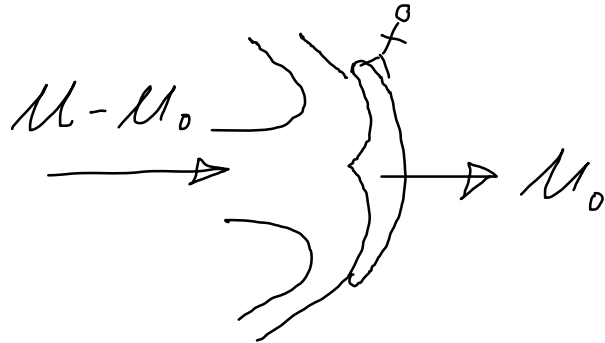
$$F = \rho M^2 h (1 + \cos \beta)$$

durch Kontinuität Einströmen.

$$U = \sqrt{2gh} \text{ bei verlustfrei Ström.}$$

14.07.2009

Mit  $U_0$  bewegte Schale  $U$ .



$$F = \rho (U - U_0)^2 h (1 + \cos \beta)$$

Umsatz pro Tiefe  $h$  /  
für die mit  $U_0$  bewegte  
Schale.

Zwei Extremwerte.

$$U_0 = 0 \quad \text{stehende Schale} \Rightarrow F = F_{\text{max}}$$

$$U = U_0$$

$$F = 0.$$

Leistung

$$P = F M_0$$

$$= \underbrace{S M_0 (M - M_0)^2}_{\leftarrow} \underbrace{h (1 + \cos \beta)}_{\leftarrow}$$

$$P_{\max} = \int \frac{1}{3} M^3 \left(\frac{2}{3}\right)^2 \times h (1 + \cos \beta)$$

$$= S M^3 \frac{1}{2} h (1 + \cos \beta) \left(\frac{2}{3}\right)^3$$

$$M_0 = 0$$

$$F = F_{\max}$$

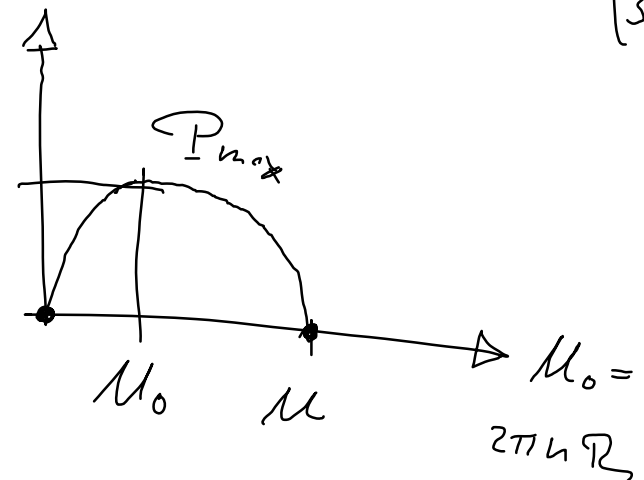
$$P = 0$$

P

$$M = M_0$$

$$F = 0$$

$$P = 0$$



maximale Leistung

$$\frac{dP}{dM_0} \stackrel{!}{=} 0$$

$$\cancel{2} M_0 (M - M_0) - \cancel{2} M_0 (M - M_0)^2 = 0$$

$$M - M_0 - 2M_0 = 0 \quad \leadsto \quad M_{0, \text{opt}} = \frac{1}{3} M$$

Leistung, die zur Verfracht von  $h$

$$P_{\text{prop}} = \frac{\rho}{2} v^3 h$$

$$\frac{P}{P_{\text{prop}}} := \lambda, C_p$$

Leistungskoeffizient | Coefficient of  
the Propulsion | Performance

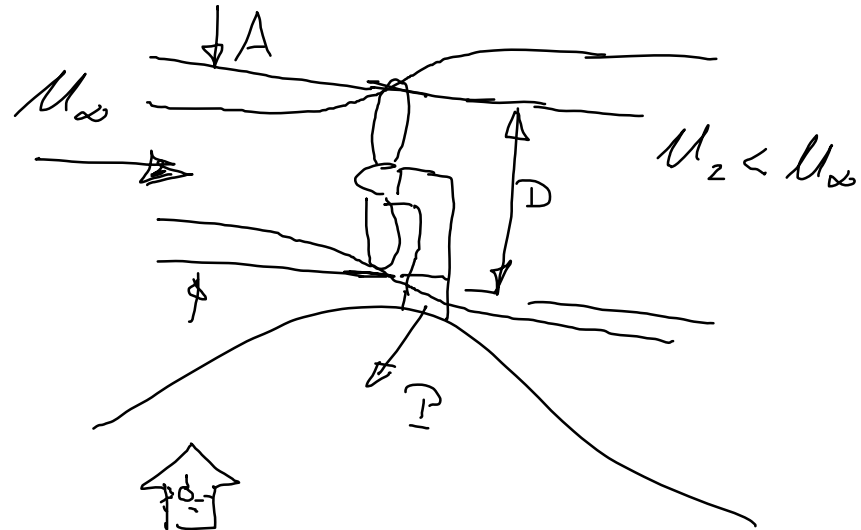
$$\lambda_{\text{opt.}} = (1 + \cos \beta) \left( \frac{2}{3} \right)^3$$

$\lambda_{opt}$  für eine Vordrallmaschine

$$\lambda_{opt} = \frac{16}{27} = 0.59$$

$$\lambda_{opt} \underline{P}_{moyd} = \underline{P}_{opt}$$

$\frac{\rho}{2} M_\infty^2$  kinetische Energie einer Luftteil.

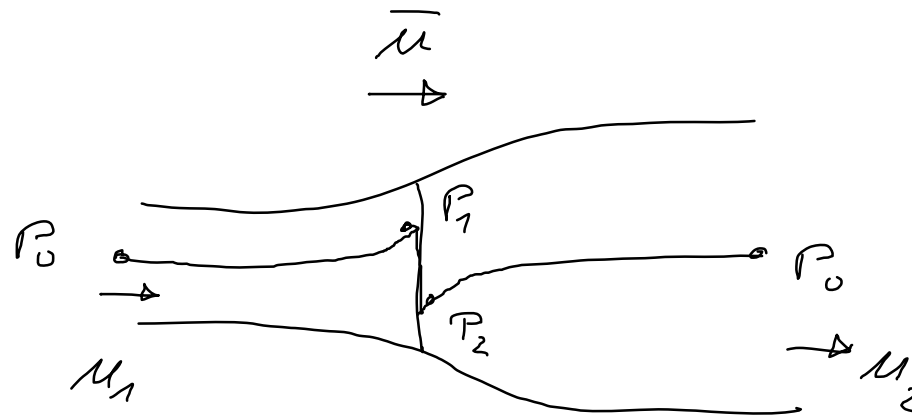


$$A = \frac{\pi D^2}{4}$$

$$\underline{P}_{moyd} = \int_A \frac{\rho}{2} M_\infty^2 U_\infty dA = \frac{\rho}{2} M_\infty^3 A$$

$$\leadsto \underline{P}_{opt} = \lambda_{opt} \frac{\rho}{2} M_\infty^3 A$$

$$\left. \begin{aligned} \rho &= 1 \frac{\text{kg}}{\text{m}^3} \\ M_1 &= 10 \frac{\text{m}}{\text{sec}} \\ D &= 10 \text{ m} \end{aligned} \right\} \begin{aligned} P_{\text{opt}} &= 0.6 * 1 * 10^3 * 1 * 10^2 \text{ W} \\ &= 60 \text{ kW} \end{aligned}$$

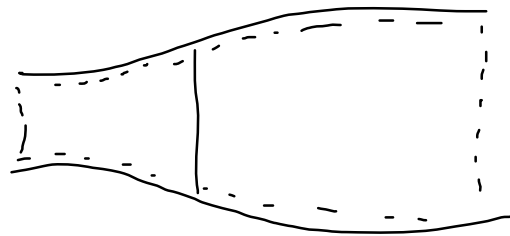


$$\left. \begin{aligned} P_0 + \frac{\rho}{2} M_1^2 &= P_1 + \frac{\rho}{2} \bar{u}^2 \\ P_2 + \frac{\rho}{2} \bar{u}^2 &= P_0 + \frac{\rho}{2} M_2^2 \end{aligned} \right\} P_1 - P_2 = \frac{\rho}{2} (M_1^2 - M_2^2)$$

Axialkraft (Schub) auf der Ventur

$$F = (P_1 - P_2) A \quad (1)$$

Alternativ über Impulsatz



$$-\rho u_1^2 A_1 + \rho u_2^2 A_2 = -F$$

$$-\rho A \bar{u} (u_1 - u_2) = -F \quad (?)$$

$$P_1 - P_2 = \rho \bar{u} (u_1 - u_2)$$

$$P_1 - P_2 = \frac{\rho}{2} (u_1^2 - u_2^2)$$

$$= \frac{\rho}{2} (u_1 - u_2) (u_1 + u_2)$$

$$\leadsto \bar{u} = \frac{1}{2} (u_1 + u_2)$$



$$P = \Delta p \bar{\mu} A$$

$$= \rho \bar{\mu}^2 (\mu_1 - \mu_2) A, \text{ mit } \bar{\mu} = \frac{1}{2} (\mu_1 + \mu_2)$$

$$= \frac{\rho}{2} \mu_1^3 A \left(1 + \frac{\mu_2}{\mu_1}\right)^2 \left(1 - \frac{\mu_2}{\mu_1}\right)$$

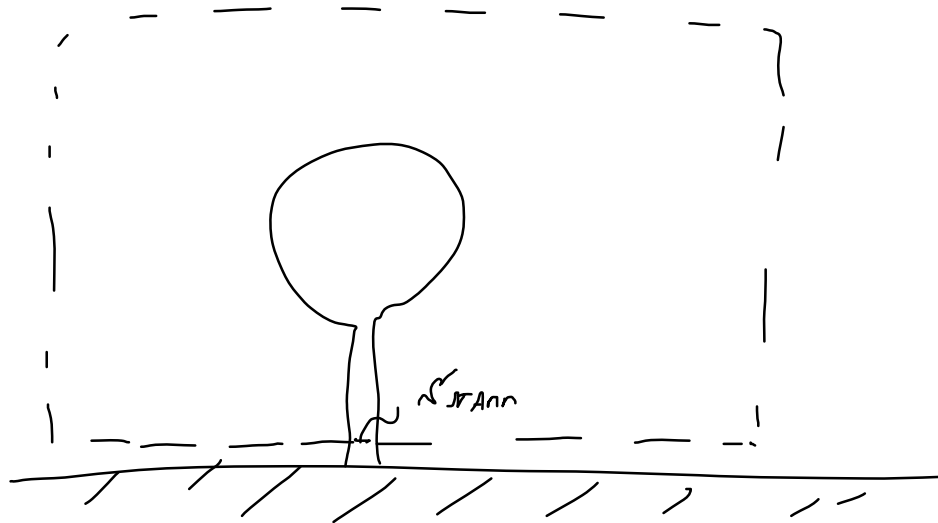
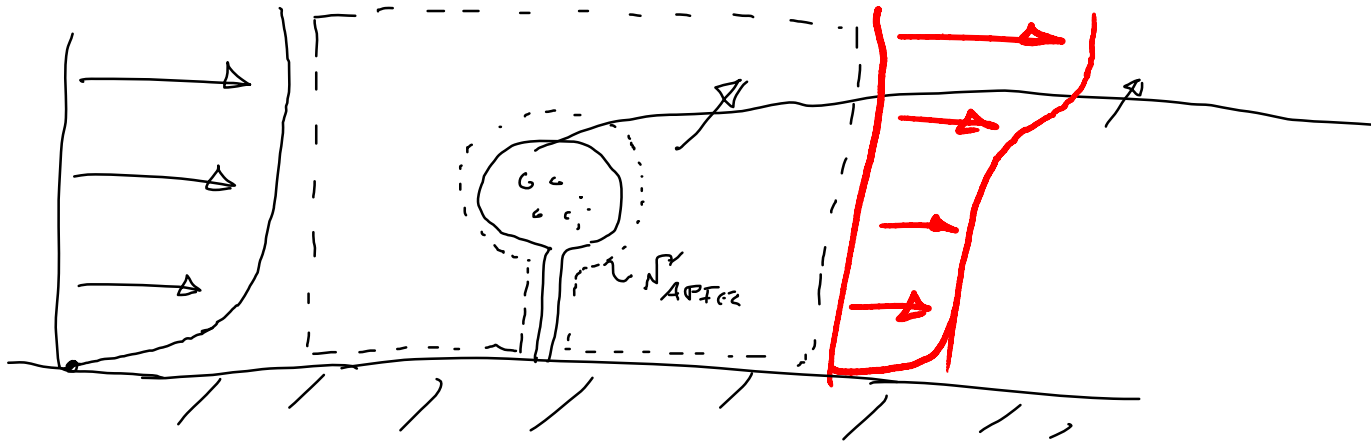
$$\lambda = \left(1 + \frac{\mu_2}{\mu_1}\right)^2 \left(1 - \frac{\mu_2}{\mu_1}\right)$$

$$\lambda_{\text{opt}} = \frac{16}{27}$$

1920 Albert Bethe.

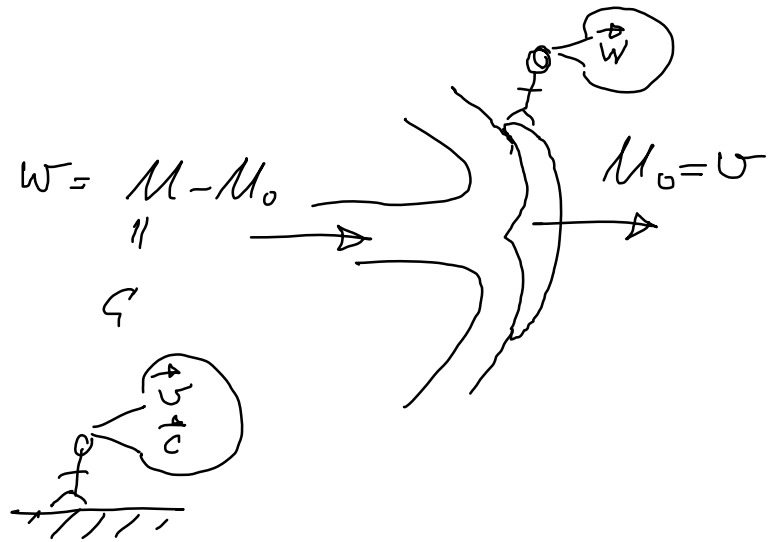


$$\frac{d\lambda}{d(\mu_2/\mu_1)} \stackrel{!}{=} 0$$



$$\int_{N_{\text{STEN.}}} \vec{E} d\vec{S}' = \int_{S_{\text{APFEZ}}} \vec{E} d\vec{S}'$$

im statisch Gleichwicht.



Richtbl.:

Faktor  $\omega$  und  $\omega_{rel}$  war  $R_L$   
 Beweg. Syst.

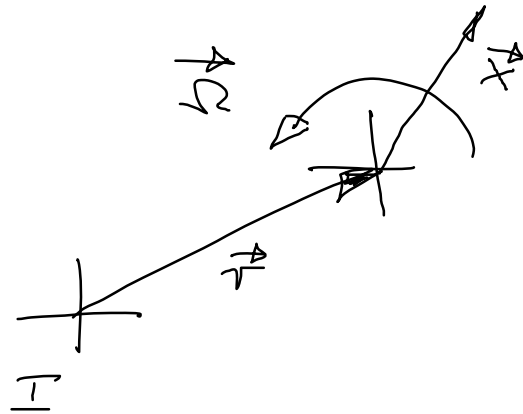
$\leadsto$  keine Tragheitskräfte,  
 da gleichförmige Beweg.

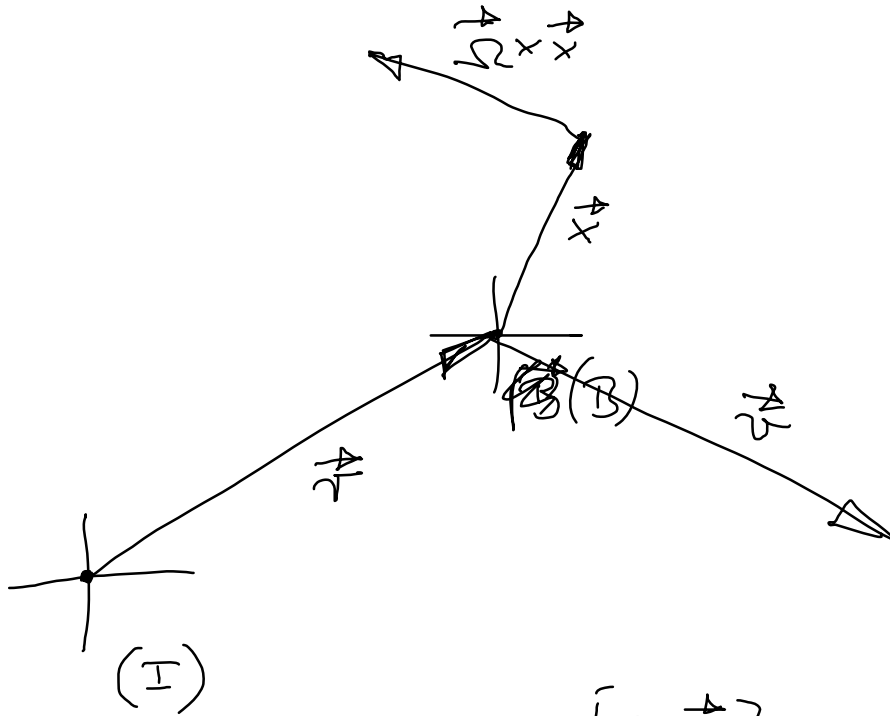
$\vec{W}$  relative Geschw.

$\vec{U}$  Führungsgeschw.

$\vec{C} = \vec{W} + \vec{U}$  Absolute Geschw.

$+ \vec{\Omega} \times \vec{x}$





$$\vec{v}_B = \left[ \begin{array}{c} \frac{D\vec{x}}{Dt} \\ \vec{x} \end{array} \right]_B$$

(I) Inertialsystem

(B) beschleunigt Syst.

