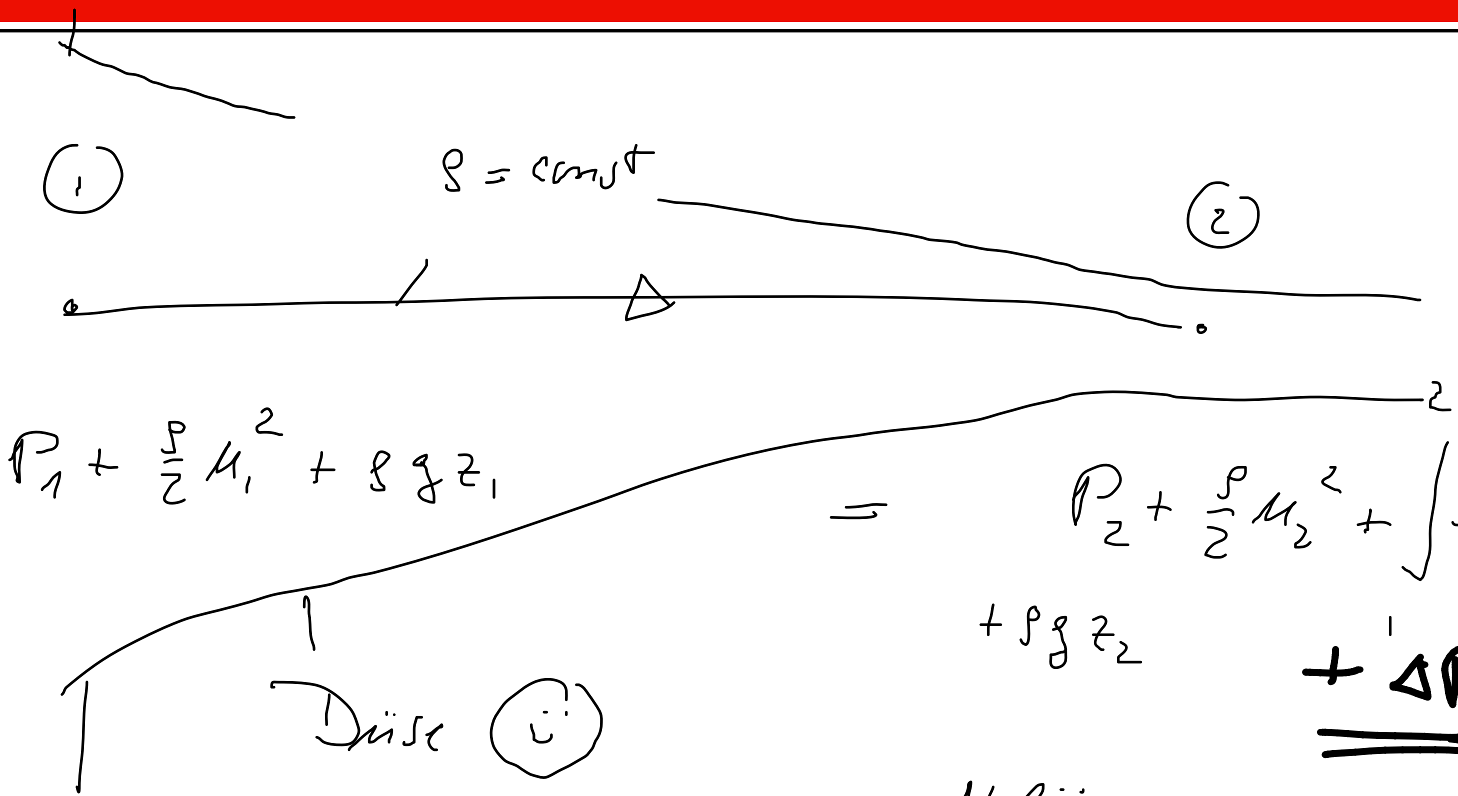
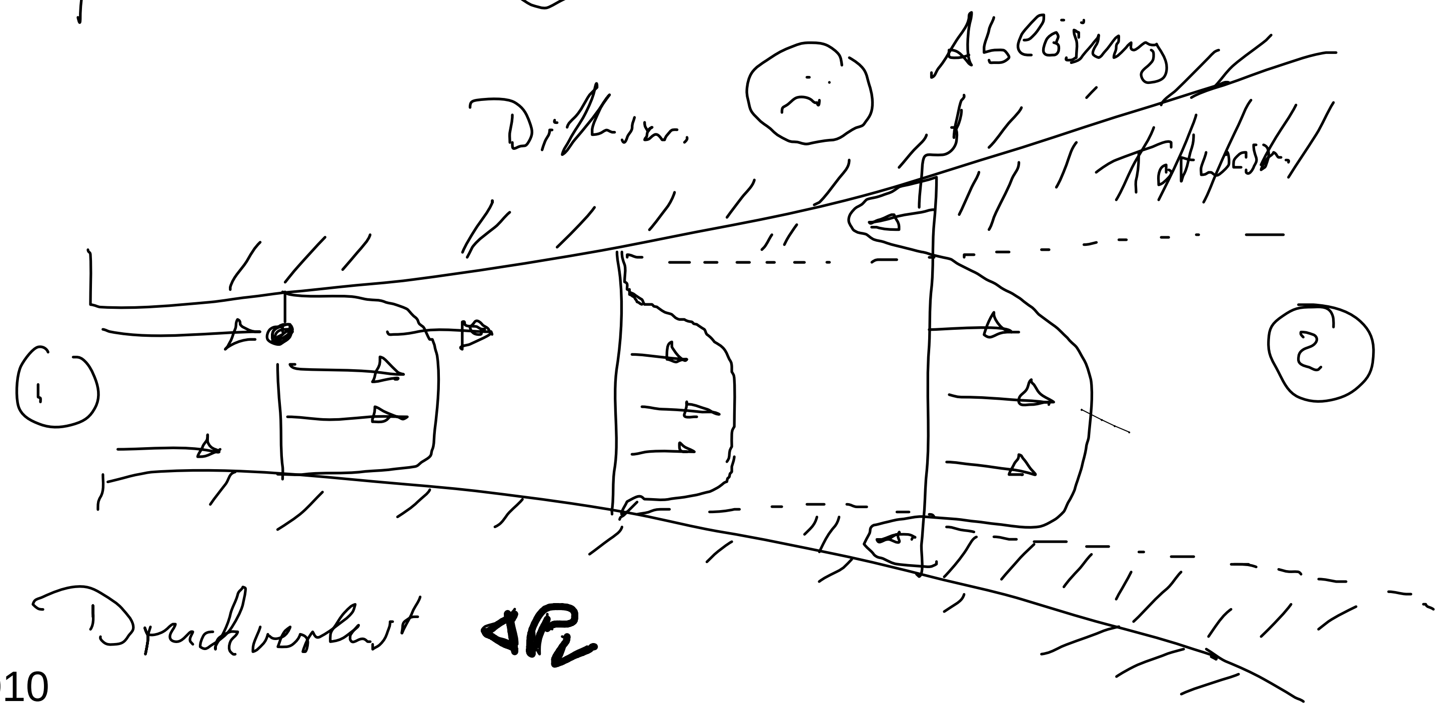


$$p_1 + \frac{\rho}{2} u_1^2 + \rho g z_1 = p_2 + \frac{\rho}{2} u_2^2 + \int \rho \frac{\partial u}{\partial t} ds + \rho g z_2 + \Delta p_v$$



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Sommersemester 2010
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Vorlesung 9

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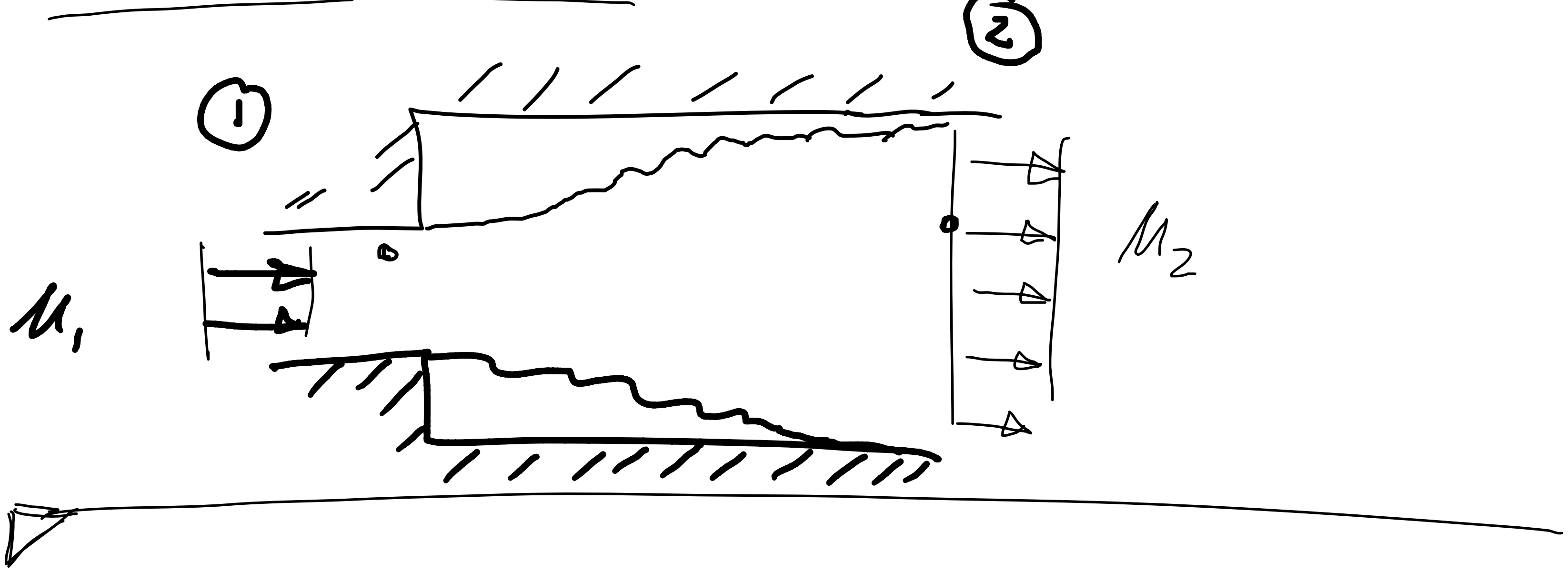
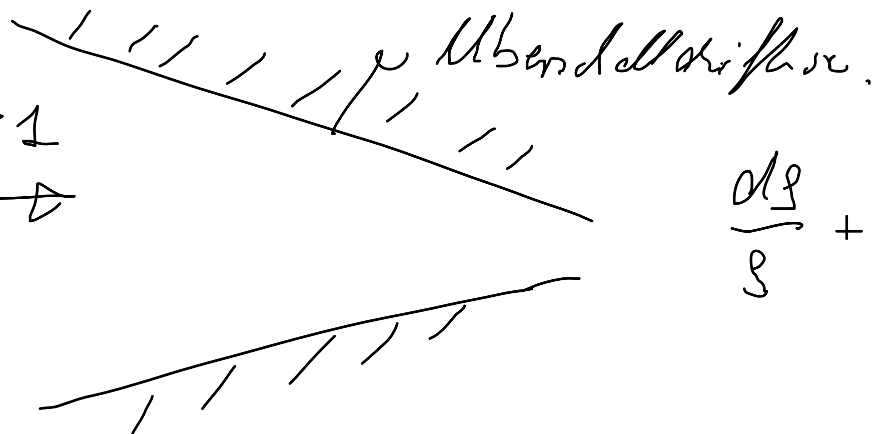
Ideal schlecht Diffusor. Unterschalldiffusor.



$$M = \frac{u}{a} > 1$$



$$\frac{d\rho}{\rho} + \frac{du}{u} + \frac{dA}{A} = 0.$$

$$\dot{m} = \rho u A = const.$$

$$d\dot{m} = d\rho\, u A + \rho\, du\, A + \rho u\, dA = 0.$$

TECHNISCHE UNIVERSITÄT DARMSTADT

FLUID SYSTEM TECHNIK

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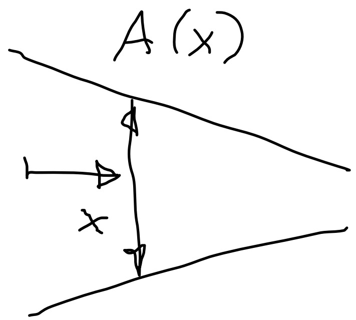
Sommersemester 2010
Strömungslehre für Mechatroniker
Vorlesung 9