$\vec{v}_I$  Führungswelt.

$$\vec{v}_I = \left[ \begin{array}{c} \frac{D\vec{x}}{Dt} \\ \vec{x} \end{array} \right]_I \quad \text{Führungswelt.}$$

$$\left[ \begin{array}{c} \frac{D\vec{x}}{Dt} \\ \vec{x} \end{array} \right]_B$$

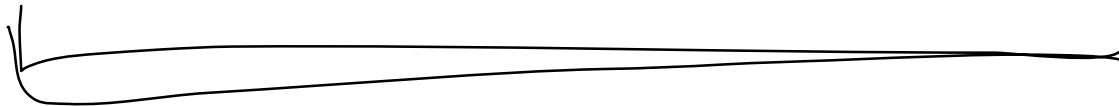


$$\vec{v}_I = \left[ \begin{array}{c} \frac{D(\vec{x} + \vec{r})}{Dt} \\ \vec{x} + \vec{r} \end{array} \right]_I = \left[ \begin{array}{c} \frac{D\vec{x}}{Dt} \\ \vec{x} \end{array} \right]_I + \vec{v} = \vec{w} + \vec{v} + \vec{\omega} \times \vec{x}$$

$$\left[ \frac{D \vec{x}}{Dt} \right]_{\mathcal{I}} = \left[ \frac{D \vec{x}}{Dt} \right]_{\mathcal{B}} + \vec{\Omega} \times \vec{x}$$

$\vec{b}$  ist ein beliebiger Tensor

$$\left[ \frac{D \vec{b}}{Dt} \right]_{\mathcal{I}} = \left[ \frac{D \vec{b}}{Dt} \right]_{\mathcal{B}} + \vec{\Omega} \times \vec{b}$$



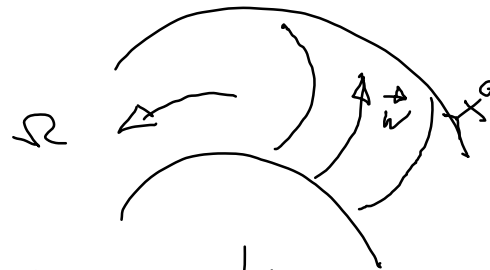
Beschleunigung eines Flüssigkeitsteilchens.

$$\underbrace{\left[ \begin{array}{c} \frac{D\vec{c}}{Dt} \\ \frac{D\vec{c}}{Dt} \end{array} \right]}_{\hat{=} \text{ Besch.}} = \underbrace{\vec{\nabla} \cdot \vec{T}}_{\hat{=} \text{ Kraft.}}$$

$\hat{=} \text{ Norm}$

Newton.

$$\vec{c} = \vec{w} + \vec{v} + \vec{\Omega} \times \vec{x}$$



$$\begin{aligned} \left[ \begin{array}{c} \frac{D\vec{c}}{Dt} \\ \frac{D\vec{c}}{Dt} \end{array} \right]_I &= \left[ \begin{array}{c} \frac{D\vec{w}}{Dt} \\ \frac{D\vec{w}}{Dt} \end{array} \right]_I + \left[ \begin{array}{c} \frac{D\vec{v}}{Dt} \\ \frac{D\vec{v}}{Dt} \end{array} \right]_I + \left[ \begin{array}{c} \frac{D}{Dt} (\vec{\Omega} \times \vec{x}) \\ \frac{D}{Dt} (\vec{\Omega} \times \vec{x}) \end{array} \right]_I \\ &= \left[ \begin{array}{c} \frac{D\vec{w}}{Dt} \\ \frac{D\vec{w}}{Dt} \end{array} \right]_B + \vec{\Omega} \times \vec{w} + \vec{a} + \underbrace{\left[ \begin{array}{c} \frac{D}{Dt} (\vec{\Omega} \times \vec{x}) \\ \frac{D}{Dt} (\vec{\Omega} \times \vec{x}) \end{array} \right]}_{\vec{\Omega} \times \vec{x}} + \vec{\Omega} \times \left( \vec{w} + \vec{v} + \vec{\Omega} \times \vec{x} \right) \end{aligned}$$

$$\left[ \frac{D \vec{c}}{Dt} \right]_{\mathcal{I}} = \left[ \frac{D \vec{w}}{Dt} \right]_{\mathcal{B}} + 2 \vec{\Omega} \times \vec{w} + \vec{\Omega} \times (\vec{\Omega} \times \vec{x}) + \dot{\vec{\Omega}} \times \vec{x} + \vec{a}$$

$$\mathcal{P} \left[ \frac{D \vec{c}}{Dt} \right]_{\mathcal{I}} = \mathcal{V} \cdot \mathcal{T}$$

$$\mathcal{P} \left[ \frac{D \vec{w}}{Dt} \right]_{\mathcal{B}} = \mathcal{V} \cdot \mathcal{T} = \mathcal{P} \left( \underbrace{2 \vec{\Omega} \times \vec{w}}_{\text{Coriolis}} + \underbrace{\vec{\Omega} \times (\vec{\Omega} \times \vec{x})}_{\text{Zentrifugalk.}} + \underbrace{\dot{\vec{\Omega}} \times \vec{x} + \vec{a}}_{\substack{\text{no non,} \\ \text{Fiktion}}})$$

$$\left[ \frac{D \vec{b}}{Dt} \right]_{\mathcal{I}} = \left[ \frac{D \vec{b}}{Dt} \right]_{\mathcal{B}} + \vec{\Omega} \times \vec{s}$$

Impulssatz in isoperist. Form.

$$\left[ \frac{D\vec{I}}{Dt} \right]_I = \vec{F}$$

$$\left[ \frac{D\vec{I}}{Dt} \right]_I + \vec{\Omega} \times \vec{I} = \vec{F}$$


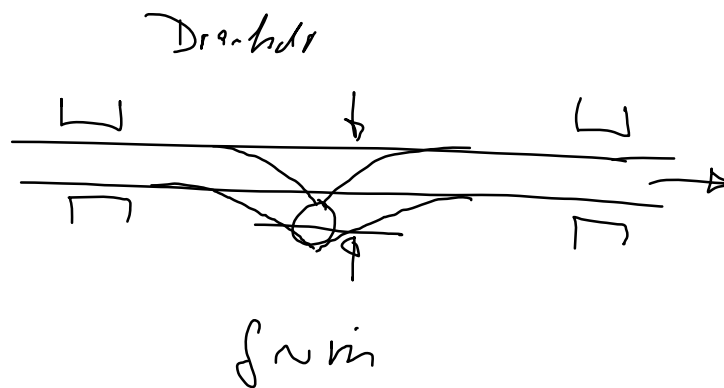
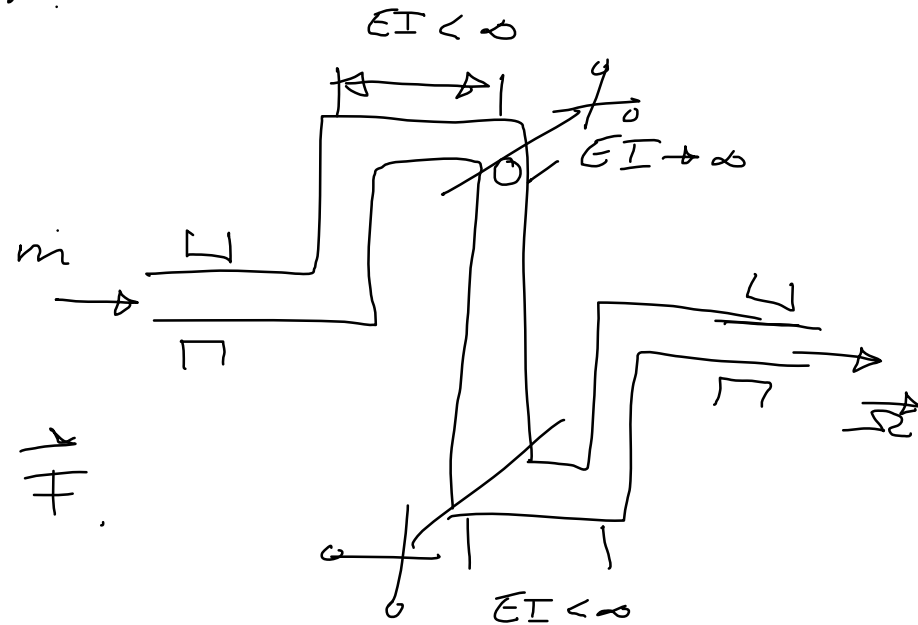


Diagram of a rod with forces  $\vec{W}$  and  $\vec{H}$  and torque  $\vec{H}$  applied.

$$\vec{\Omega} \times \vec{I} = \vec{F}$$

$$\vec{H} = \int_V \rho \vec{c}^+ dV$$



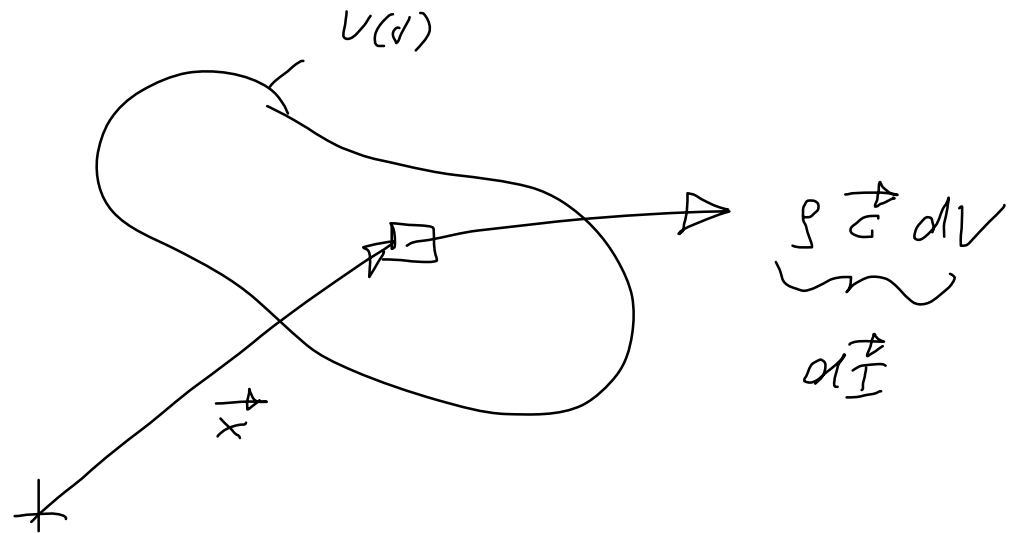


Dipolgesetz

$$\vec{D} = \int_{V(d)} d\vec{D}$$

$$d\vec{D} = \vec{x} \times d\vec{I}$$

$$\vec{D} = \int_{V(d)} \vec{x} \times \rho \vec{c} dV$$

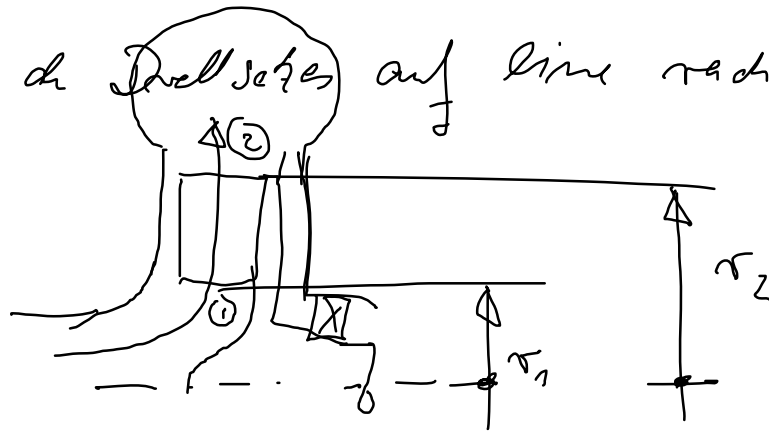


Dipolgesetz : Die zeitliche Änderung der Dipolmoment ist gleich der Strom  $a$   $\int$  der Körper.

$$\frac{D}{Dt} \vec{D} = \vec{H} \quad \text{Poisson.}$$

$$\vec{H} = \underbrace{\oint_{S'} \vec{x} \times \vec{t} dS'}_{\text{Oberflächent}} + \underbrace{\int_V \vec{x} \times \rho \vec{h} dV}_{\text{Volumenwert}}$$

Anwendung des Biot-Savart auf eine radiale Strömungsmaschine.



- a) feststehende UV, Schallbewegung in UV  
 b) bewegliche UV, wie Schall. ~ Beschleunigung fest.



$$\left. \begin{array}{l} \vec{v} = \dot{s} \\ \vec{p} = \rho \dot{s} \end{array} \right\} P = \vec{p} \cdot \vec{v}$$

$\frac{1}{T}$

- a) feststehende Kontrollvolumen.

$$\frac{d}{dt} \int_V \vec{x} \times \rho \vec{c} dV + \oint_{\partial V} \vec{x} \times \rho \vec{c} \cdot \vec{n} dS = + \vec{M}$$

~~Normal der  
 El.  $\vec{v}$   $\vec{p}$~~

b)

$$\oint_{\partial V} \vec{x} \times \rho \vec{c} \cdot \vec{w} \cdot \vec{n} dS = + \vec{M}$$

$$\underbrace{\left(\vec{x} \times \rho \vec{c}\right)_1}_{-V_1} \int_{A_1} \vec{c} \cdot \vec{n} dA + \underbrace{\left(\vec{x} \times \rho \vec{c}\right)_2}_{V_2} \int_{A_2} \vec{c} \cdot \vec{n} dA = \vec{\Gamma}$$

$$\vec{\Gamma} = \left(\vec{x} \times \rho \vec{c}\right)_2 V_2 - \left(\vec{x} \times \rho \vec{c}\right)_1 V_1 \quad \text{Euler's Transient}$$

bei stationärer Strömung:

$$\rho_2 V_2 = \rho_1 V_1$$



$$\vec{\Gamma} = m (\tau_2 c_{u2} - \tau_1 c_{u1}) \vec{e}_2$$

Umsatz  $L_e$

$c_u$  Umfangskomponente der Abschl.  $v$

Wenn hier Name auf die Flussigkeit einsetzen wird,  
dann

$$\Gamma = m \left( r_2 c_{\theta 2} - r_1 c_{\theta 1} \right) = \frac{m}{2\pi} \left( \Gamma_2 - \Gamma_1 \right)$$

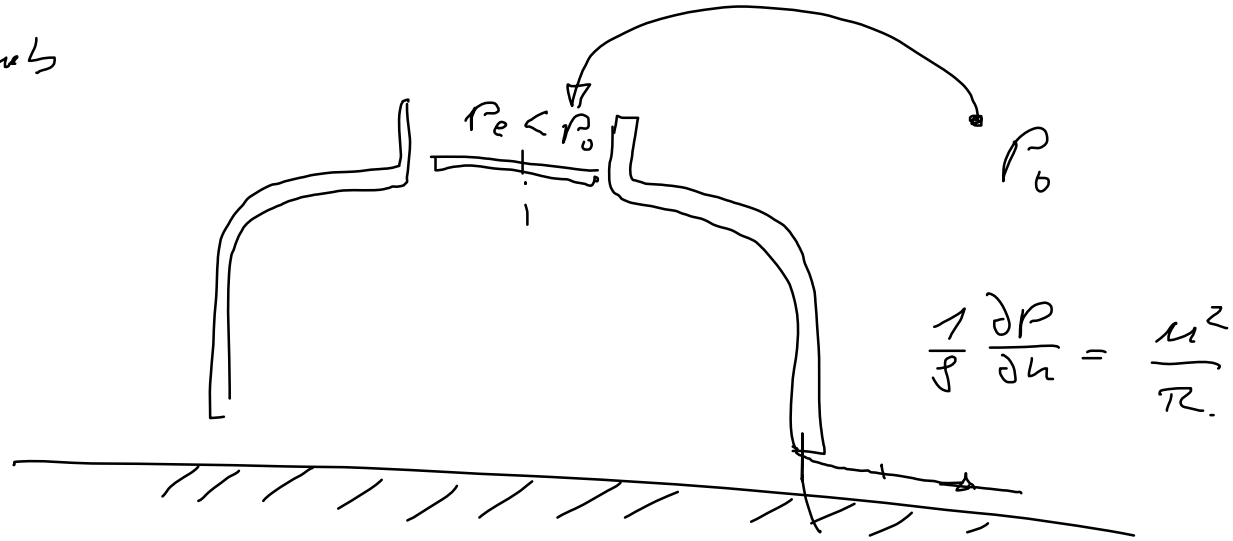
$$\leadsto r_2 c_{\theta 2} = r_1 c_{\theta 1} \quad \text{Dreh bleibt erhalten.}$$

Wichtig bei Wirbeln.

Stärke des Wirbels

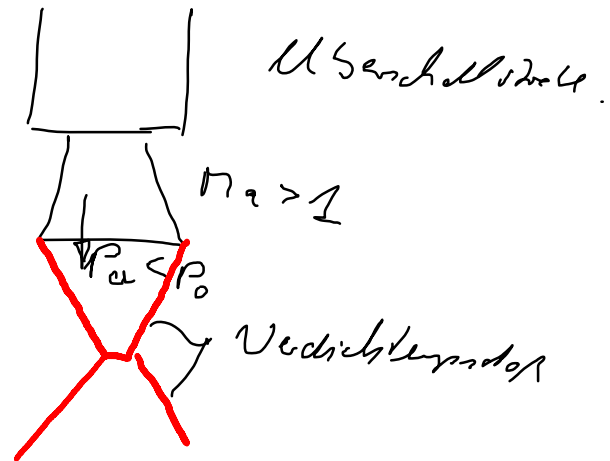
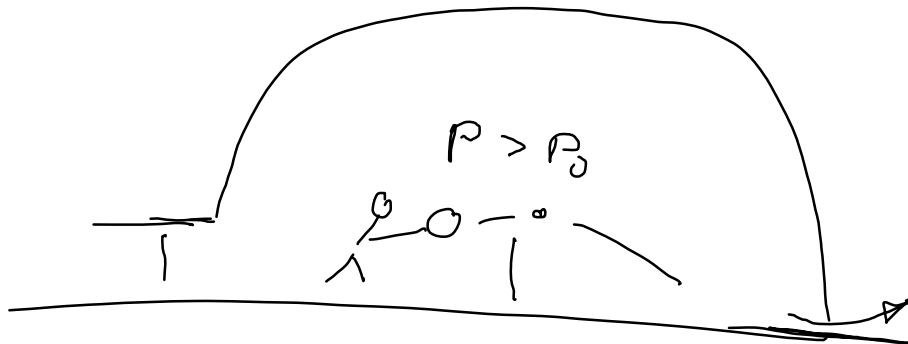
$$\Gamma = \oint \vec{c} \cdot d\vec{x} = 2\pi r c_{\theta} \quad \text{Zirkulation.}$$


Übungen 1 & 2

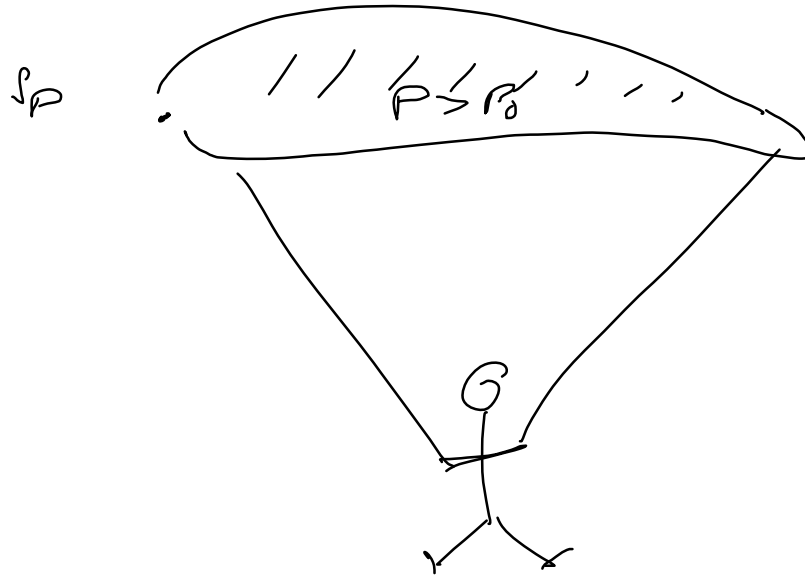


$P_0$

$P_a = P_0$ , da  
Unterschied freistell.



$$\frac{\rho}{z} \mu_0 + \rho_0$$




# Bewegte Koordinatensystem

$\vec{b}$

$$\begin{bmatrix} \overline{D \vec{b}} \\ \overline{D t} \end{bmatrix}_I = \begin{bmatrix} \overline{D \vec{b}} \\ \overline{D t} \end{bmatrix}_B + \underbrace{\vec{\Omega} \times \vec{b}}_{\text{Zentrifugalkraft, Coriolis ...}}$$

Zentrifugalkraft,  
Coriolis ...

Massen  $\times$  Beschl = Kraft

$$S \begin{bmatrix} \overline{D \vec{c}} \\ \overline{D t} \end{bmatrix}_I = \vec{D} \cdot \vec{T}$$



Stationäre Strömung, Drehsatz im integrierten Form.