Machzahl $Ma := \frac{u}{a}$



$$\frac{d\rho}{\rho} + \frac{du}{u} + \frac{dA}{A} = 0 \;\Big|\; \frac{1}{dx}$$

$$\underbrace{\frac{1}{\rho}\frac{d\rho}{dx}}_{\frac{1}{\rho a^2}\frac{dp}{dx}} + \frac{1}{u}\frac{du}{dx} + \frac{1}{A}\frac{dA}{dx} = 0 \qquad \Big| \quad -\frac{u}{a}\frac{du}{dx} + \frac{a}{u}\frac{du}{dx} + \frac{a}{A}\frac{dA}{dx} = 0 \;\Big|\; \frac{u}{a}$$

$$\rho u \frac{du}{dx} = -\frac{dp}{dx}$$

Impulssatz (x-Eulergleichung)

$$a^2 := \frac{dp}{d\rho}\Big|_{s=\text{const.}}$$

Quadrat der Schallgeschwindigkeit

$$a^2 = \gamma R T$$

$$p = C\rho^{\gamma}; \quad \gamma = \frac{c_p}{c_v}$$

$$p = \rho R T$$

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FLUID SYSTEM TECHNIK

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Sommersemester 2010
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Vorlesung 9

$$\frac{1}{u}\frac{du}{dx}\left(1-M^2\right) = -\frac{1}{A}\frac{dA}{dx}$$

	$\frac{dA}{dx} < 0$	$\frac{dA}{dx} > 0$
$M < 1$	$\frac{du}{dx} > 0$ Düse	$\frac{du}{dx} < 0$ Diffusor
$M > 1$	$\frac{du}{dx} < 0$ Diffusor	$\frac{du}{dx} > 0$ Düse

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Sommersemester 2010
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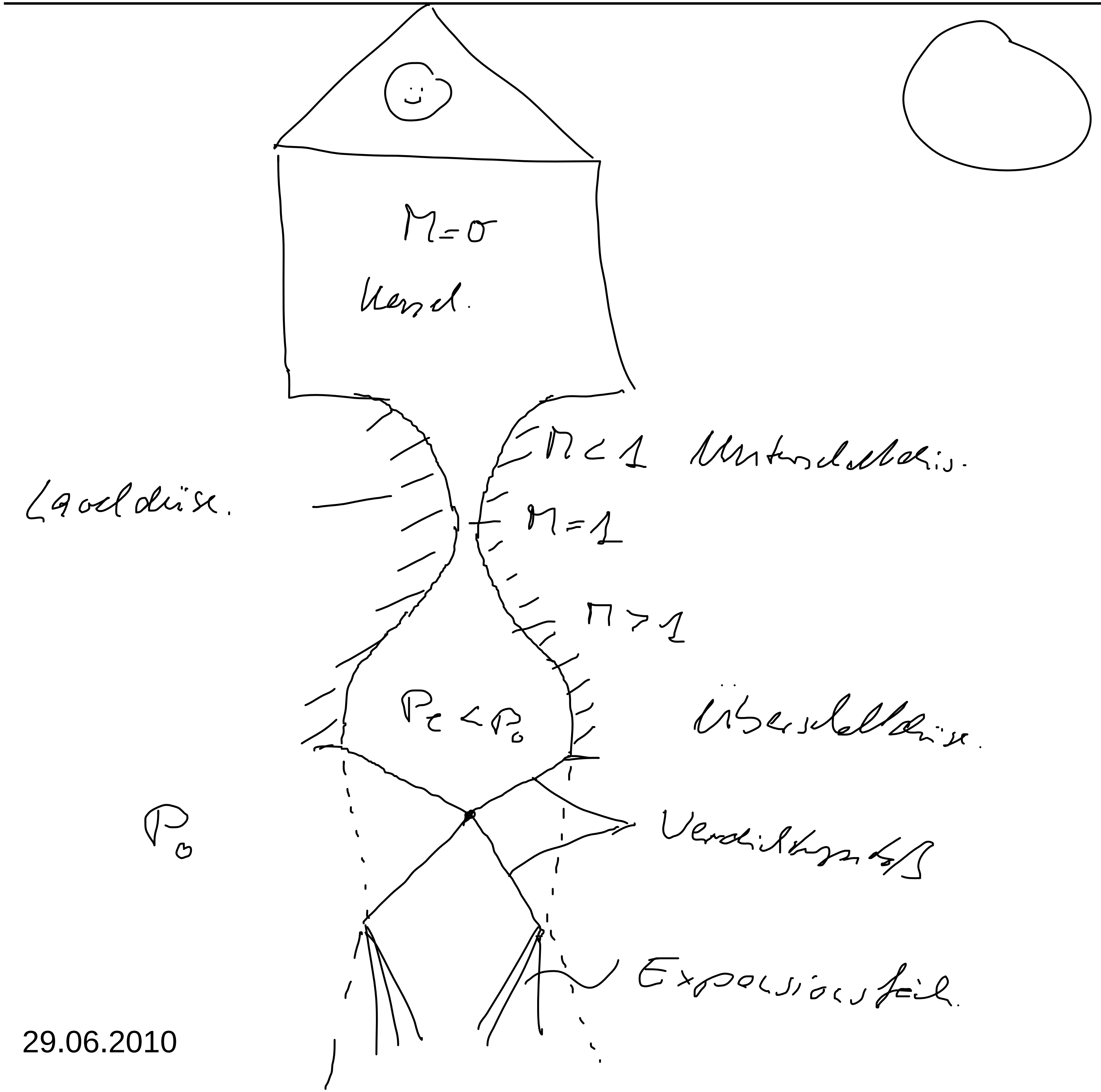
Vorlesung 9



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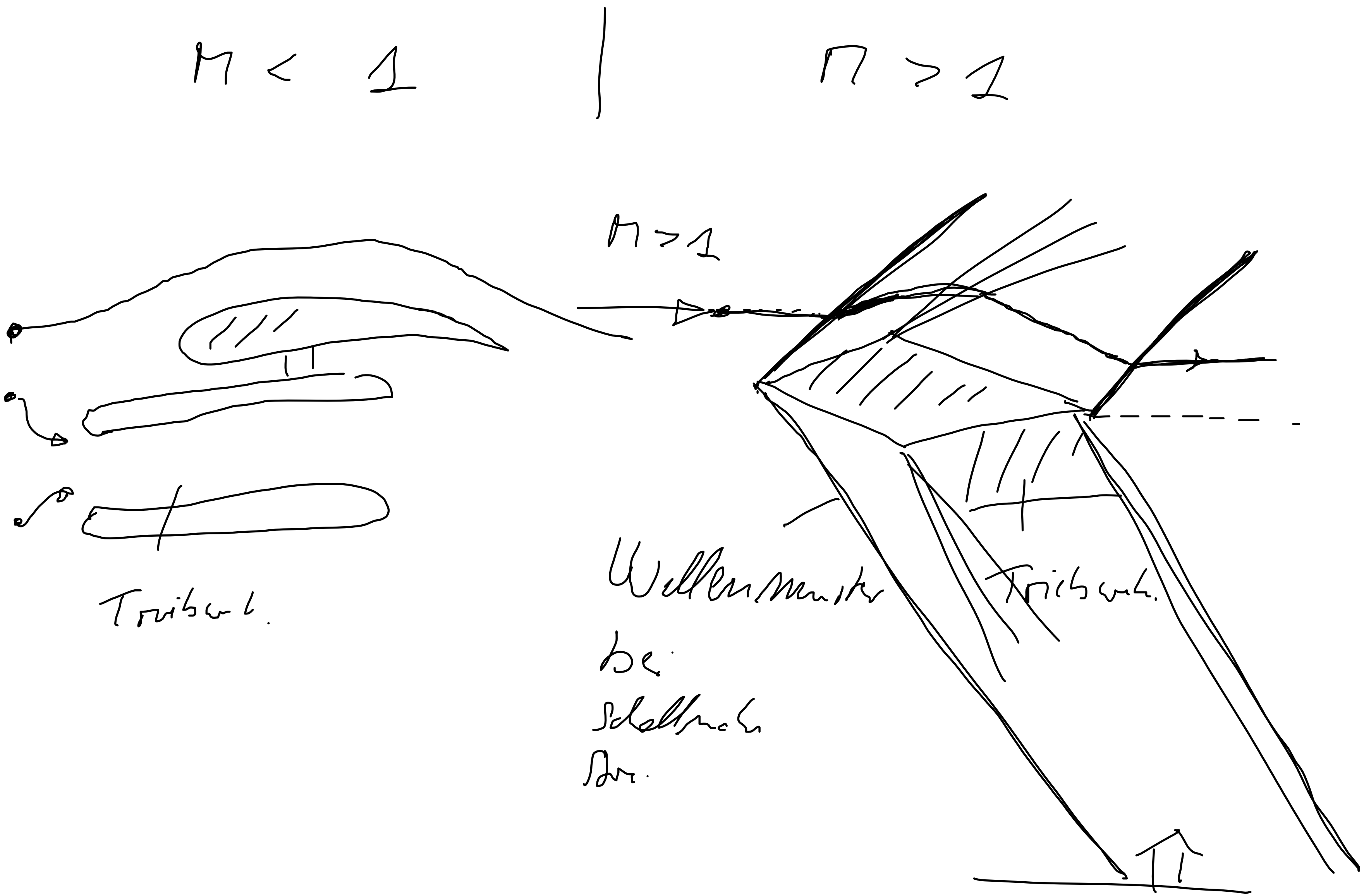
Vorlesung 9



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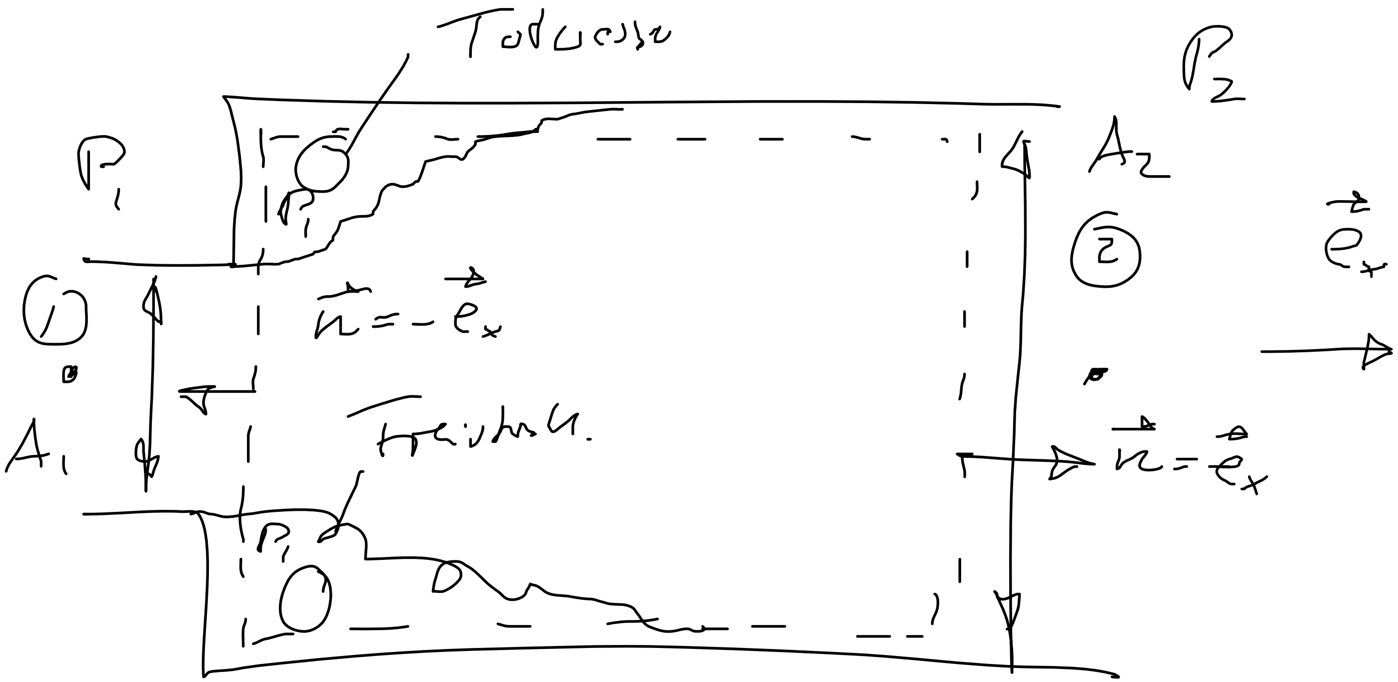
Vorlesung 9



Carnotsch Stoßverlust. $\rho = \text{const.}$ $Ma^2 \ll 1$.

$$p_1 + \frac{\rho}{2} u_1^2 = p_2 + \frac{\rho}{2} u_2^2 + \Delta p_V$$

$$(p_2 - p_1)_{ideal} = \frac{\rho}{2}\left(u_1^2 - u_2^2\right) = \frac{\rho}{2} u_1^2 \left(1 - \left(\frac{A_1}{A_2}\right)^2\right)$$