Drehsatz

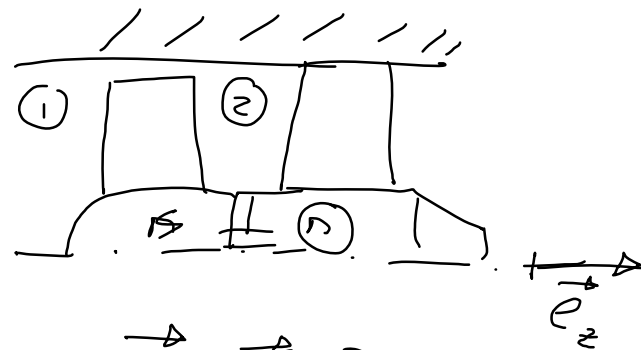
$$\frac{D\vec{D}}{Dt} = \vec{M}$$

Dreht.

$$\vec{D} = \int_V \vec{x} \times \rho \vec{c} dV$$

Euler'sche Turbinengleichung

$$m (\tau_2 c_{m2} - \tau_1 c_{m1}) = M_z$$



$$\left. \begin{aligned} P &= \vec{M} \cdot \vec{\Omega} \\ \vec{\Omega} &= \Omega \vec{e}_z \end{aligned} \right\} P = \Omega M_z$$

Spezialfall:  $\Gamma_2 \equiv 0$ .  $\Gamma_2 = \int (\tau_2 c_{\mu 2} - \tau_1 c_{\mu 1})$

$\leadsto \tau_2 c_{\mu 2} = \tau_1 c_{\mu 1} = \text{const.} \rightarrow \frac{\Gamma}{2\pi}$

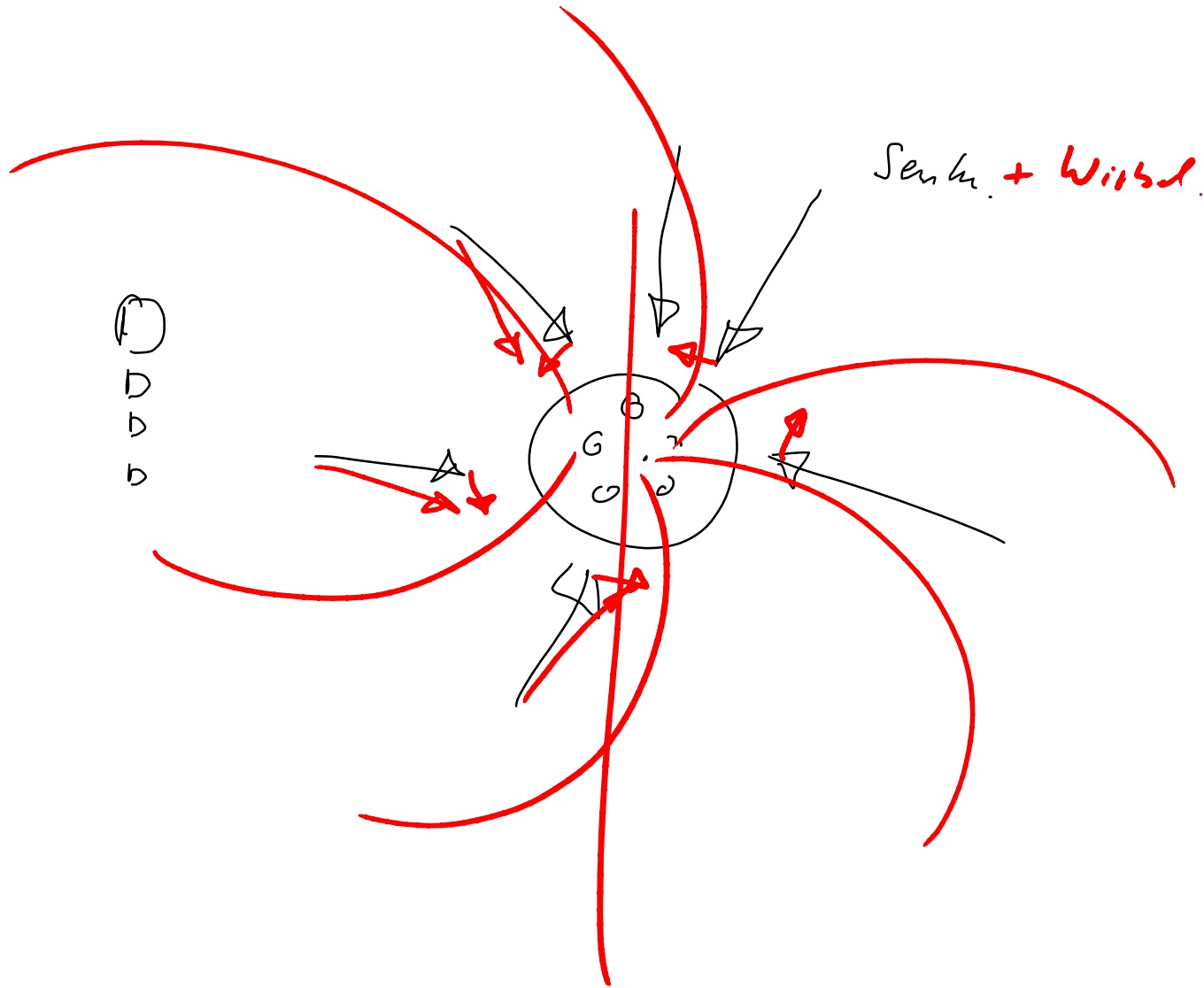
Dreh eines Flüssigkeitskitts bleibt erhalten, wenn kein Moment auf die Flüssigkeit wirkt.

$\nabla_{\mu} M^{\mu} = \hat{=} \nabla_{\mu} \varphi^{\mu}$

Zirkulation  $\Gamma$  ist ein Maß für die Drehstärke

$\Gamma := \oint_{\mathcal{C}} \vec{c} \cdot d\vec{x}$   $\Rightarrow \Gamma = \int_0^{2\pi} \tau d\varphi c_{\varphi} = 2\pi \tau c_{\varphi}$

z.B.  $\mathcal{C}$  ist ein Kreis  $d\vec{x} = r d\varphi \vec{e}_{\varphi}$



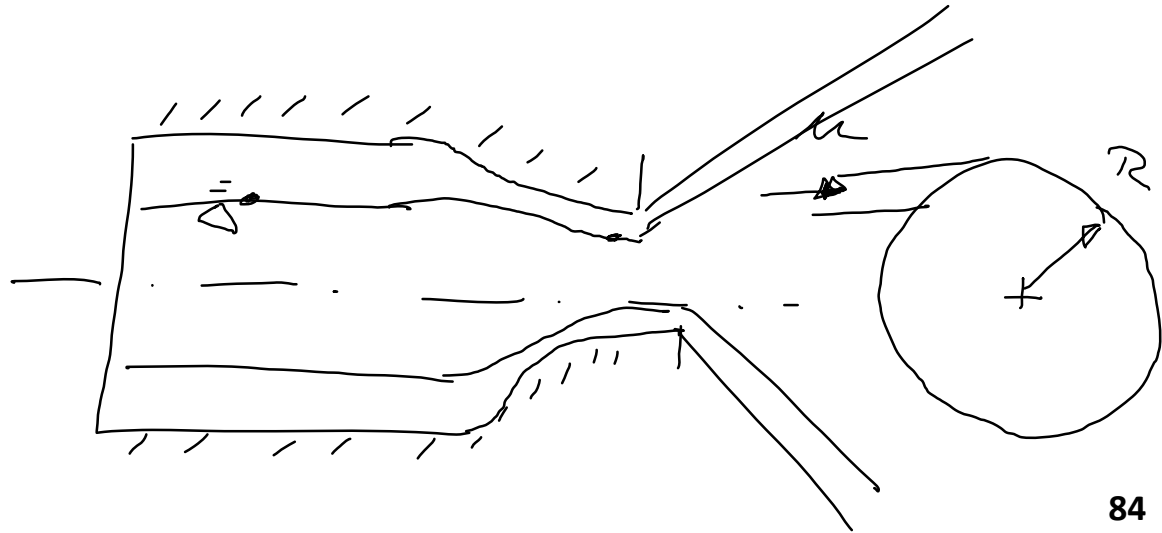
# Wirbel Senke

$$\vec{C} = C_r \vec{e}_r + C_\varphi \vec{e}_\varphi$$

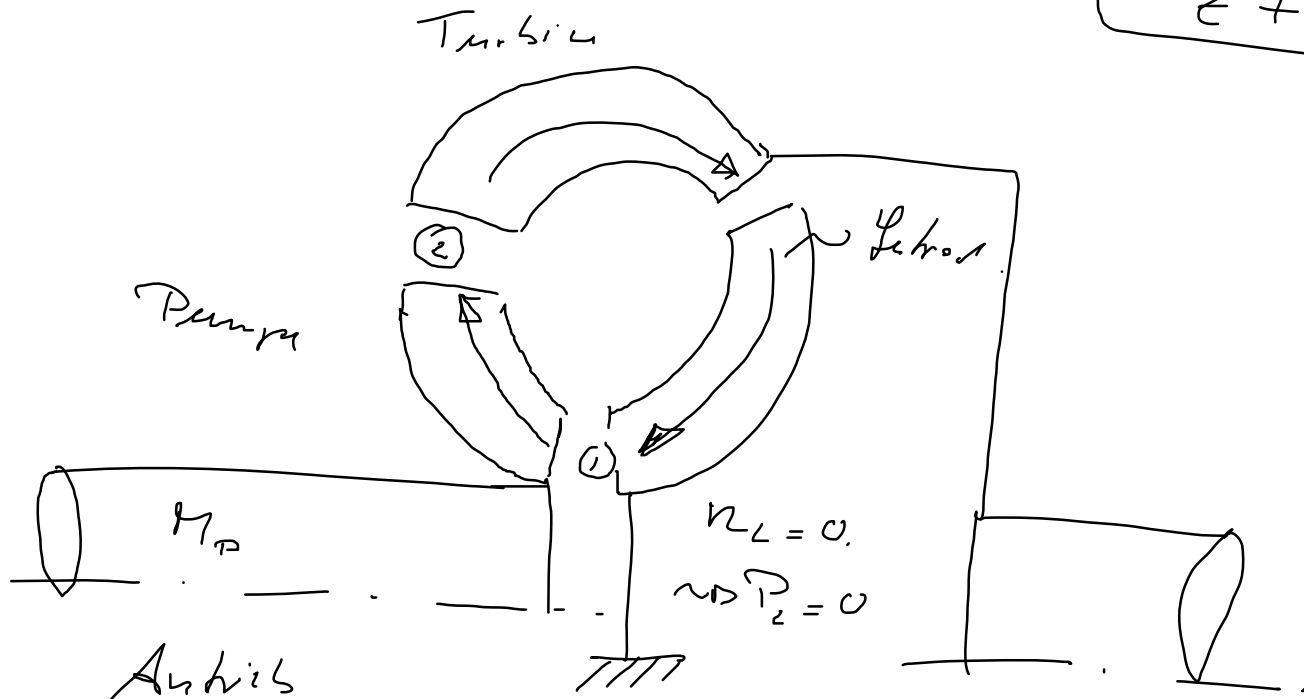
$$C_r = -\frac{\dot{V}}{2\pi r h} \sim \frac{1}{r}$$

$$C_\varphi = \frac{\Gamma}{2\pi r} \sim \frac{1}{r}$$

Ölbrennerdüse



Hydrodynamische Getriebe (Fotolyer, Voith)

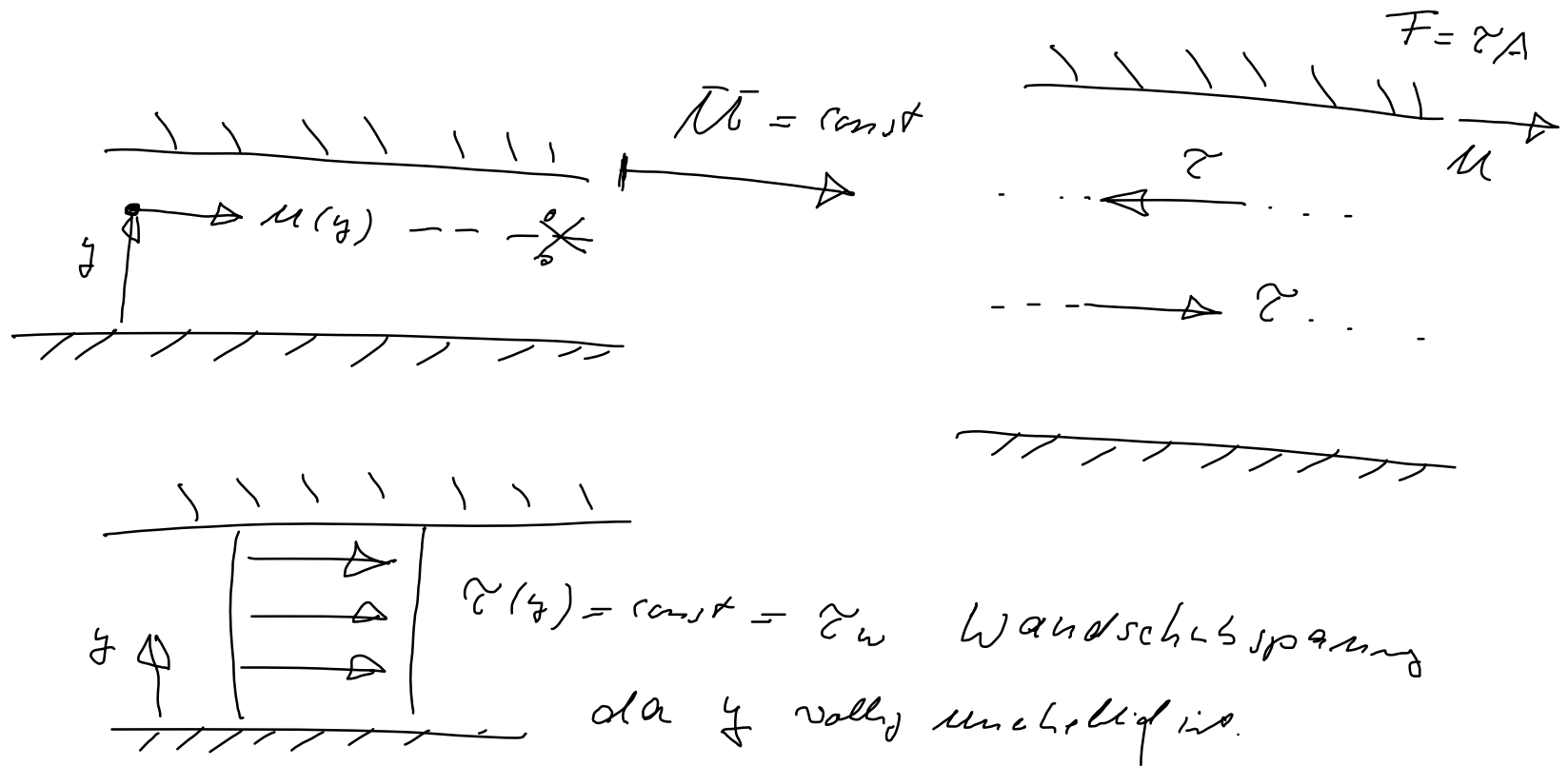


$M_P, M_T, M_L$  wird jeweils über die  
 $M_P + M_T + M_L = 0$

$M_P \Omega_P + M_T \Omega_T = 0 \quad \eta_f \approx 1$

# Schichtenströmung, viskose Strömung

Reibungsdominierte Strömungen.



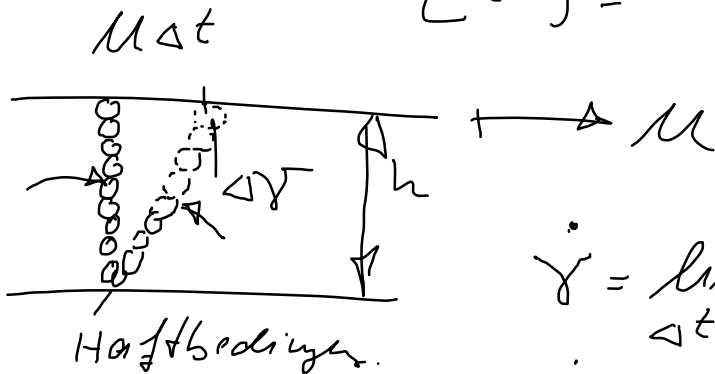
# Newtonsche Flüssigkeit

$$\tau = \eta \dot{\gamma}$$

Proportionalitätsfaktor  $\eta$  ist die dynamische Viskosität

$$\eta_{\text{H}_2\text{O}} = 1 \text{ mPa s}$$

$$[\dot{\gamma}] = \frac{1}{\text{s}}$$



$$\dot{\gamma} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \gamma}{\Delta t} = \frac{U \Delta t}{h \Delta t} = \frac{U}{h}$$

$$\dot{\gamma} = \frac{dU}{dy} \quad \text{verallgemeinert.}$$

$$\tau = \tau_w = \mu \frac{dU}{dy}$$

$$y \frac{\tau_w}{\mu} + C_1 = U(y)$$

Haftbed  $U(y=0) = 0 \Rightarrow C_1 = 0$ .

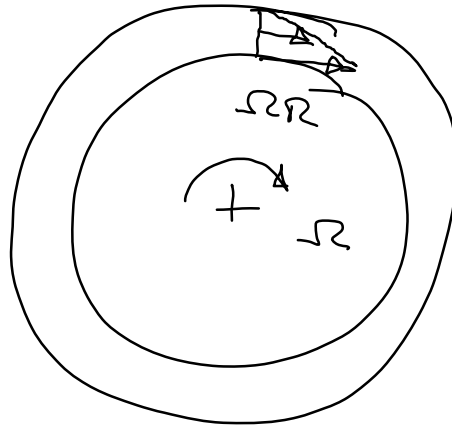
$$U(y) = \frac{\tau_w}{\mu} y$$

Schleppströmung  
Couette-Strömung.



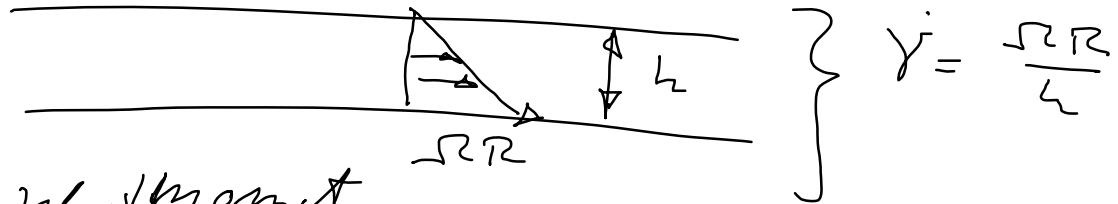
Anwendung

Tisch B



Wenn die Exzentrizität  $e$  zwischen Zylinder und Bohrer  $\equiv 0$

in der Abw. d.