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FLUID SYSTEM TECHNIK

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Sommersemester 2010
Strömungslehre für Mechatroniker
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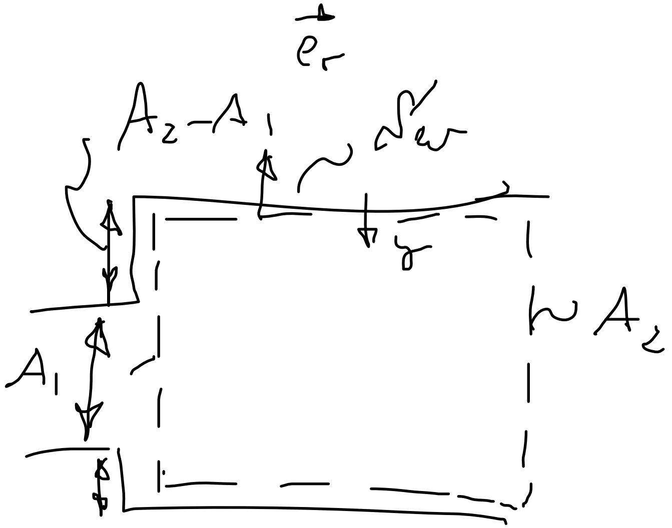
Reale Druckänderung $(p_2 - p_1)_{real}$ folgt aus dem Impulssatz.

$$-\rho u_1^2 A_1 + \rho u_2^2 A_2 = \int_{A_1, A_2} \vec{t} \cdot \vec{e}_x \, dS + \int_{A_2 - A_1} \vec{t} \cdot \vec{e}_x \, dS + \int_{S_W} \vec{t} \cdot \vec{e}_x \, dS$$

mit $\vec{t} = -p\vec{n}$ auf A_1, A_2 und $\vec{t} = -p_1(-\vec{e}_x)$ auf $A_2 - A_1$



$$\vec{t} = \vec{e}_r \cdot \underset{\sim}{T} \quad \Big| \cdot \vec{e}_x$$

$$= \tau_{rx} = \eta \frac{du}{dy} \approx 0,$$

da ein Totwasser vorhanden ist

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FLUID SYSTEM TECHNIK

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Sommersemester 2010
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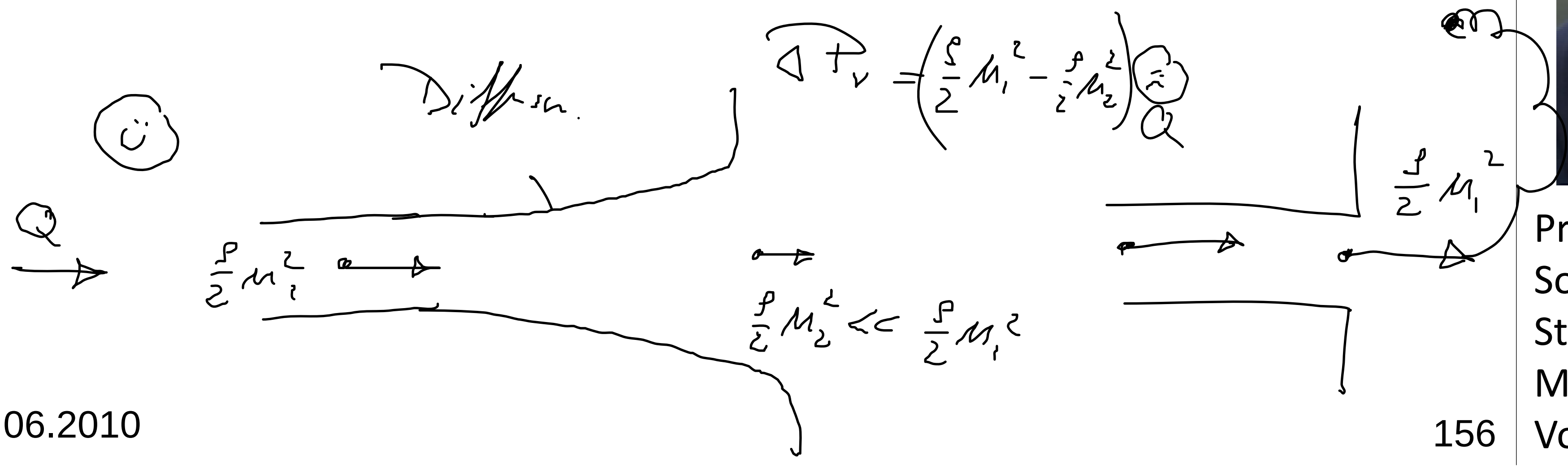
Vorlesung 9

$$(p_2 - p_1)_{ideal} - (p_2 - p_1)_{real} = \Delta p_v = \frac{\rho}{2}(u_1 - u_2)^2$$

Bernoulli — Impulssatz

$$\boxed{\Delta p_{vc} = \frac{\rho}{2}(u_1 - u_2)^2}$$



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Sommersemester 2010
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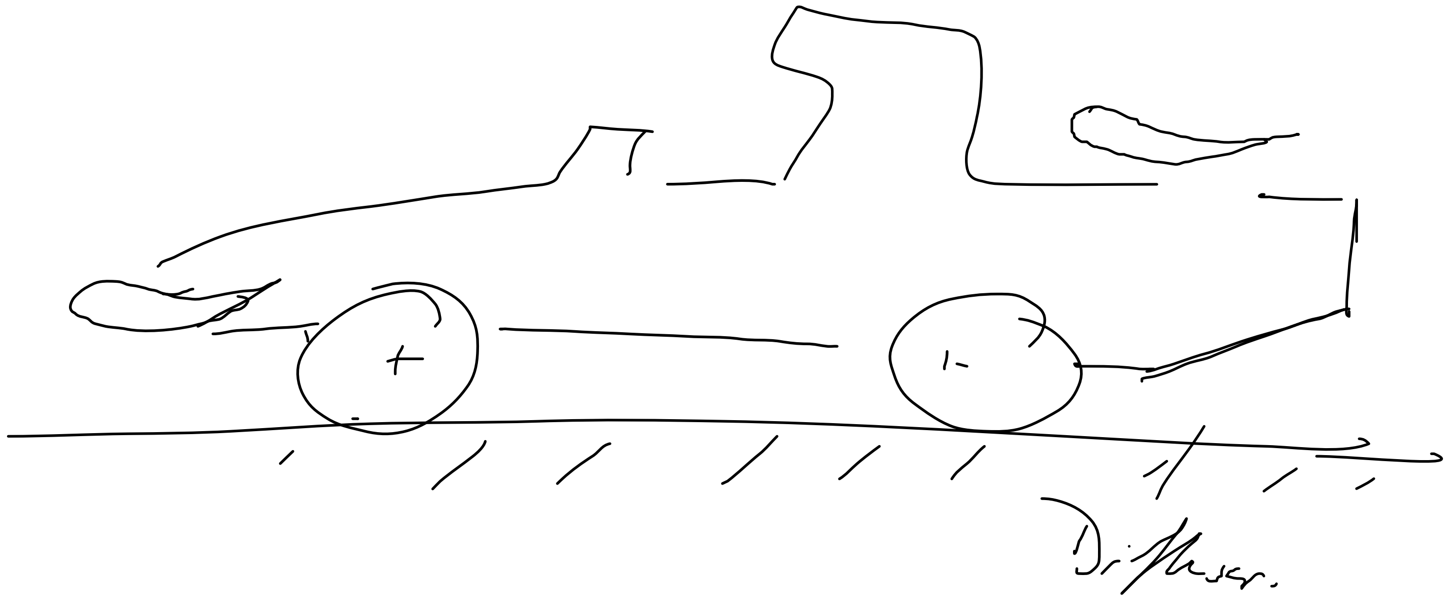
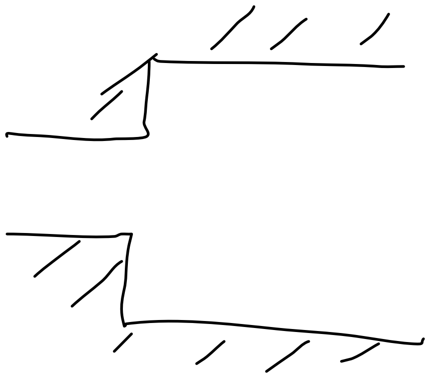
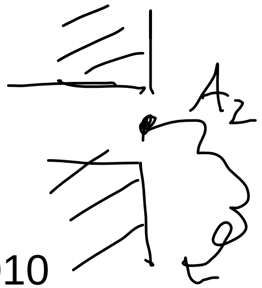
Vorlesung 9





$$\Delta p_{V_c} = \frac{\rho}{2} u_1^2 \left(1 - \frac{u_2}{u_1}\right)^2$$



$$= \frac{\rho}{2} u_1^2 \left(1 - \frac{A_1}{A_2}\right)^2$$

$$\lim_{A_1/A_2 \to 0} \Delta p_V = \frac{\rho}{2} u_1^2 .$$ Ausströmverlust.

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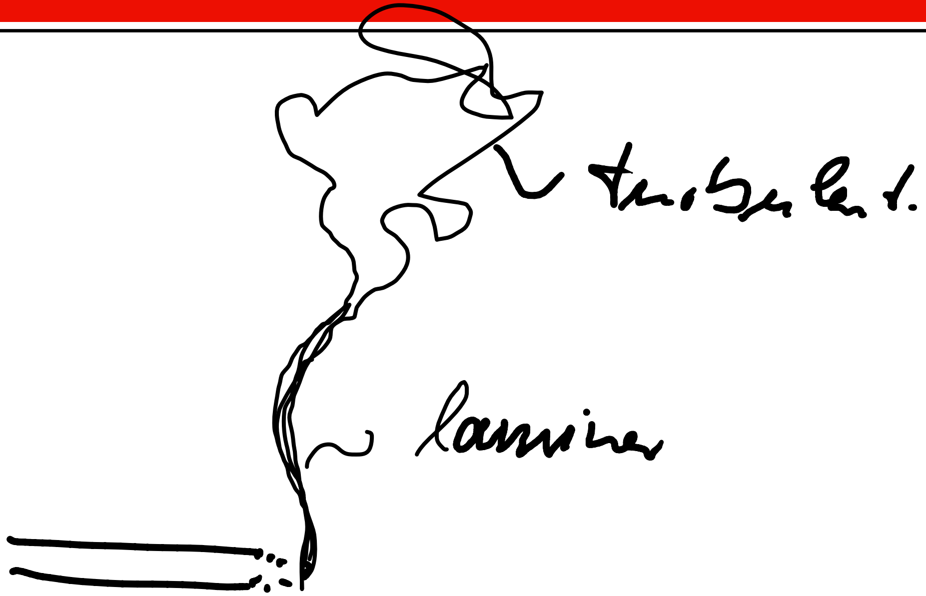
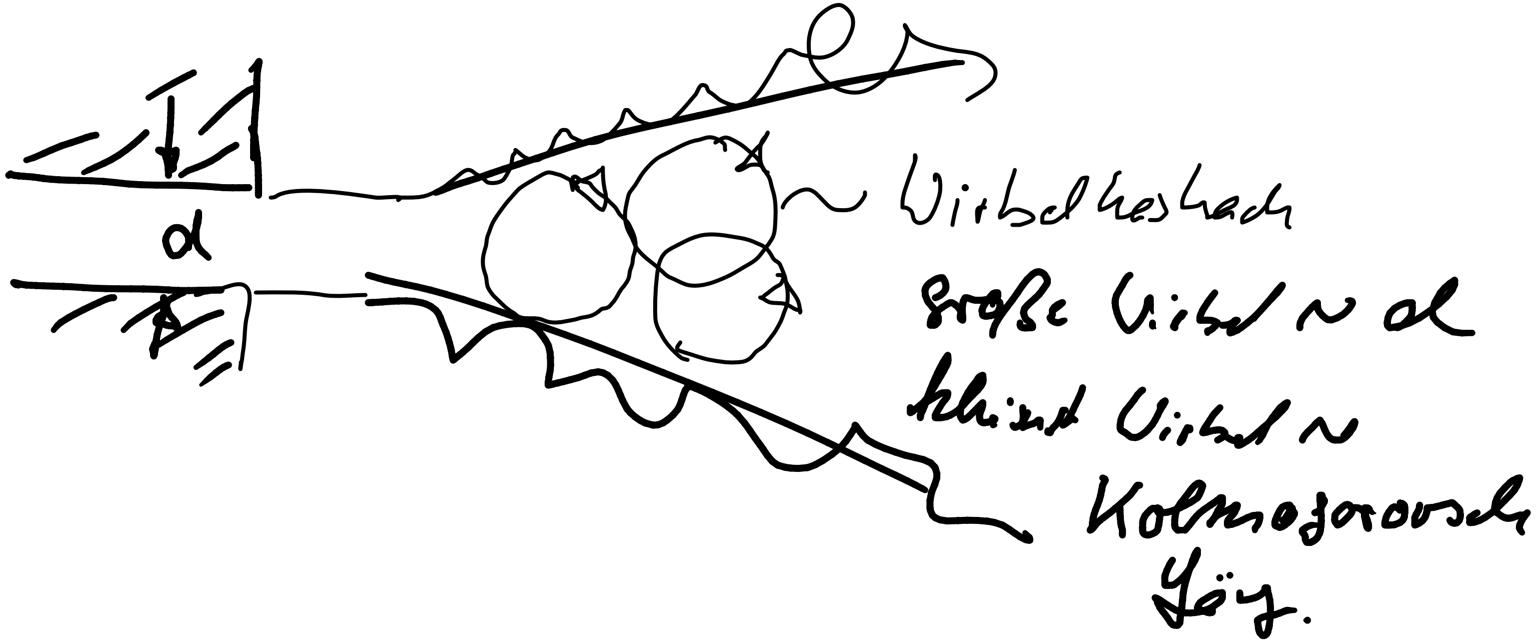
Vorlesung 9