Moment = Verformmoment

$$M = \tau_w 2\pi R^2 B$$

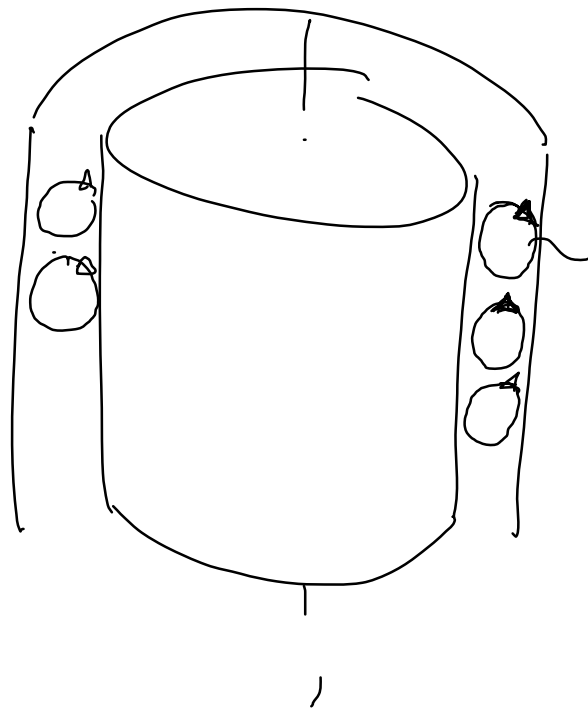
mit  $\tau_w = \gamma_w = \gamma \frac{\Omega R}{\omega}$

$$M = \int \frac{\Omega^2 R^3}{\omega} 2\pi B$$

Reynoldszahl

$$\frac{R^2 \Omega}{\nu} \approx \underbrace{\left( \frac{\Omega R^2}{\nu} \right)}_4 \approx \frac{\text{Trägheitkr.} \sim \rho (\Omega R)^2}{\text{visk. Kr.} \sim \eta \Omega}$$

$$\nu = \frac{\eta}{\rho}$$

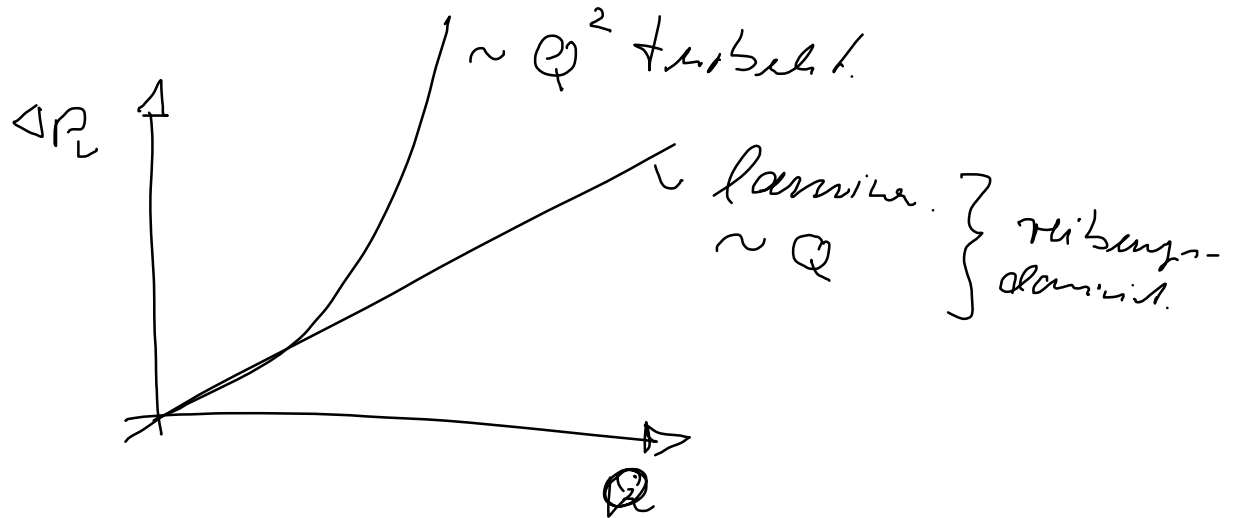
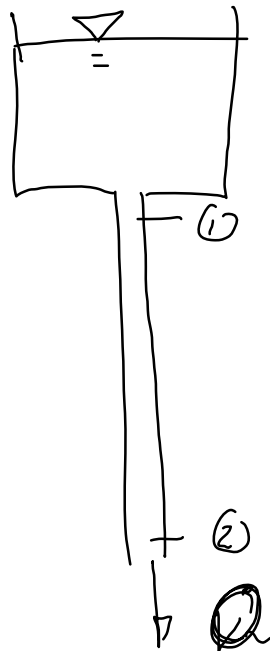


Taylor-Wirbel

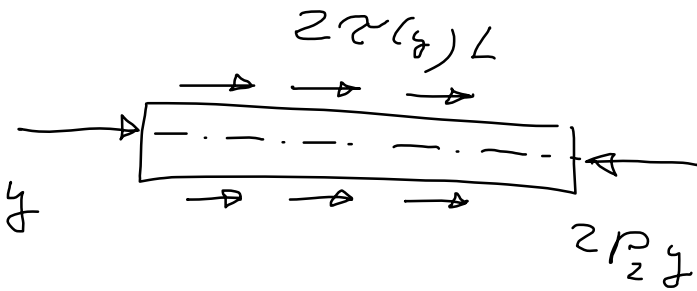
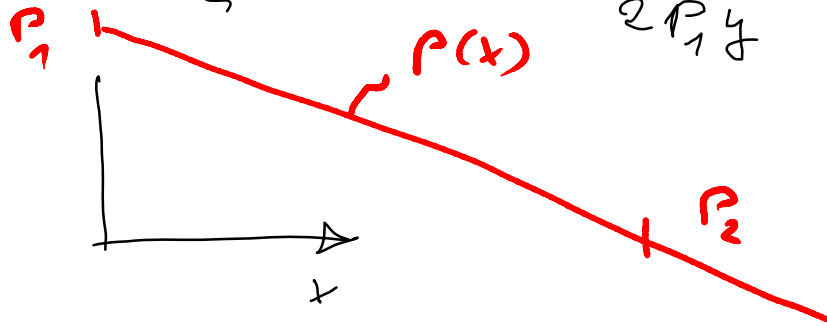
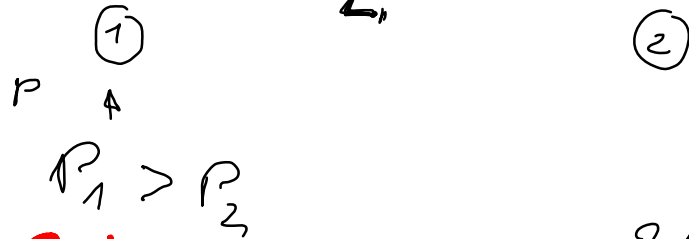
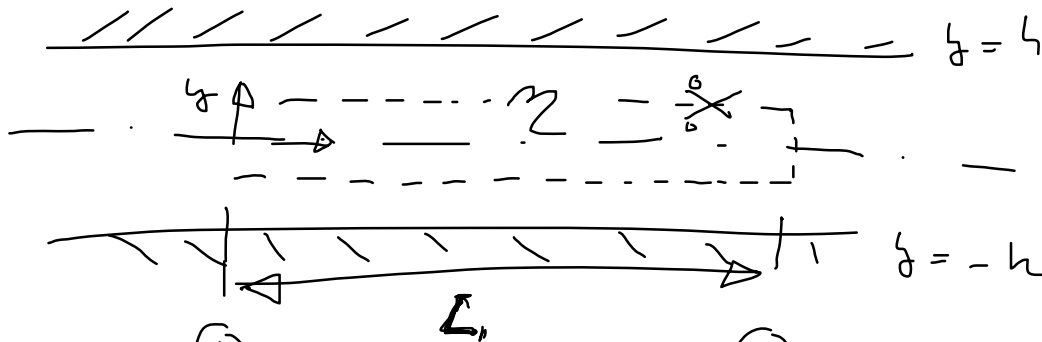
G.I. Taylor

Druckgetriebene Strömung Poiseuille-Strömung.

Frage. Antwort im 19ten Jahrhundert.



ebene Spalt / Kanal



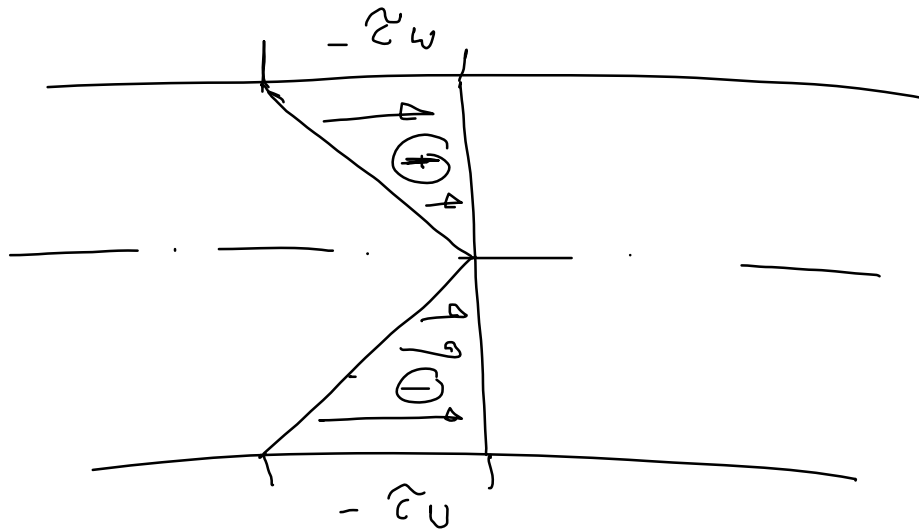
An der Wand  $y = \pm h$

$$\tau(\pm h) = - \frac{P_1 - P_2}{L} (\pm h)$$

$$\tau_h := \frac{P_1 - P_2}{L} h$$

$$2P_1 y + 2\tau(y)L - 2P_2 y = 0$$

$$\tau(y) = - \frac{P_1 - P_2}{L} y$$



$$\tau = \eta \dot{\gamma} = -\eta \frac{dM}{dy} \quad \text{Newtonsche Material.}$$

$$\tau = -\tau_w \frac{y}{h} \quad \text{Gleichschritt / Bewegung.}$$

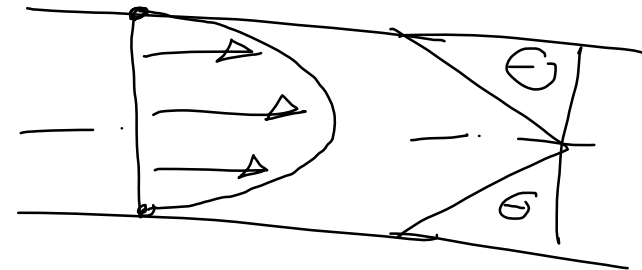
$$\frac{dM}{dy} = -\frac{\tau_w}{\eta} \frac{y}{h}$$

$$M = \frac{h^2 \omega}{2 \hbar} \left( \frac{y}{h} \right)^2 + C_1$$

Randbedingung

$$M(y=h) = 0 \quad \leadsto \quad C_1 = - \frac{h^2 \omega}{2 \hbar}$$

$$M(y) = \frac{h^2 \omega}{2 \hbar} \left( 1 - \left( \frac{y}{h} \right)^2 \right)$$



Volumenstrom pro Tiefenprofil

$$Q = 2h \int_0^1 u(y) d(y/h)$$

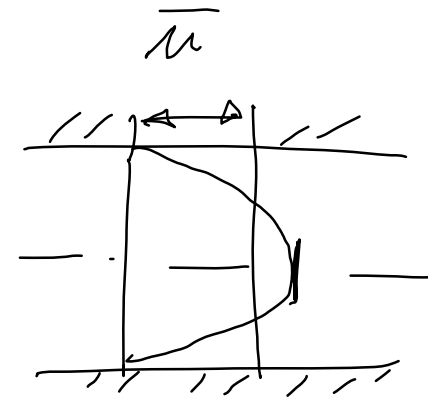
$$= 2h \int_0^1 \frac{h}{2} \frac{\tau_w}{\tau} \left[ 1 - \left( \frac{y}{h} \right)^2 \right] d\left( \frac{y}{h} \right)$$

$$= \frac{h^2 \tau_w}{\tau} \left( 1 - \frac{1}{3} \right)$$

$$= \frac{h^2 \tau_w}{\tau} \frac{2}{3}$$

mit der Bräunungsd.

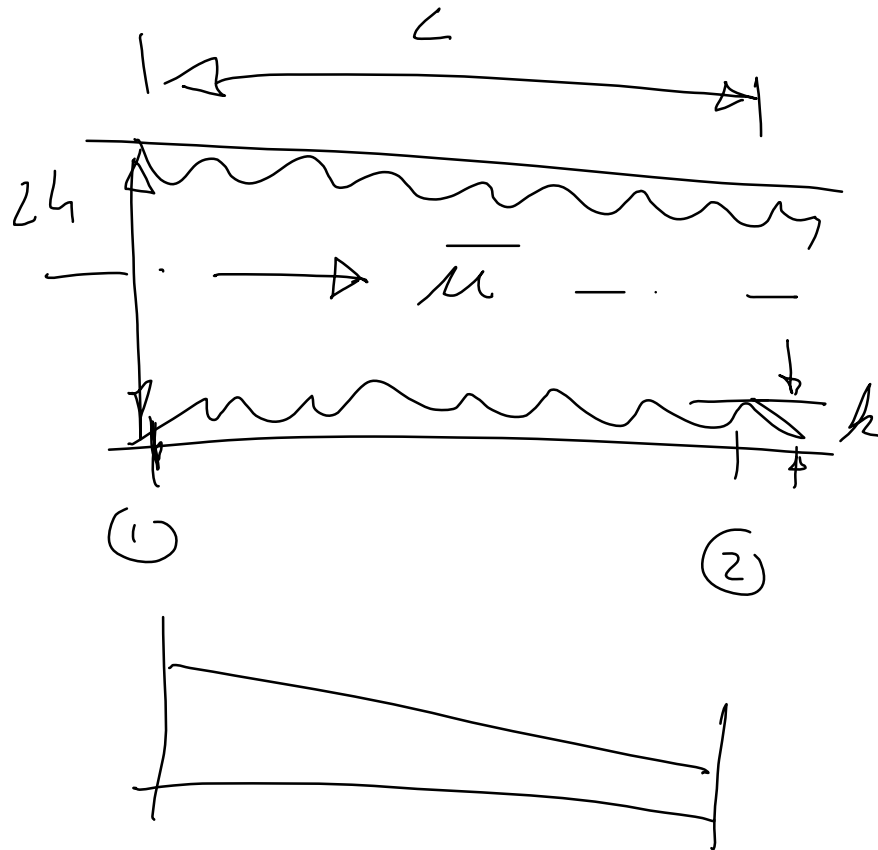
$$\bar{u} = \frac{Q}{2h} = \frac{1}{3} \frac{\tau_w h}{\tau}$$



$$u_{\max} = u(0) = \frac{1}{2} \frac{\tau_w h}{\tau}$$

$$\frac{u_{\max}}{\bar{u}} = \frac{3}{2} = 1.5$$

Dimensionslos Dentelley in Form  
 eines Widerstands  $\zeta$



$$\frac{(P_1 - P_2)_v}{\frac{\rho}{2} \bar{u}^2} = \zeta \left( \frac{L}{h}, \frac{\bar{u}}{c}, \frac{h}{h} \right)$$

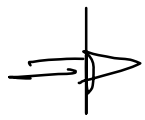
$$\zeta := \frac{P_1 - P_2}{\frac{\rho}{2} \bar{u}^2}$$

Vel.,  $h$ ,  $\mu$ .



$$\frac{(P_1 - P_2)_v}{\frac{\rho}{2} \bar{u}^2} := \int \left( \frac{L}{h}, \frac{\bar{u} h}{2}, \frac{h}{L} \right)$$

bei einer Schichtströmung ist der Druckverlust  $\sim L$



$$\frac{(P_1 - P_2)_v}{\frac{\rho}{2} \bar{u}^2} := \frac{L}{h} \underbrace{\int \left( \frac{\bar{u} h}{2}, \frac{h}{L} \right)}_{\text{Widerstandszahl}} \quad (1)$$

$$\bar{u} = \frac{1}{3} \frac{\sigma_w h}{\nu} \quad (2)$$



$$2L(P_1 - P_2) = 2\sigma_w L \approx \dots$$

$$\sigma_w = \frac{(P_1 - P_2)}{L} h \quad (3)$$

$$(1) \rightsquigarrow \frac{\tau_w}{\frac{\rho}{2} u^2} \frac{\cancel{h}}{\cancel{h}} = \frac{\cancel{h}}{\cancel{h}} \lambda \left( Re, \frac{h}{L} \right)$$

$$Re = \frac{\bar{u} L}{\nu}$$

Widerstandszahl  $\lambda \hat{=}$  dimensionslose  
Verdrickung

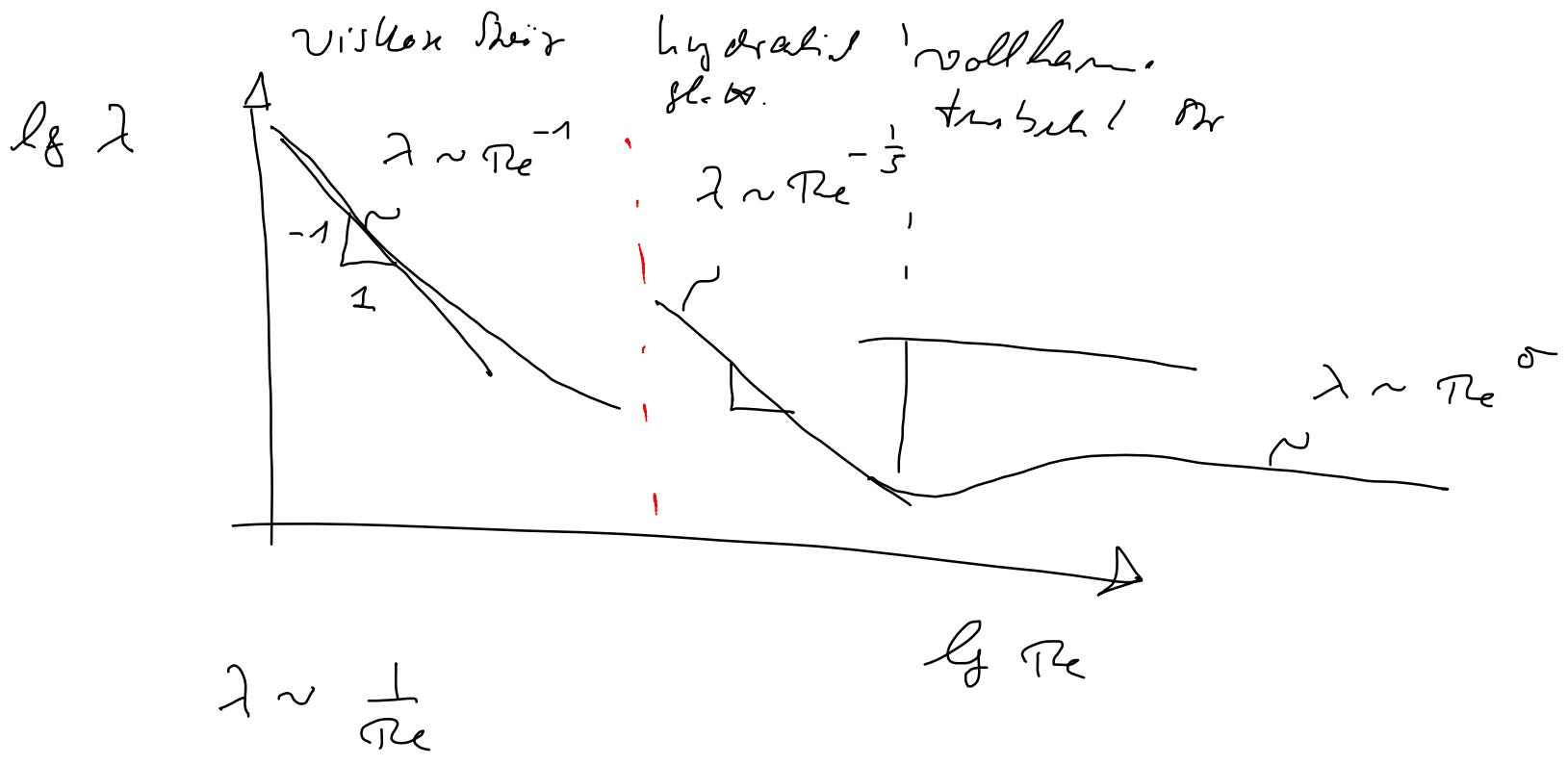
Für da oben nach.

$$(2) \quad \tau_w = \frac{\bar{u}^3 \zeta}{h}$$

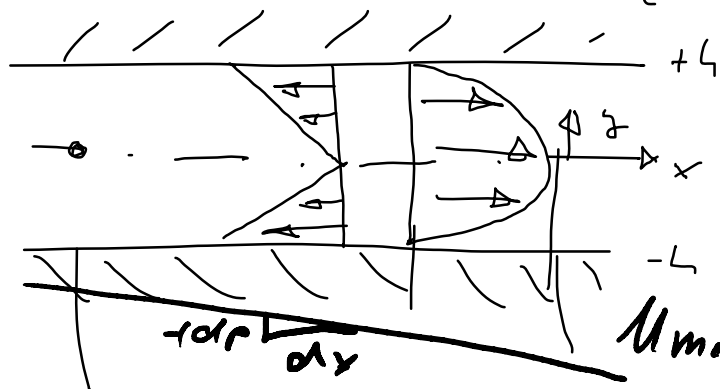
$$(2) \text{ in } (1) \quad \frac{2 \cancel{\bar{u}^3 \zeta}}{h \cancel{\rho} \bar{u}^2} = \lambda \left( Re, \frac{h}{L} \right)$$

$$\lambda = \frac{6}{\frac{\bar{u} h}{2}}$$

$$\lambda = \frac{6}{Re}$$



ebene Wand  $\tau(y) = \tau_w \frac{y}{h}$



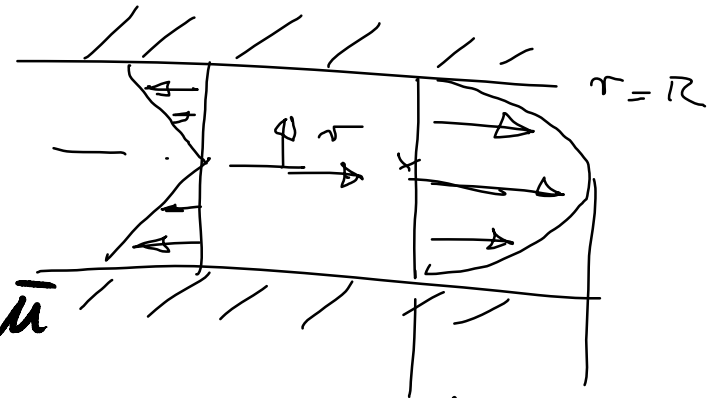
(1)

$$\frac{dp}{dx} < 0$$

(2)

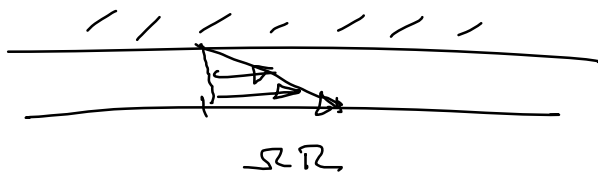
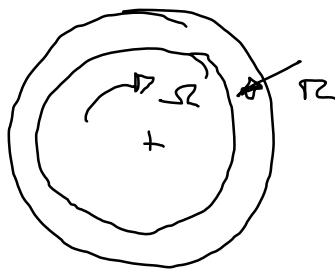
$$u_{\text{mov}} = \frac{3}{2} \bar{u}$$

Rohr  $\tau(r) = \tau_w \frac{r}{R}$



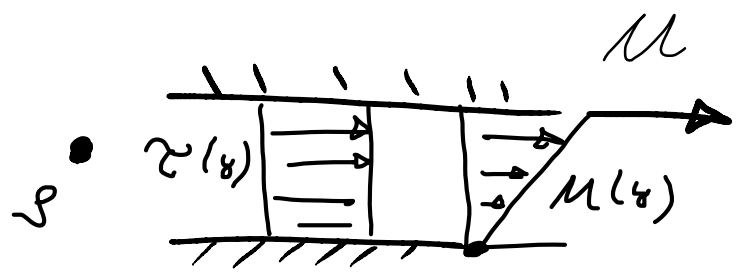
$$u_{\text{mov}} = 2 \bar{u}$$

Anwendung für ein Schleppkorn  
Tiefe  $\gg R$

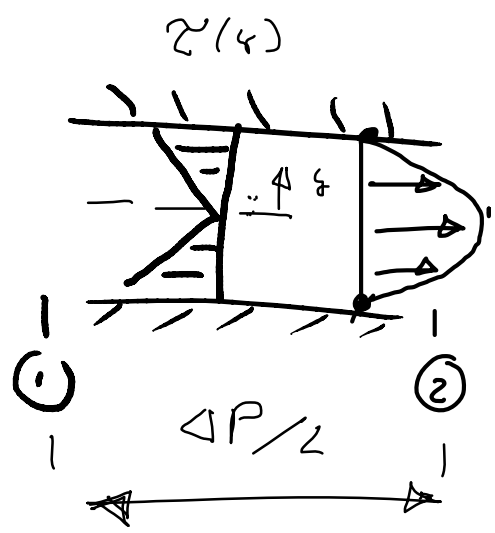


# Laminare, stationäre Strömung.

## Schleppströmung



## Druckströmung

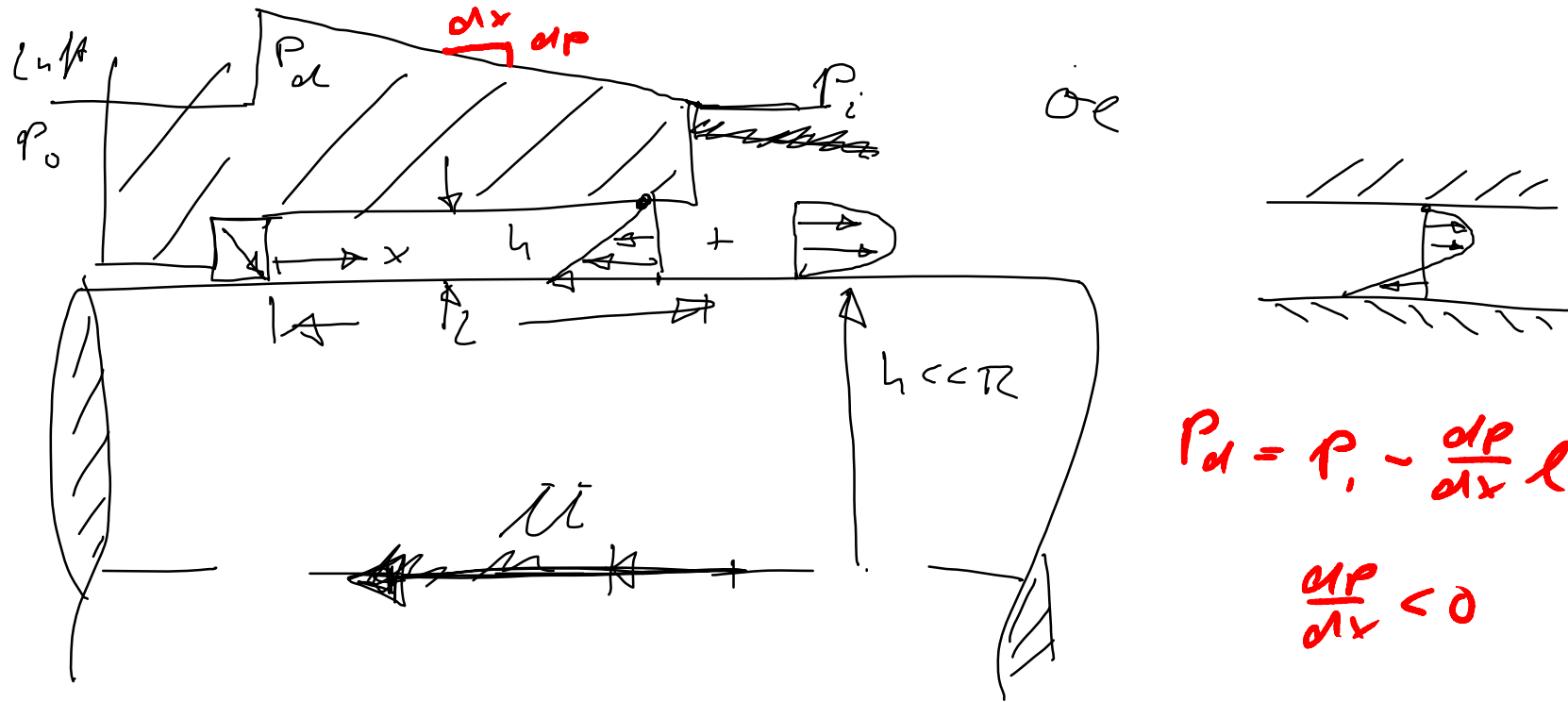


### Superposition

da Strömungsformel ist möglich,  
da lineare Bewegungsgleichung verbleibt

$$\rho \frac{D\mu}{Dt} = -\frac{dP}{dx} + \eta \frac{d^2\mu}{dy^2}$$

$$\frac{dP}{dx} = \eta \frac{d^2\mu}{dy^2}, \quad \text{da } \frac{D\mu}{Dt} \equiv 0.$$



$$P_d = P_i - \frac{dP}{dx} l$$

$$\frac{dP}{dx} < 0$$

1. Hypothese: Reine Schleppbewegung.

Haftbedingung im vorderen Ende ✓

Kontinuität im hinteren Ende.

$$Q = 2\pi R \frac{1}{2} \mu h^2 > 0 \quad \text{☹}$$

Leben

Gesamt volumenkons pro Tiefenheit

$$Q = \underbrace{-\frac{1}{2} \mu h^2}_{Q_{\text{canon.}}} + Q_{\text{Pois.}}$$

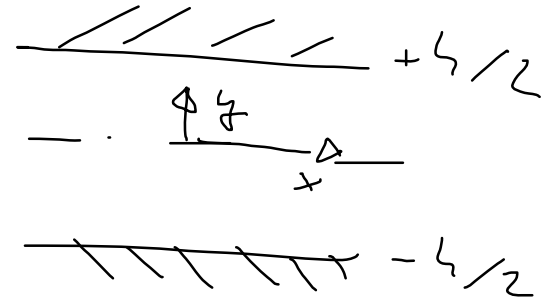
~~$$\frac{dP}{dx} = \mu \frac{d^2 M}{dy^2}$$~~

Bewegungsgleichung für die durchgehenden Str...

~~$$\frac{\mu}{2} \frac{dP}{dx} + \zeta_1' = \frac{dM}{dy}$$~~

~~$$\frac{1}{2} \frac{\mu^2}{\mu} \frac{dP}{dx} + \zeta_1' \mu + \zeta_2' = \mu(y)$$~~

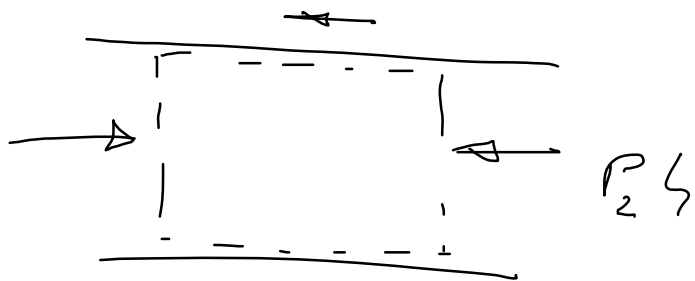
$$\mu(y = h/2) \equiv 0.$$



$$\tau_w = \frac{3 \bar{\mu} \eta}{h/2}$$

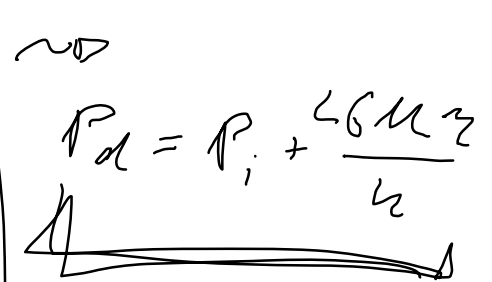
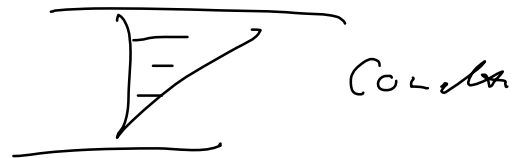
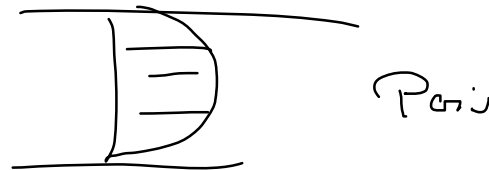
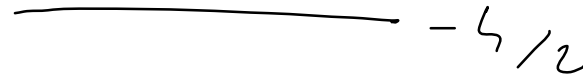
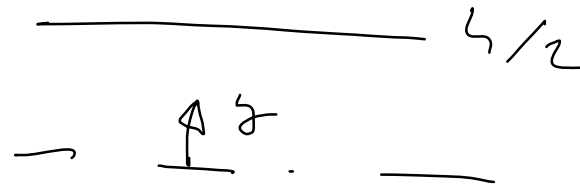
$$Q_{\text{Pois}} = \frac{\tau_w (h/2) h}{3 \eta}$$

$$Q_{\text{Pois}} = \frac{\tau_w h^2}{6 \eta}$$



$$P_1 h - P_2 h = 2 \tau_w L$$

$$\Rightarrow \frac{P_1 - P_2}{L} = \frac{2 \tau_w}{h}$$



$$Q_{\text{Pois}} = \frac{h^3}{2} \frac{P_1 - P_2}{L} \frac{1}{6 \eta}$$

$$\frac{1}{2} \mu h = \frac{h}{6 \eta} \frac{P_d - P_i}{L} \frac{1}{6} \quad 104$$



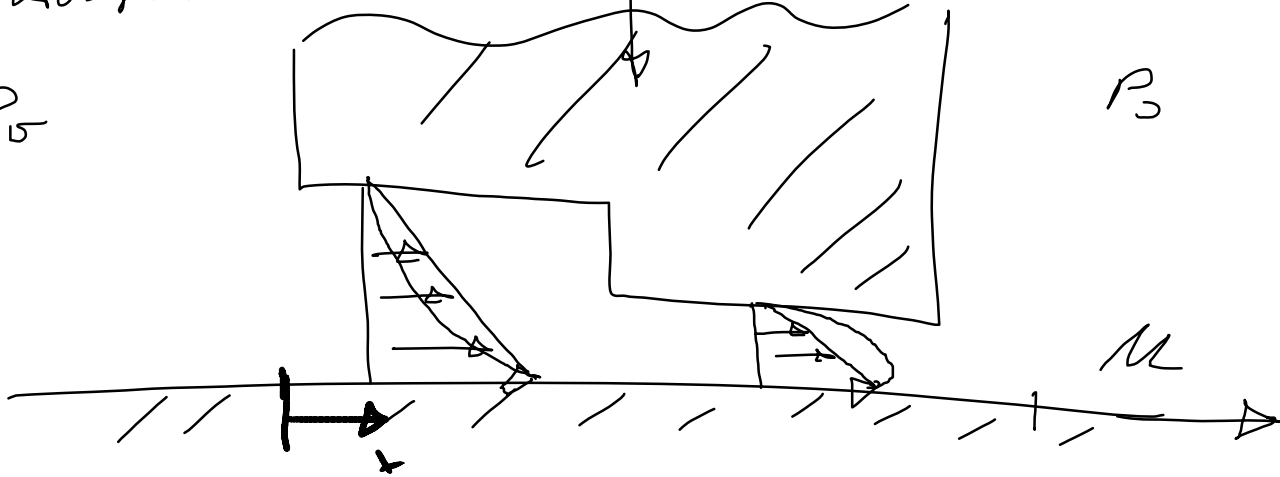
Stufenlex.

einfachste Hydrodynamisches  
Lex.

$P_5$

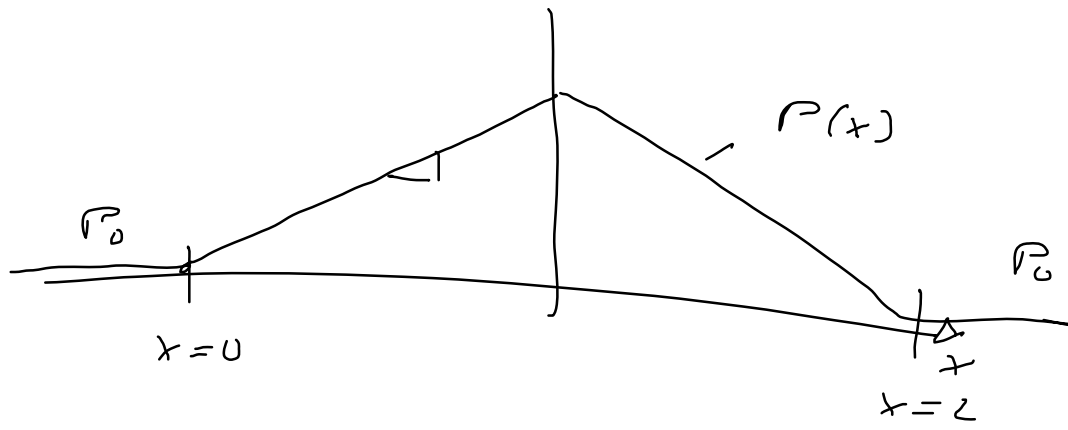
$$F = \int_0^L p(x) dx$$

Kraft pro  
Teileneinheit



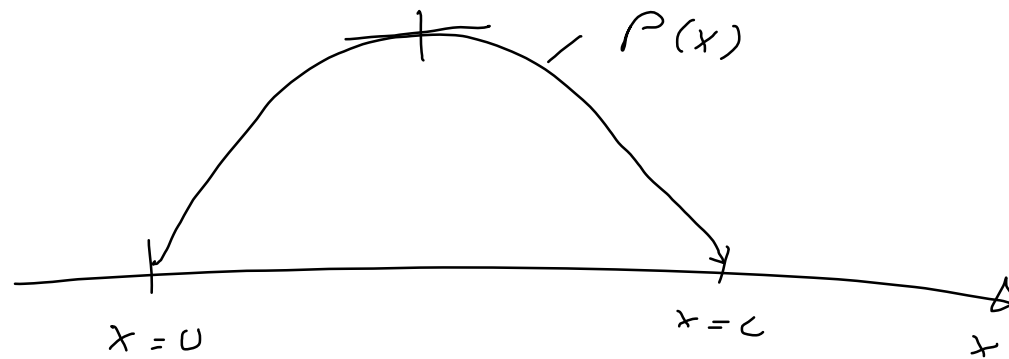
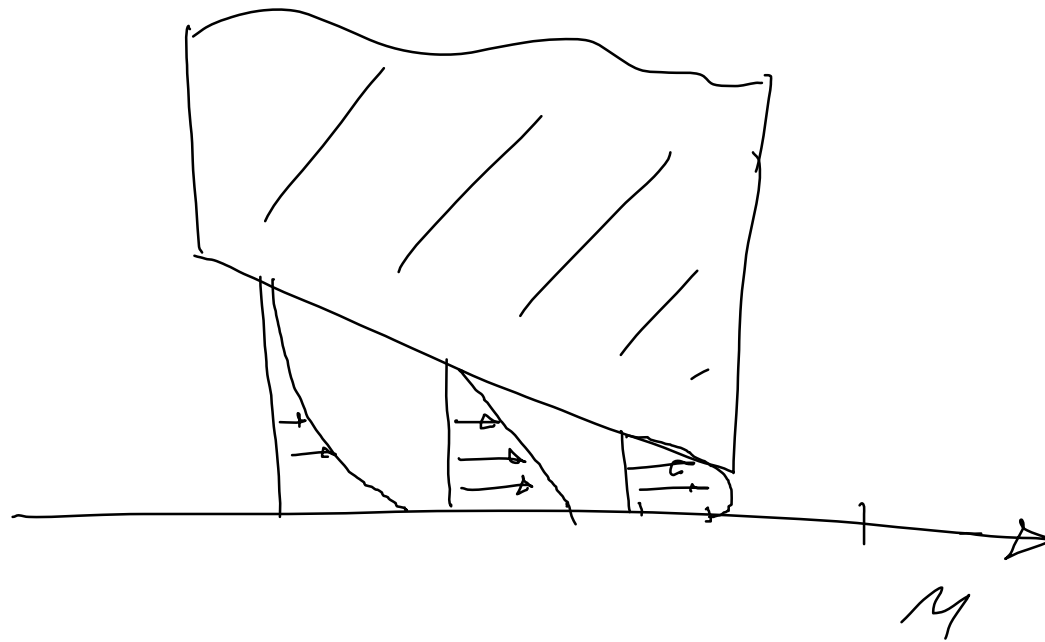
$$\frac{dp}{dx} > 0$$