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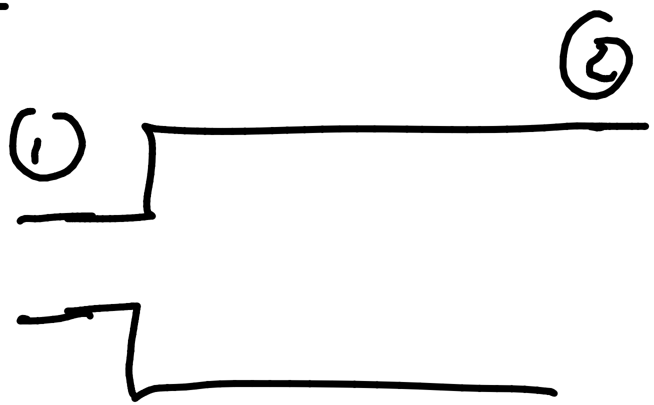
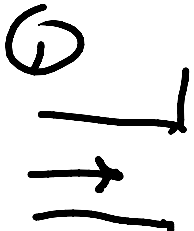
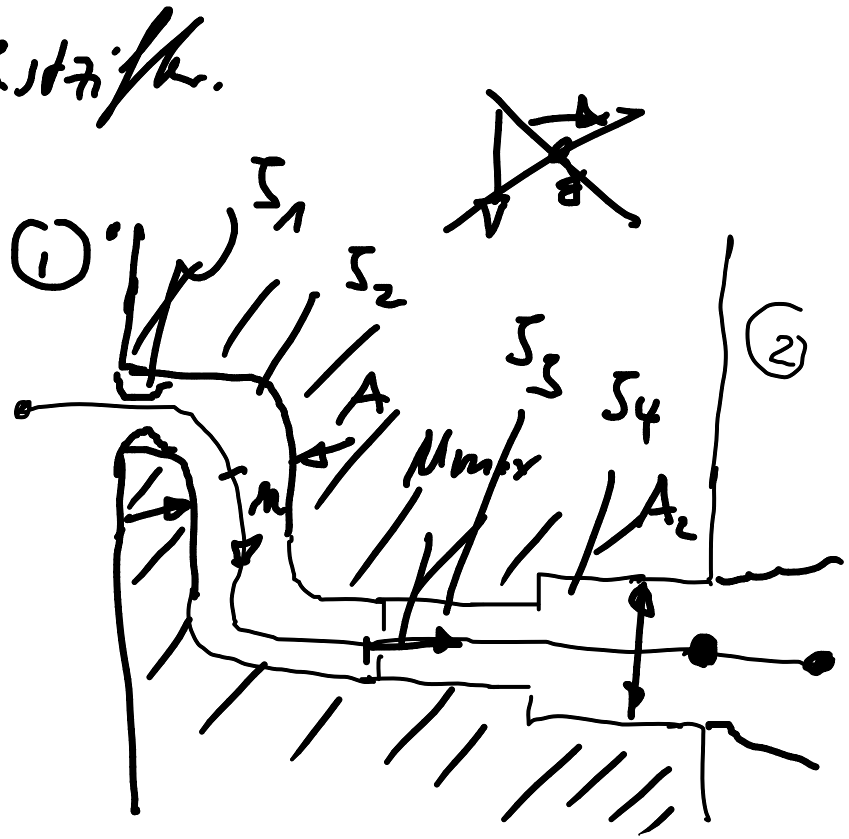
$$\frac{\Delta p_V}{\frac{\rho}{2} u_1^2} := \zeta \qquad \text{Verlustziffer.}$$



$$\zeta_c = \left(1 - \frac{A_1}{A_2}\right)^2$$



$$\zeta_A = 1$$



$$p_1 = p_2 + \frac{\rho}{2} u_2^2 + \Delta p_V$$

$$\Delta p_V = \frac{\rho}{2} u^2 \sum_{i=1}^{4} \zeta_i$$

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FLUID SYSTEM TECHNIK

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Vorlesung 9

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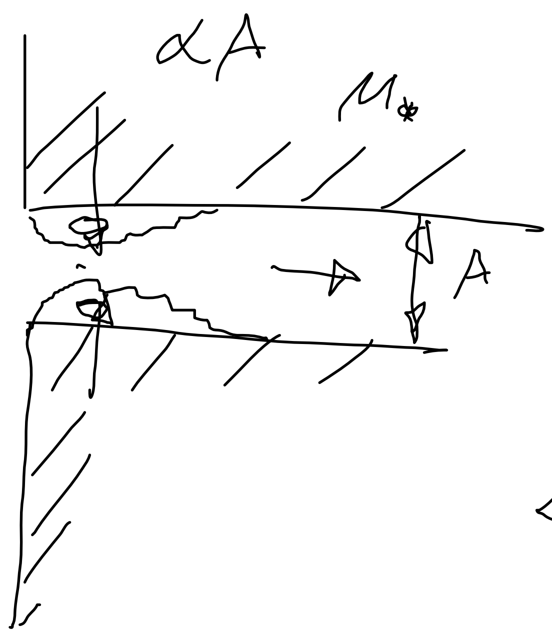
Ideal schlechte Einlauf / Düse.