$$\frac{dp}{dx} < 0$$

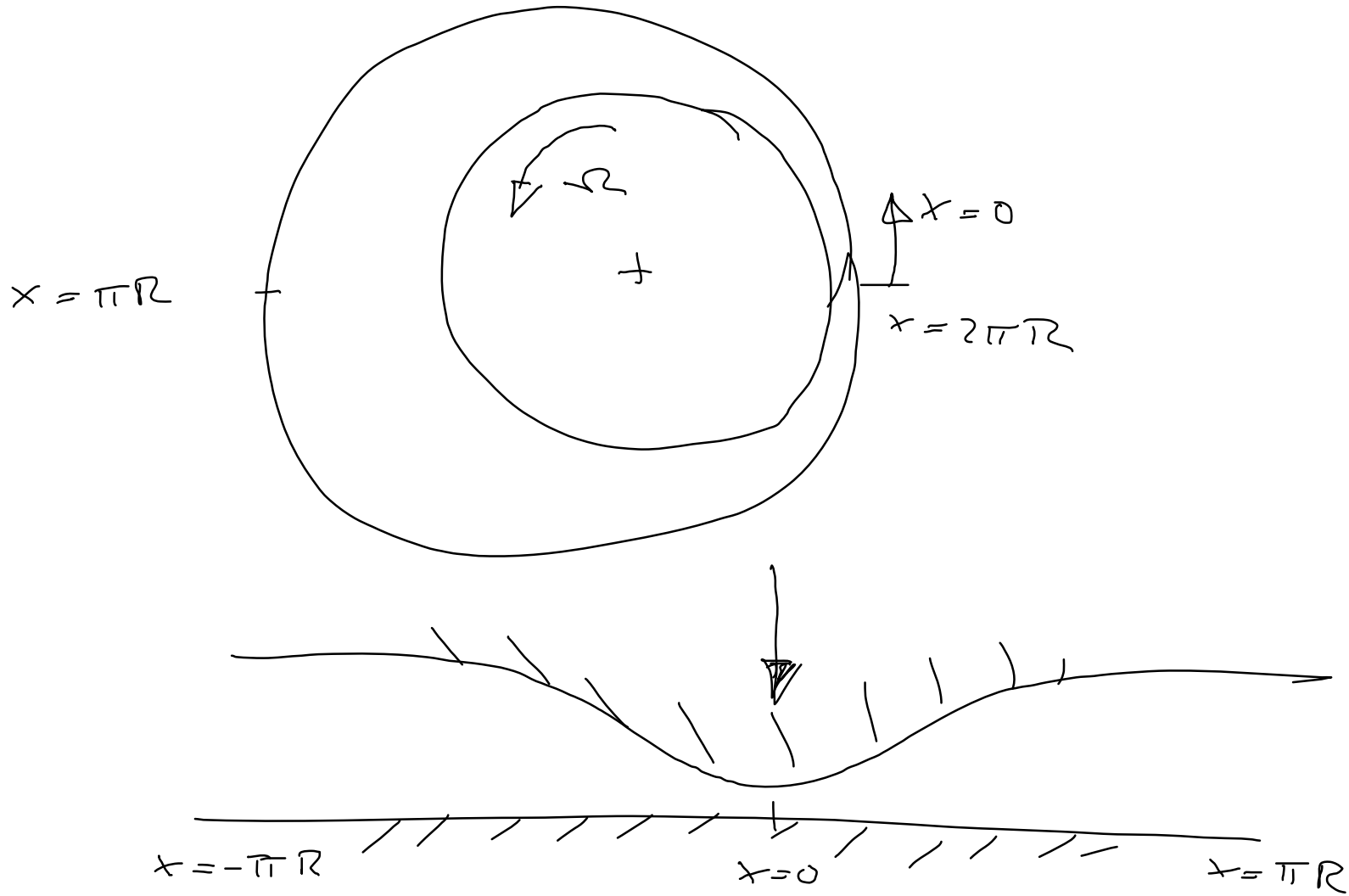


Veilleg

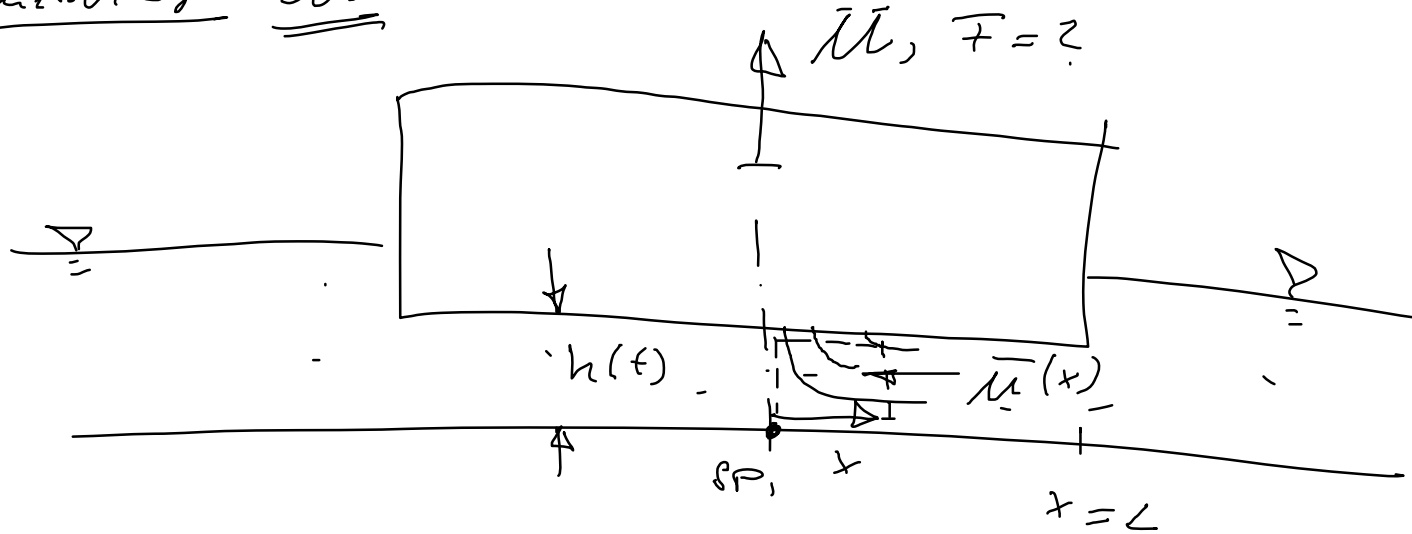
Anwendung  
Axialleg.



Radialtyp.



Querschnitt eben



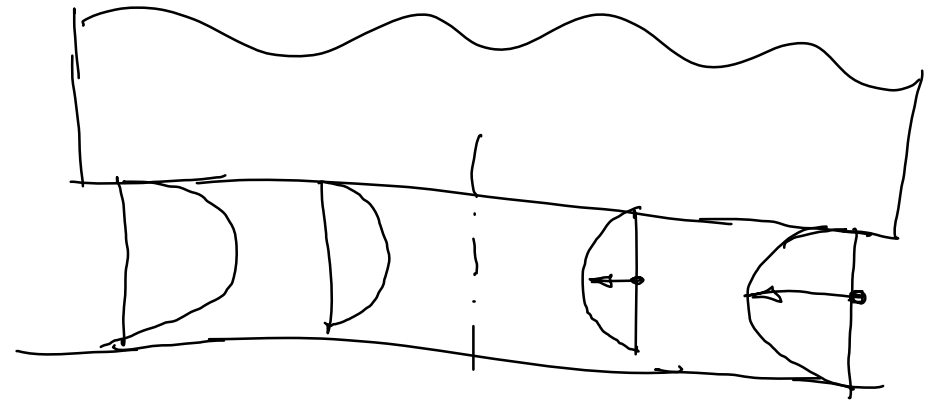
$$\frac{dh}{dt} = M = \dot{h}$$

$$\bar{M}(x) h(t) = x M$$

$$\bar{M}(x,t) = x \frac{\dot{h}}{h}, \text{ mit } M = \dot{h}$$

$$\rho \frac{DM}{Dt} = -\frac{dP}{dx} + \sum \sigma_{j1}$$

$$\leadsto \tau_w(x,t) \approx \frac{\bar{M}(x,t) \sum \sigma_{j1}}{h(t)}, \text{ sofern } \rho \frac{DM}{Dt} \ll \left| \frac{dP}{dx} \right|$$



Annahme: Gelad (x) und zeitl. (t)  
ausgehende Lösung wird  
angenommen.

$$\rho \dot{h}^2 \ll \eta \frac{\dot{h}}{h}$$

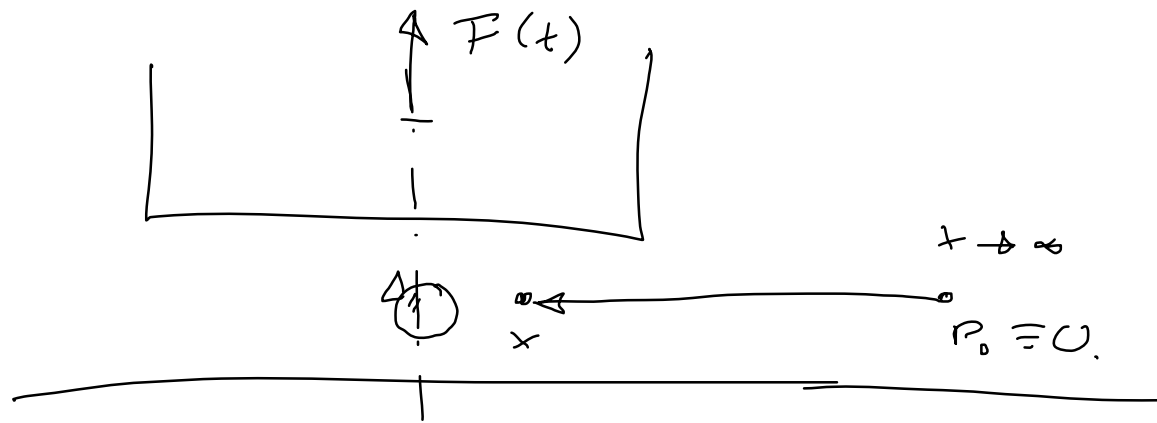
$$\frac{\rho \dot{h}^2 h}{\eta \dot{h}} = \frac{\rho \dot{h} h}{\eta} \ll 1$$

Tregheit  $\ll$  viskose Spannung.

Reynoldszahl  $Re := \frac{\rho U L}{\eta}$

$$\tau_w \sim \rho(x,t) \sim F(t)$$

Wenn  $Re \gg 1$   $\sim$  Bernoulli:

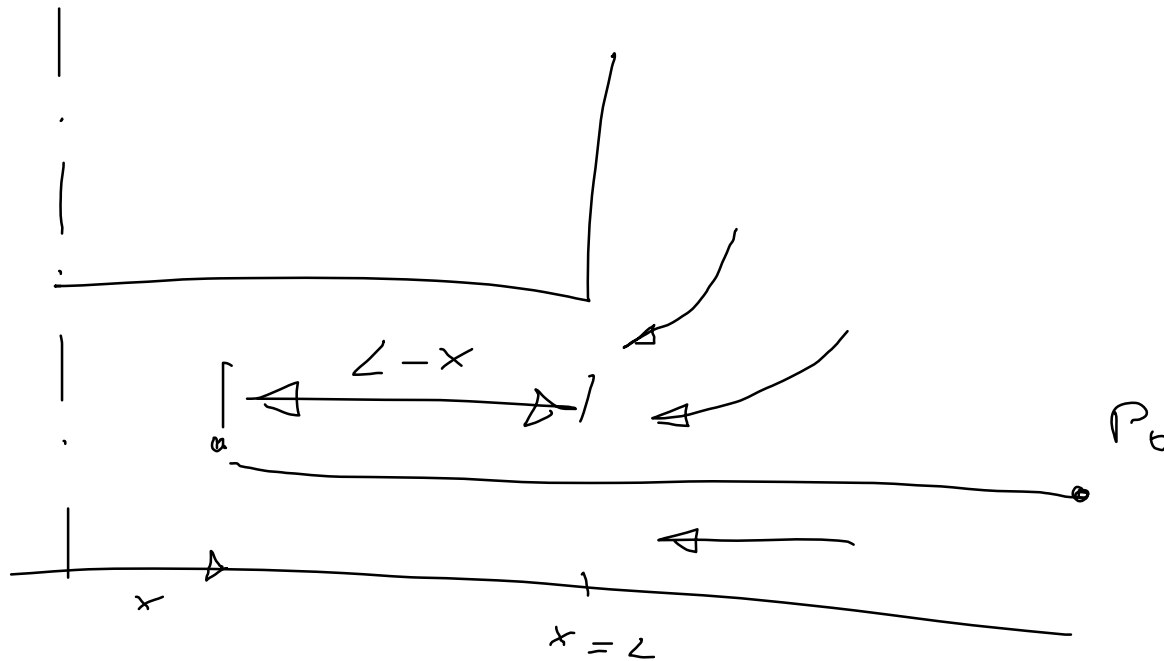


$$gh^2 \gg \nu \frac{h}{L}$$

$$\bar{u}(x,t) = x \frac{h}{L}$$

$$p(x) + \frac{\rho}{2} \bar{u}^2(x,t) = \sigma$$

$$\leadsto p(x,t) \leadsto F(t)$$

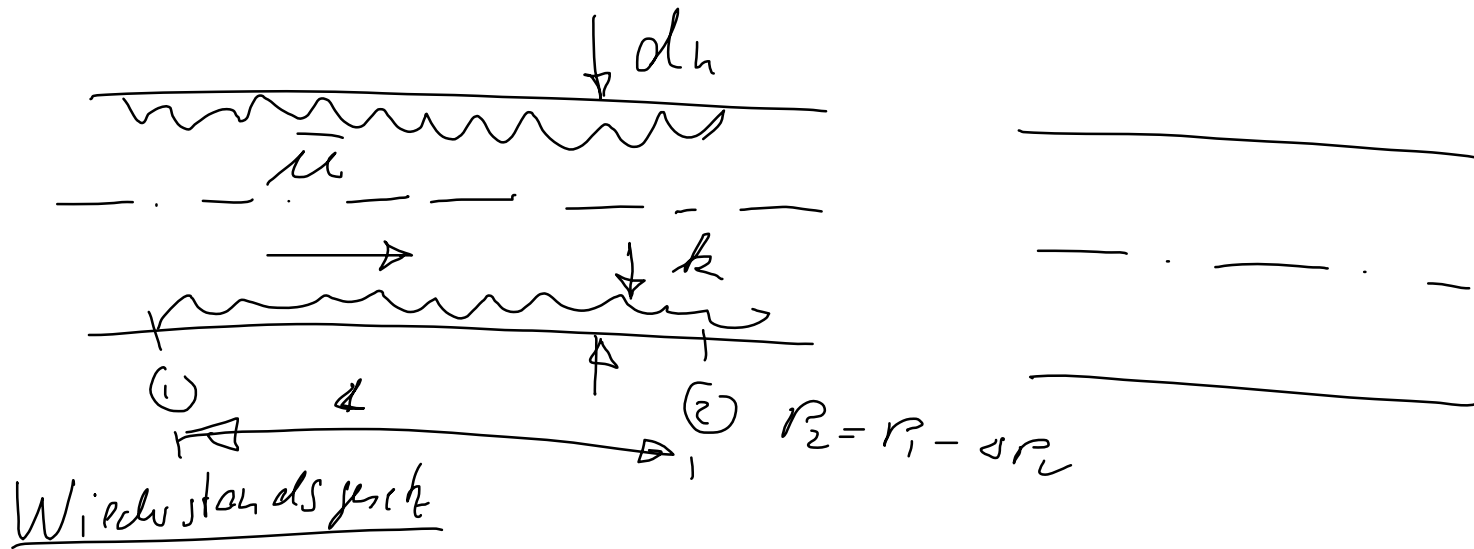


$$P_0 + \frac{\rho}{2} U_0^2 = \rho(x,t) + \frac{\rho}{2} \bar{u}(x,t)^2 + \int_{x=L}^x \rho \dot{u}(x,t) dx$$

$$\bar{u}(x,t) = x \frac{\dot{u}}{L}$$

$$\left(\frac{\dot{u}}{L}\right)^2 = \frac{\ddot{u}^2}{L} - \frac{\dot{u}^2}{L^2}$$

$$\rho \left(\frac{\dot{u}}{L}\right)^2 (L-x) +$$



$$\lambda = \frac{\Delta P_v}{L} \frac{dh}{\frac{\rho}{2} \bar{u}^2} = \lambda(Re, \frac{k}{d})$$

Kreisrohr.

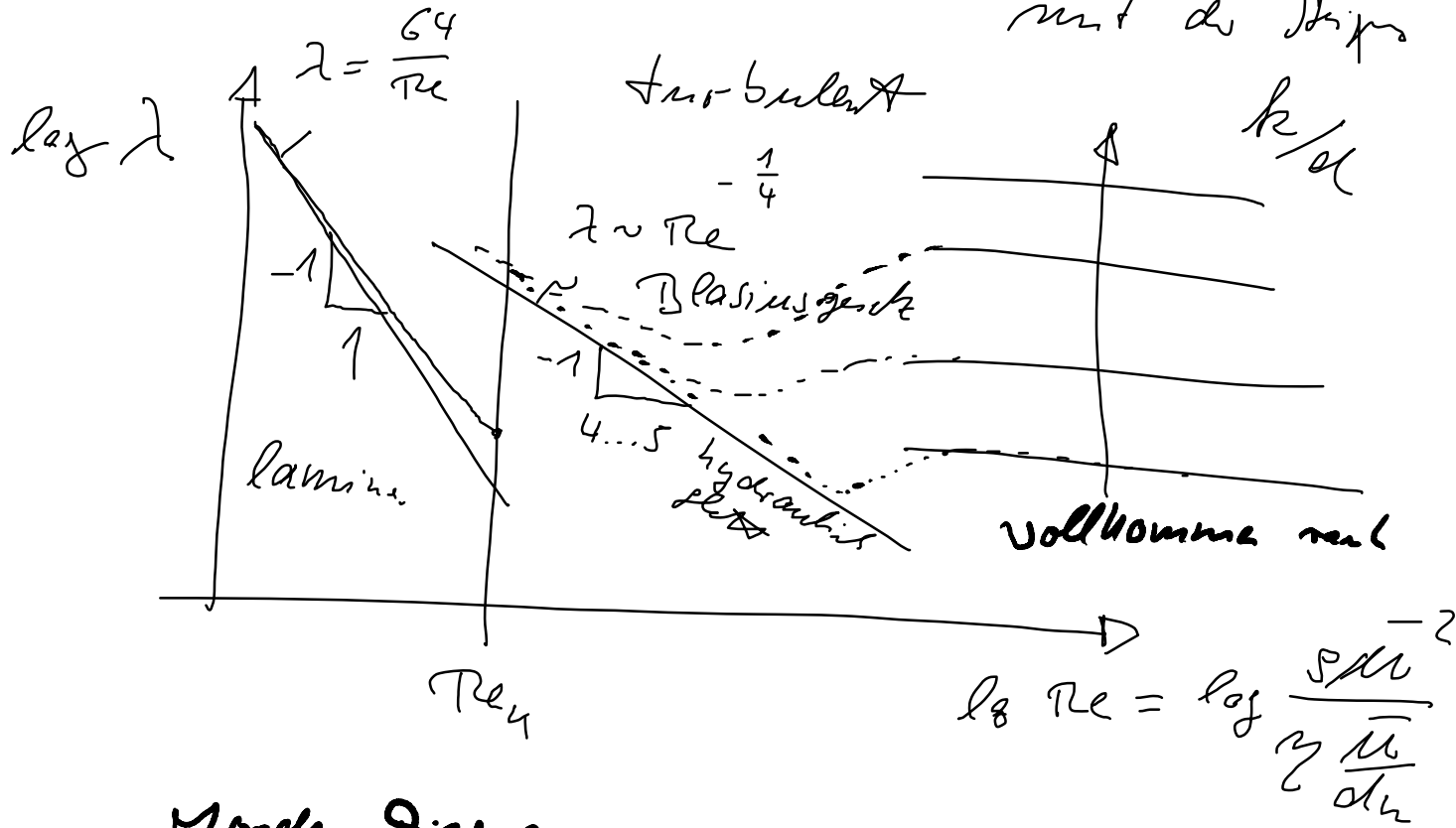
$\lambda = \frac{64}{Re}$

für den ~~oben~~ Kanal bei laminar Ström.

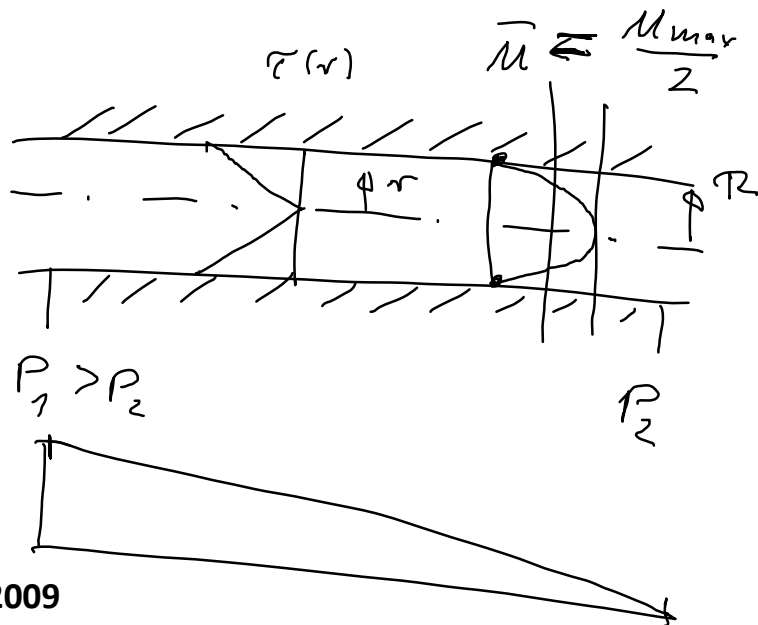
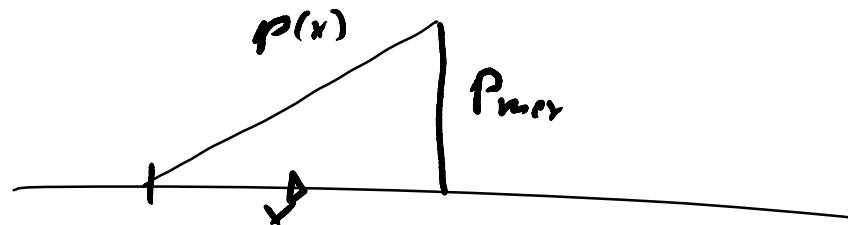
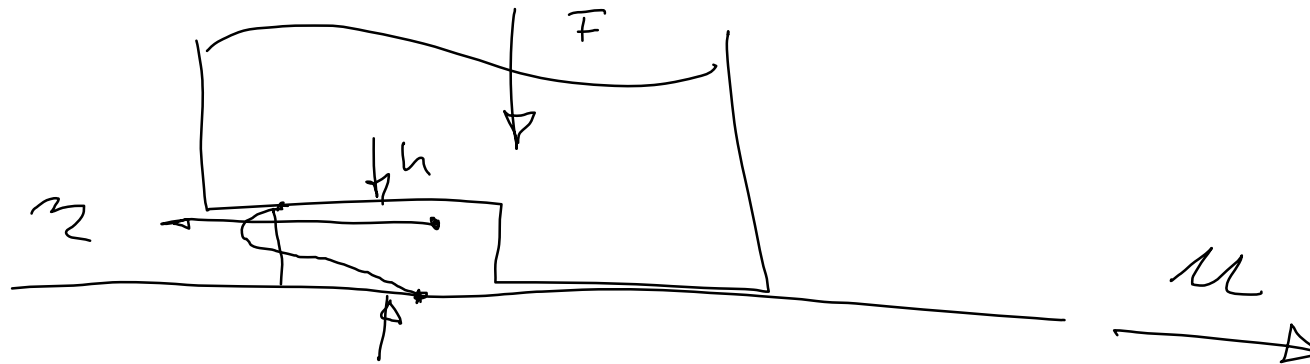
Bei laminar Strömung spielt die Rauigkeit  $k/d$  keine Rolle!

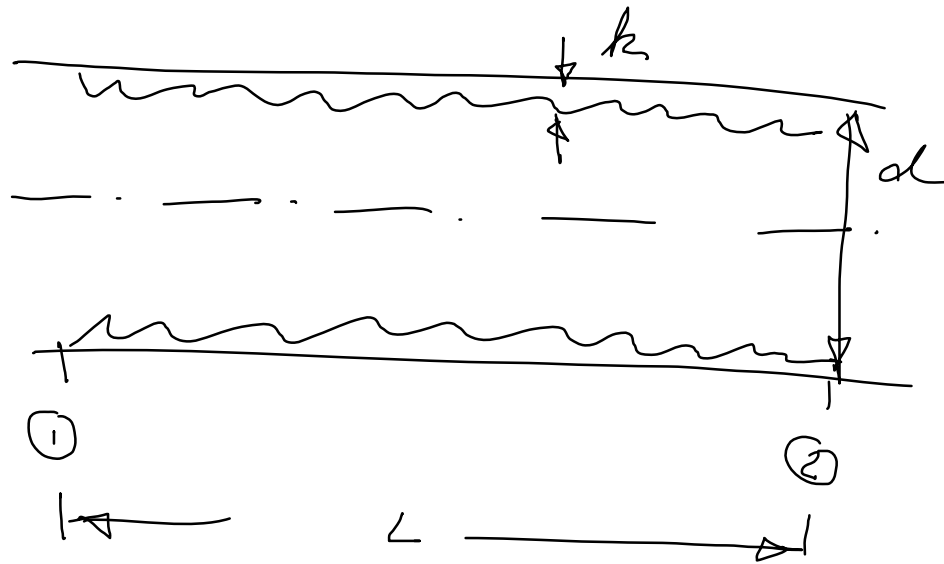


$\lg \lambda = \lg 64 - \lg Re.$  lineare Funktion  
 mit der Steigung  $-1$



Mody Diagramm.





$$\frac{\Delta P_v}{\frac{\rho}{2} \bar{u}^2} \frac{dk}{L} = \lambda \left( Re, \frac{k}{d} \right)$$

$$Re = \frac{\frac{\rho}{2} \bar{u}^2}{\gamma \frac{\bar{u}}{d}} = \frac{\rho \bar{u} d}{\gamma}$$

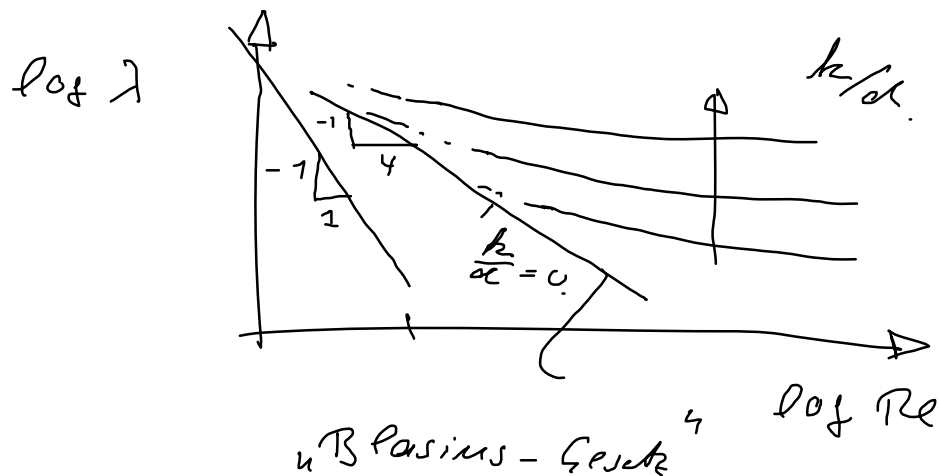
viskose Grenzfall

$$\lim_{Re \rightarrow 0} \lambda(Re, \frac{k}{\alpha}) = \frac{const}{Re} \quad \Delta P_v \neq f_v(\dot{q})$$

$$\lambda \sim \frac{1}{Re} \quad \lambda = \frac{64}{Re} \text{ für das Kreisrohr.}$$

turbulenter Grenzfall

$$\lim_{Re \rightarrow \infty} \lambda(Re, \frac{k}{\alpha}) = \lambda(\frac{k}{\alpha}) \quad \Delta P_v \neq f_v(\dot{q})$$



$$5 \cdot 10^3 \lesssim Re \leq 10^5$$

$$\lambda = 0.316 Re^{-\frac{1}{4}}$$

Blasius-Gesetz  
"Fit"

$$\frac{1}{\sqrt{\lambda}} = 2 \log(Re \sqrt{\lambda}) - 0.8 \quad \text{Prandtl-Gesetz.}$$

$$\frac{1}{\left(2 \log(Re \sqrt{\lambda^{(k)}}) - 0.8\right)^2} = \lambda^{(k+1)} \quad \lambda^{(0)} = 0.02$$

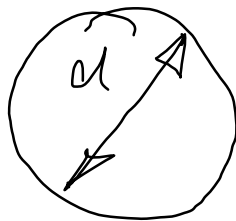
Re ✓

Fixpunktiteration.

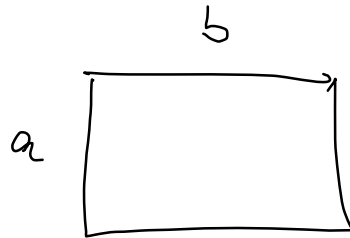
Im  $Re \rightarrow \infty$  spielt die Viskosität nur  
dann die Reynoldszahl keine Rolle

$$\lambda = \left( 2 \log \frac{d}{2k} + 1,74 \right)^{-2}$$

Kreisrohr

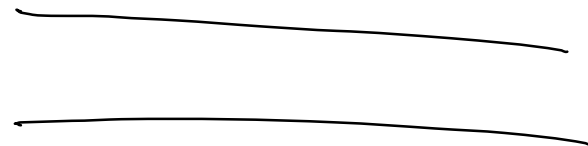


Rechteckrohr



Planrohr

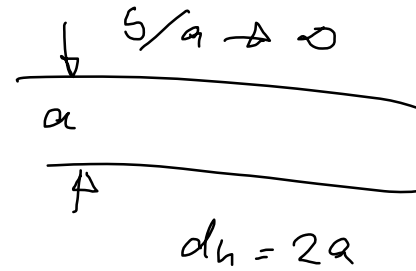
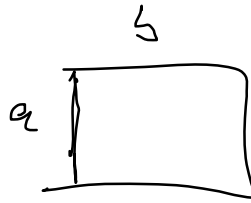
$$b/a \rightarrow \infty$$



Bei turbulenter Strömung kann die Widerstandszeit als  
Kreisrohr verwendet werden.

hydraulischer Durchmesser

$$d_h := \frac{4A}{u} = \frac{4 \text{ Querschnitt}}{\text{Umfang}}$$



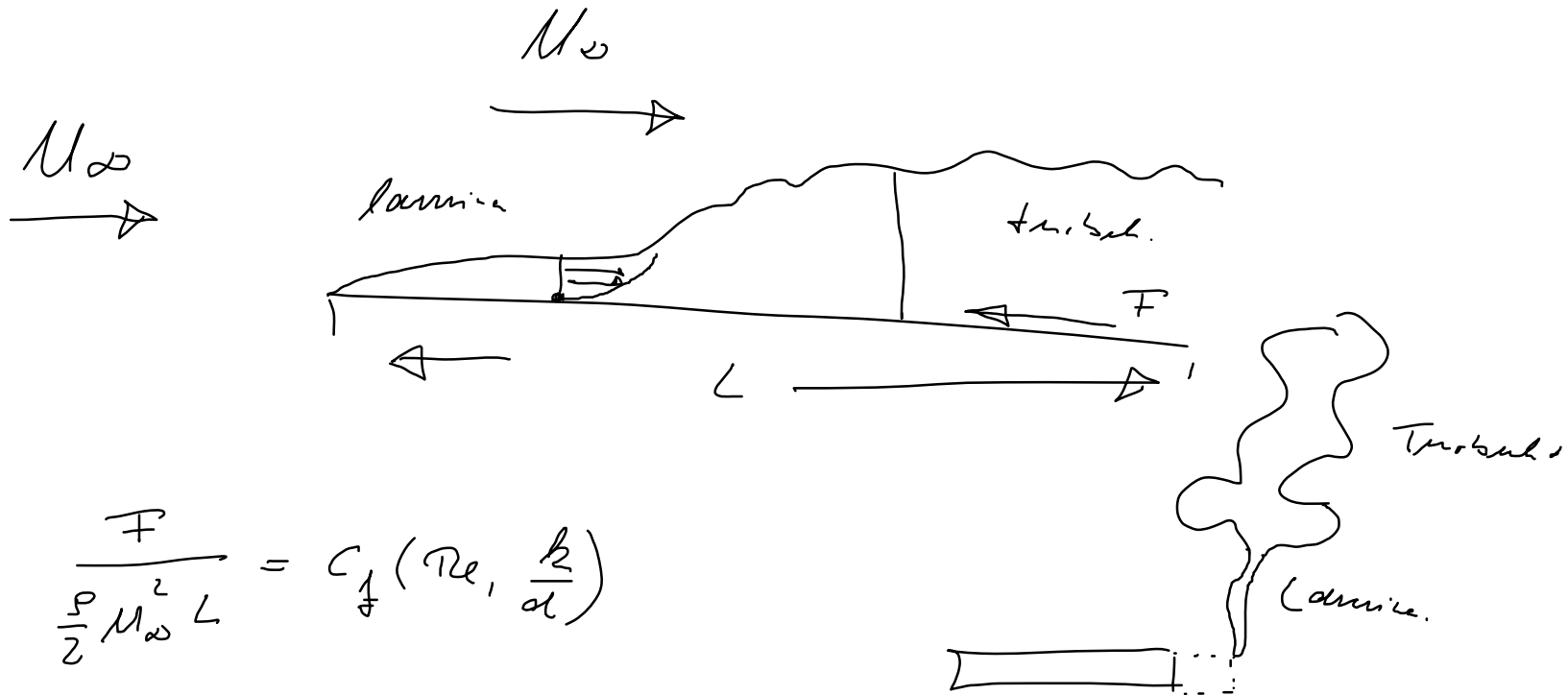
$$d_h = \frac{4 \frac{\pi}{4} d^2}{\pi d} = d \quad \checkmark$$

$$d_h = \frac{4ab}{2(a+b)}$$

$$\lim_{\frac{a}{b} \rightarrow 0} 2a \frac{1}{\frac{a}{b} + 1} = 2a$$

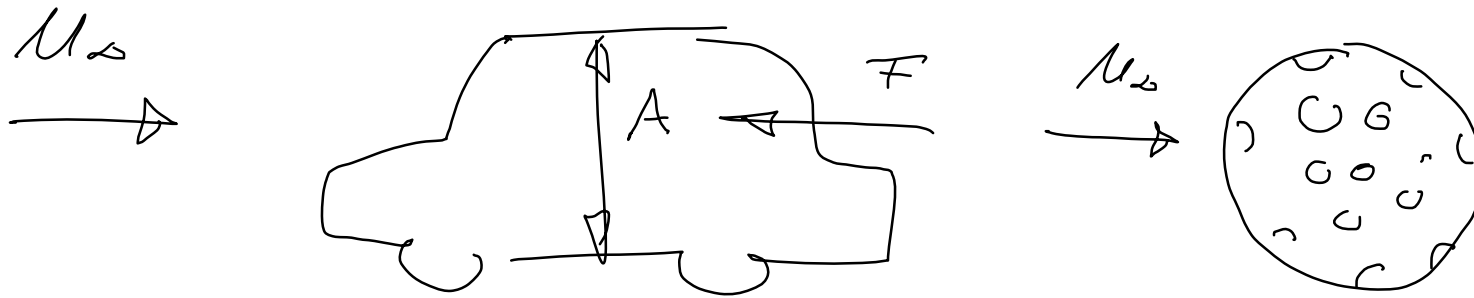
$$= 2a \frac{b}{a+b}$$

$$= 2a \frac{1}{\frac{a}{b} + 1}$$



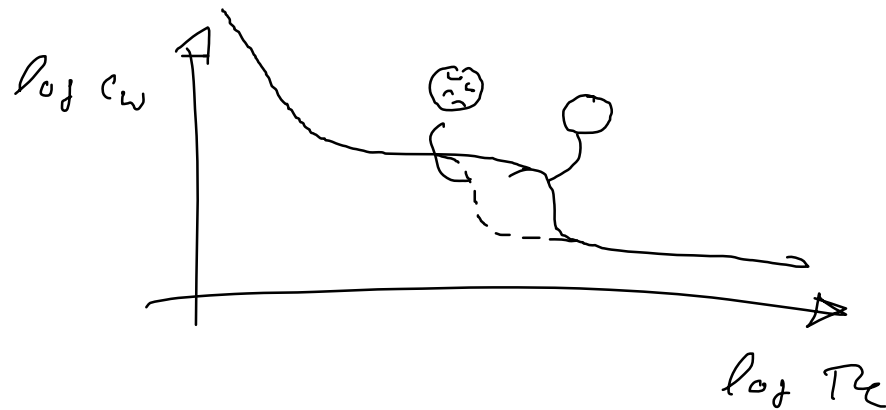
$$\frac{F}{\frac{\rho}{2} U_\infty^2 L} = C_f \left( Re, \frac{k}{\alpha} \right)$$

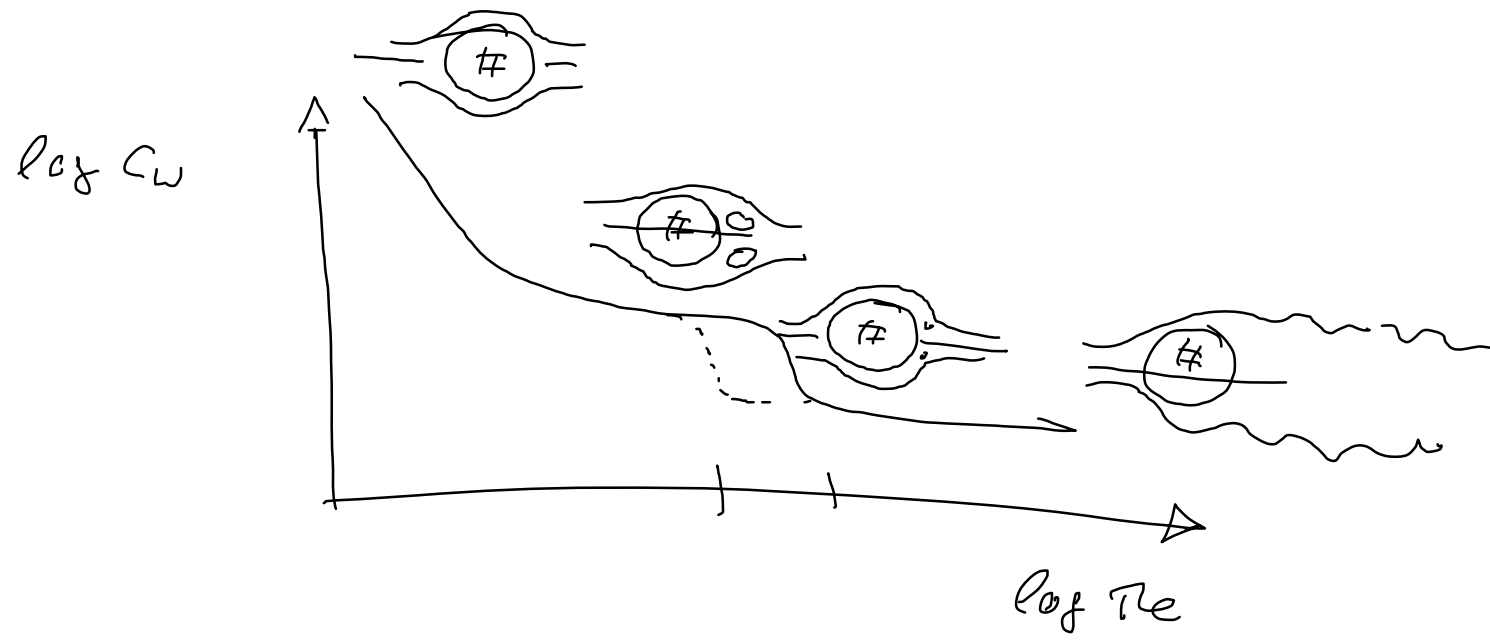


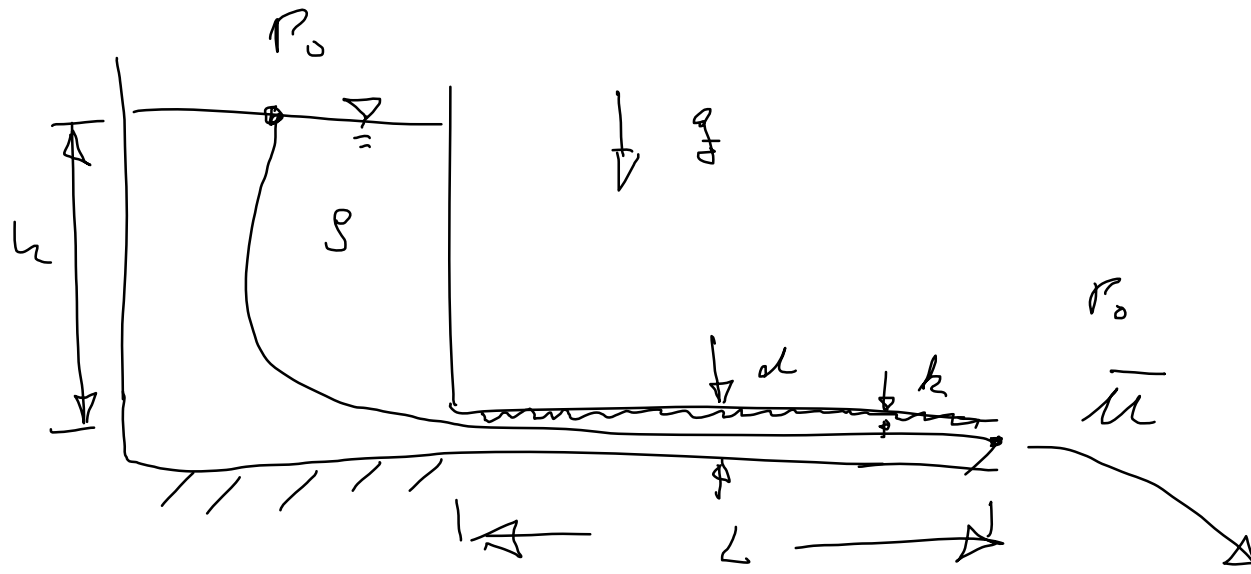


$$\frac{F}{\frac{\rho}{2} U_\infty^2 A} = C_w \left( Re, \frac{k}{L} \right)$$

$$L := \sqrt{A}$$







Verlust freier Bernoulli:

$$P_0 + 0 + \rho g h = P_0 + \frac{\rho}{2} \bar{u}^2 (+ \Delta P_v)$$

$$\overset{\Delta P_v = 0}{\leadsto} \bar{u} = \sqrt{2gh} \quad \text{oder}$$

$$\Delta P_v = \frac{\rho}{2} \bar{u}^2 \} = \frac{\rho}{2} \bar{u}^2 \frac{L}{d} \lambda(\text{Re}, \frac{k}{d})$$

$$Re \ll Re_k$$

$$Re_d = 2300$$

$$\begin{aligned} \rho g h &= \frac{\rho}{2} \bar{u}^2 + \frac{\rho}{2} \bar{u}^2 \zeta \\ &= \frac{\rho}{2} \bar{u}^2 (1 + \zeta) \end{aligned} \left. \vphantom{\begin{aligned} \rho g h &= \frac{\rho}{2} \bar{u}^2 + \frac{\rho}{2} \bar{u}^2 \zeta \\ &= \frac{\rho}{2} \bar{u}^2 (1 + \zeta) \end{aligned}} \right\} \sqrt{\frac{2 \rho g h}{1 + \zeta}} = \bar{u}$$

$$\zeta \geq 0 \quad \bar{u} < \sqrt{2 \rho g h}$$

$$Re \rightarrow \infty \quad \zeta \neq f_h(\bar{u})$$

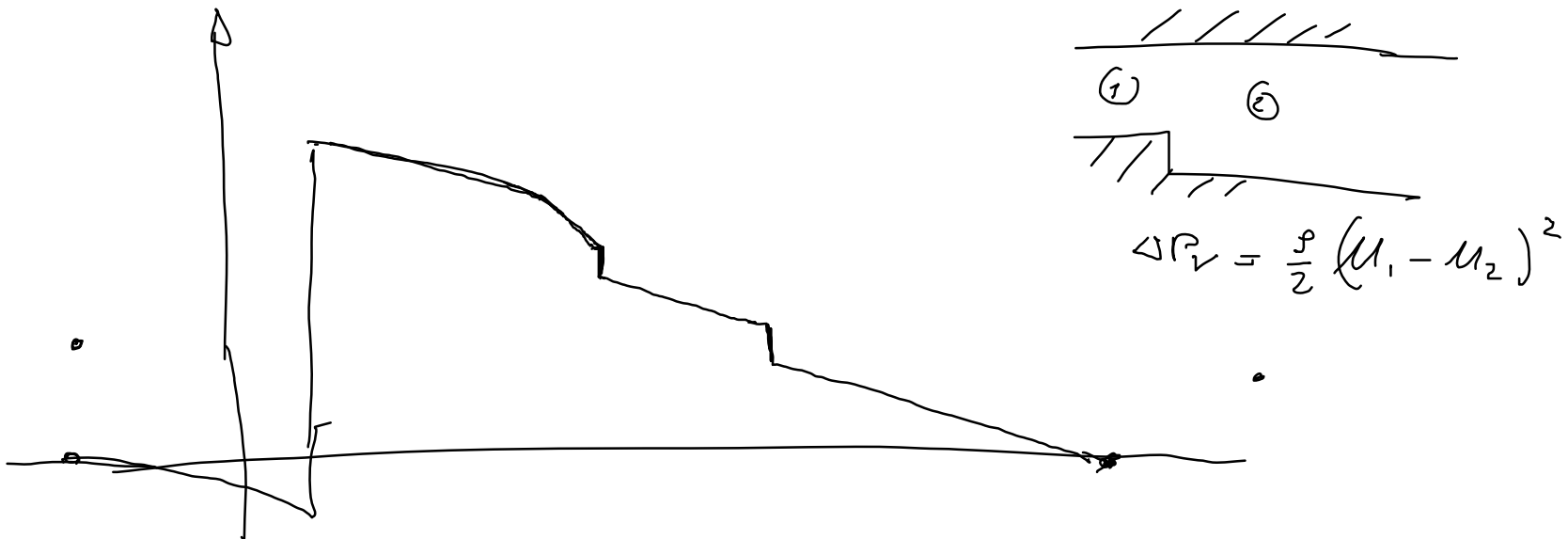
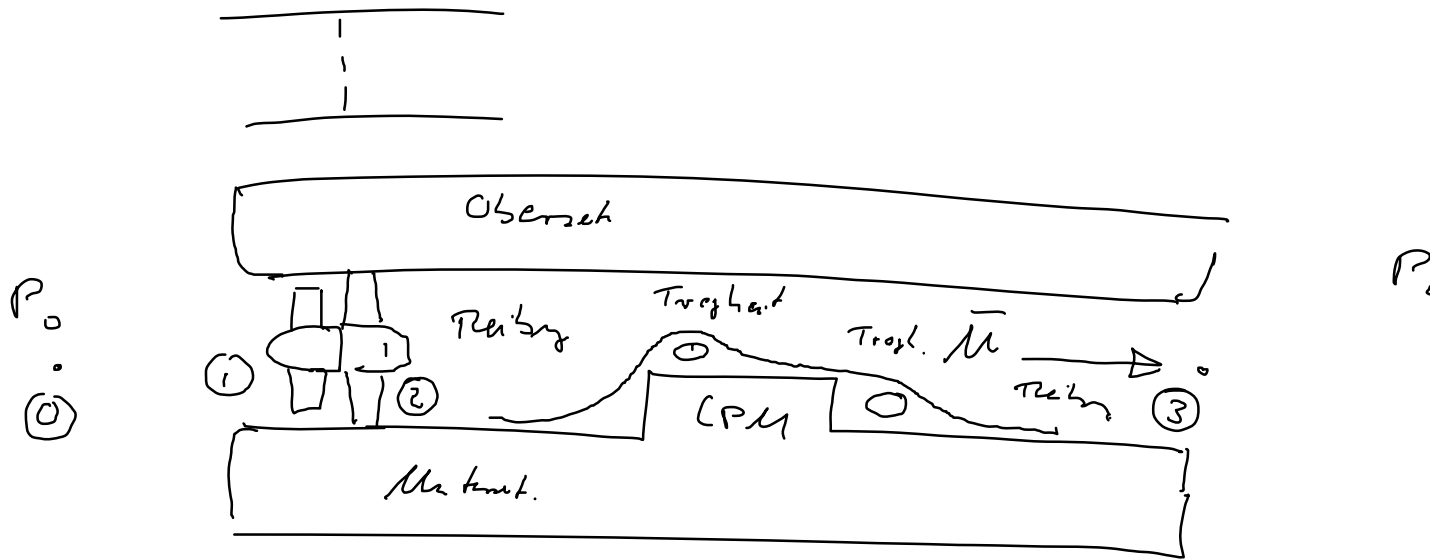
$$Re \rightarrow 0 \quad \zeta = \frac{l}{d} \frac{64}{Re} = \frac{l}{d} \frac{64}{\bar{u} d}$$

$$\rho g h = \frac{\rho}{2} \bar{u}^2 + \frac{\rho}{2} \bar{u}^2 \frac{l}{d} \frac{64}{\bar{u} d} \frac{2}{3}$$

$$SSh = \frac{5}{2} \bar{u}^2 + \bar{u} 32 \frac{l}{a^2} \eta$$

$$\bar{u}^2 + \frac{64}{\cancel{32}} \frac{\bar{u} l}{a^2} \eta - \frac{SSh}{5} = 0.$$

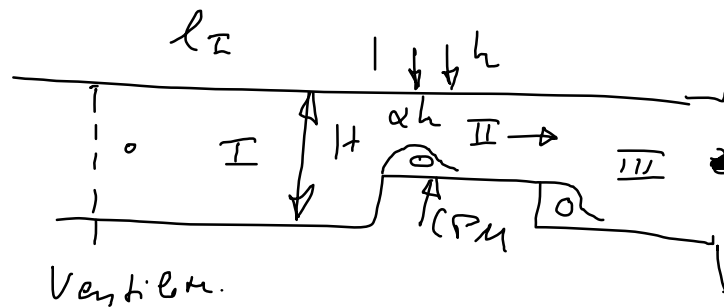
~



$$(1) \quad P_0 = \underset{=}{P_1} + \frac{\rho}{2} \underline{\underline{U}}^{-2} \quad \leadsto \quad P_1 < P_0$$

$$(2) \quad P_2 - P_1 = \int (Q) \quad \text{Ventilator Kennlinie.}$$

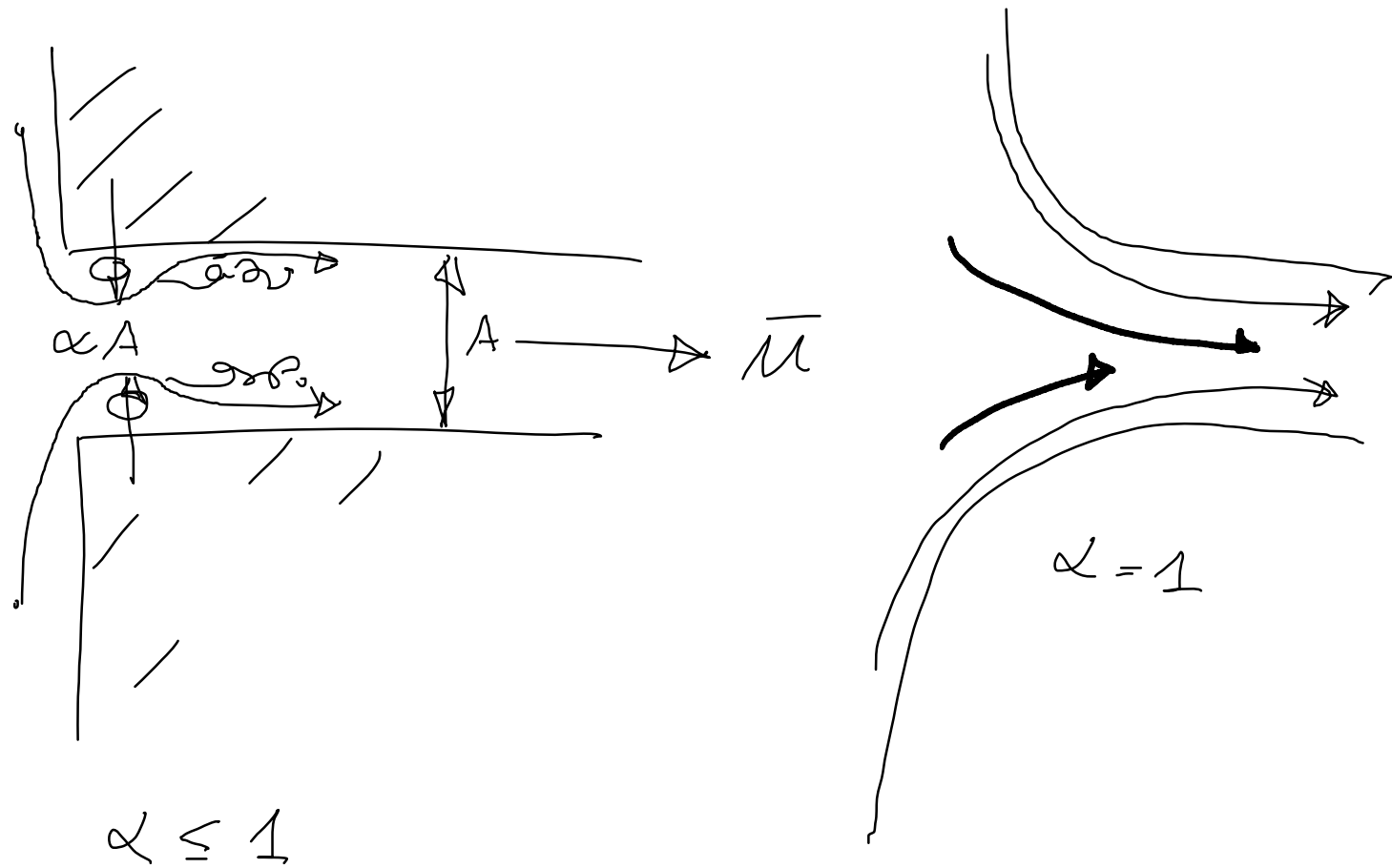
$$(3) \quad \underset{=}{P_2} + \frac{\rho}{2} \underline{\underline{U}}^{-2} = P_0 + \frac{\rho}{2} \underline{\underline{U}}^{-2} + \frac{\rho}{2} \underline{\underline{U}}^{-2} \left( \lambda_{\text{I}} \frac{l_{\text{I}} + l_{\text{II}}}{\alpha} + \left( \frac{1-\alpha}{\alpha} \right)^2 \left( \frac{H}{h} \right)^2 + \right.$$



$$\left. \lambda_{\text{II}} \frac{l_{\text{II}}}{h} \left( \frac{H}{h} \right)^2 + \left( \frac{H}{h} - 1 \right)^2 \right)$$

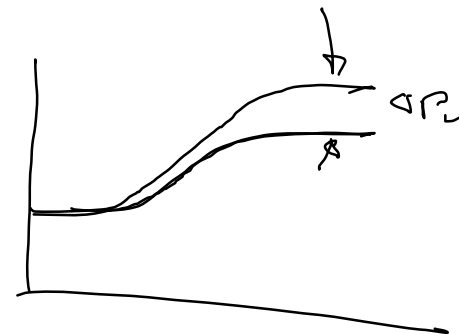
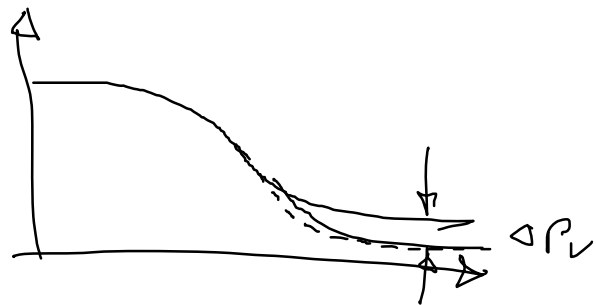
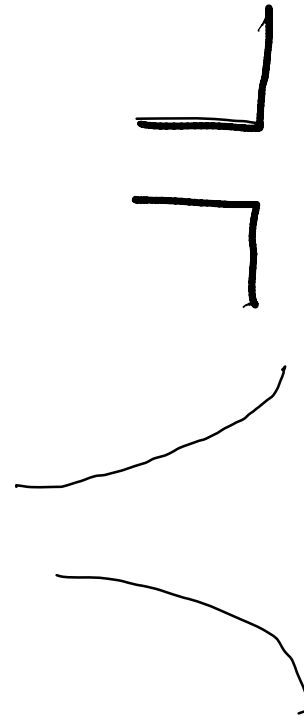
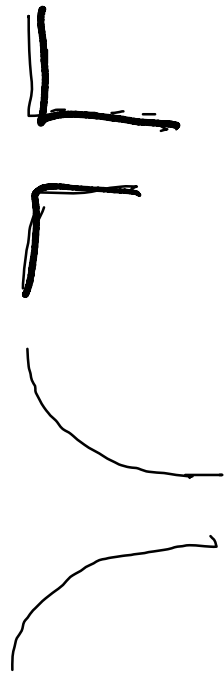
$$\Delta P_v = \frac{\rho}{2} (u_1 - u_2)^2$$

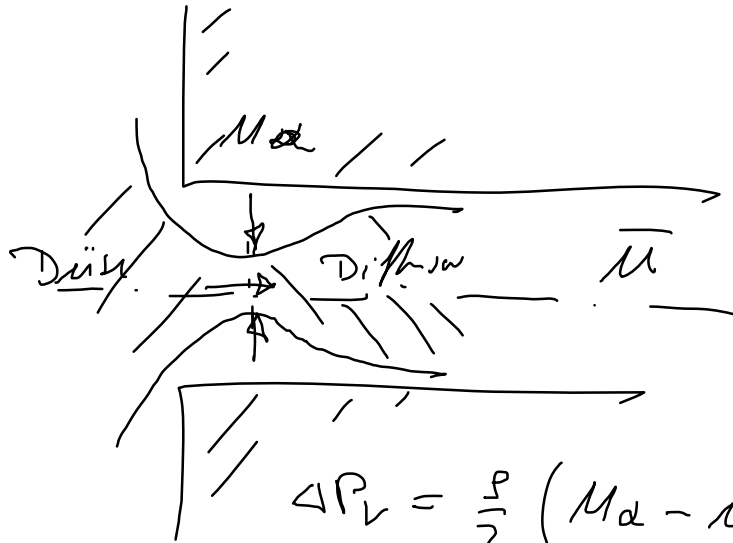
$$= \frac{\rho}{2} \underline{\underline{U}}^{-2} \left( \frac{u_1}{\underline{\underline{U}}} - 1 \right)^2$$



$$\Delta P_v = \frac{\rho}{2} \bar{u}^2 \left( \frac{1-\alpha}{\alpha} \right)^2 = \rho c \Delta T$$







$$\Delta P_L = \frac{\rho}{2} (M_0 - \bar{M})^2 = \frac{\rho}{2} \bar{M}^2 \left( \frac{1}{\alpha} - 1 \right)^2 =$$

$$2 A M_0 = A \bar{M} \Rightarrow M_0 = \bar{M} \frac{1}{\alpha}$$

$$= \frac{\rho}{2} \bar{M}^2 \left( \frac{1-\alpha}{\alpha} \right)^2$$