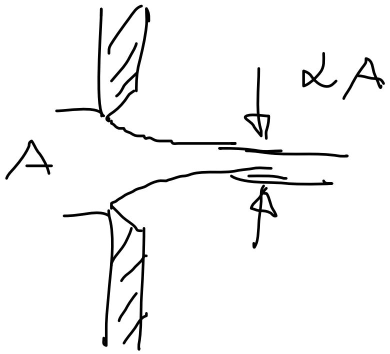
$\alpha \leq 1$

Strahlkontraktion

$$\Delta p_{vc} = \frac{\rho}{2}\left(u_{max} - u\right)^2$$

$$= \frac{\rho}{2} u^2 \left(\frac{u_{max}}{u} - 1\right)^2$$

$$= \frac{\rho}{2} u^2 \left(\frac{1}{\alpha} - 1\right)^2$$



2D $\alpha = \frac{\pi}{\pi + 2} = 0.61$

$$\zeta := \frac{\Delta p_{vc}}{\frac{\rho}{2} u^2} = \left(\frac{1-\alpha}{\alpha}\right)^2.$$

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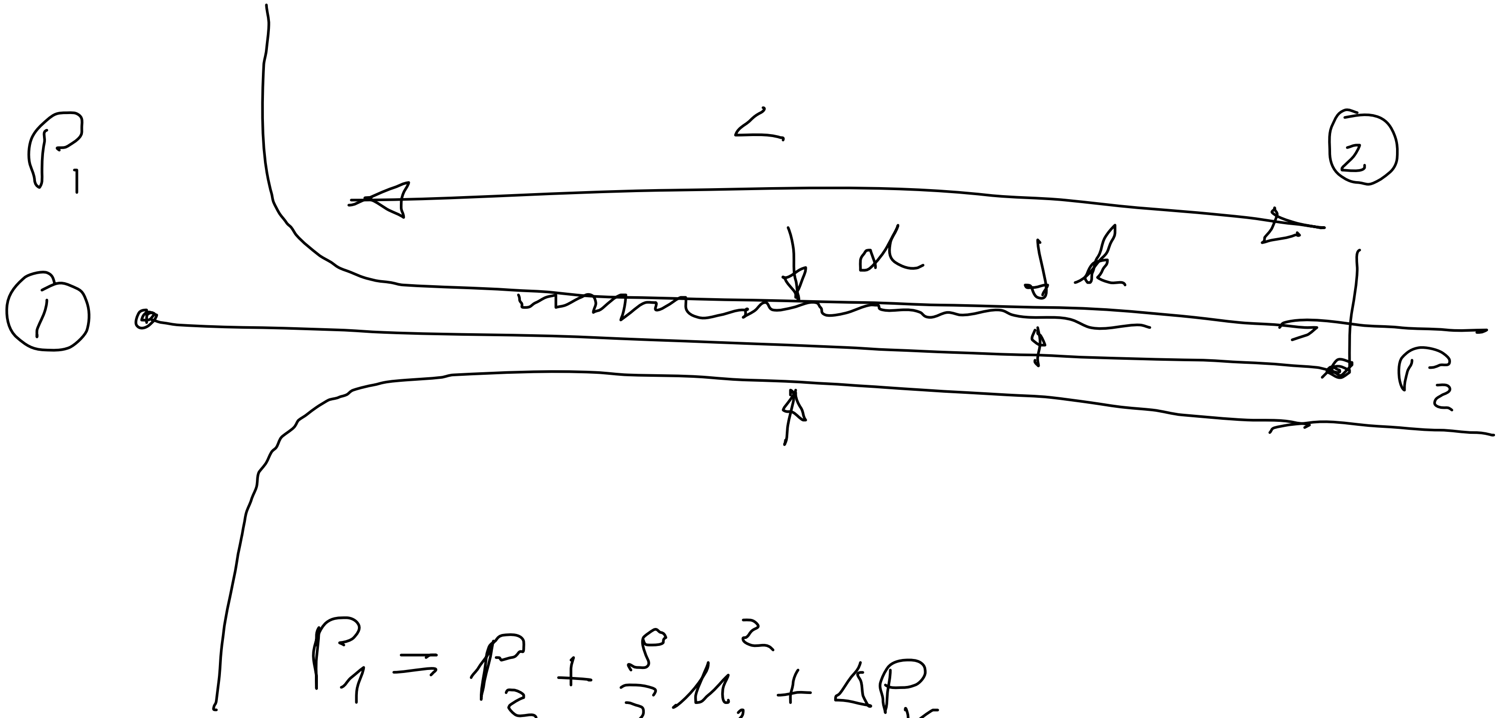
Vorlesung 9



$$p_1 = p_2 + \frac{\rho}{2} u_2^2 + \Delta p_v$$

$$\Delta p_v \sim \frac{L}{d} \frac{\rho}{2} u_2^2$$

$$\frac{d}{L} \Delta p_v = \frac{\rho}{2} u_2^2 \, \lambda(Re, k/d)$$

λ Widerstandsziff.

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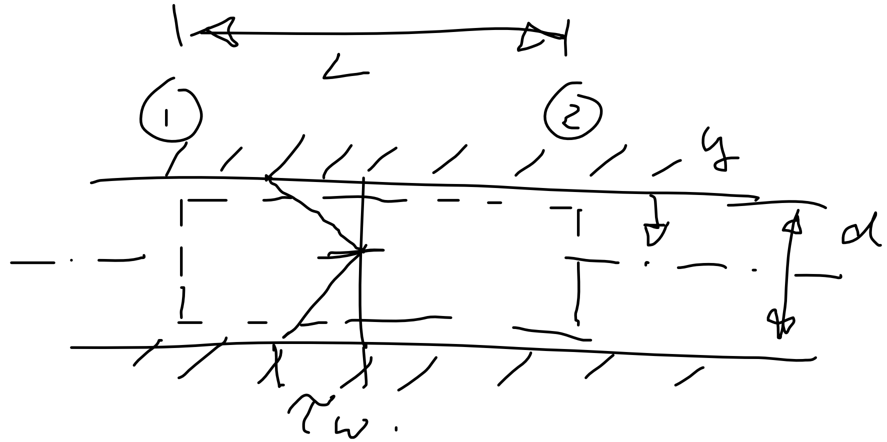
λ Druckverlust pro Längeneinheit (dimensionslos).



$$\Delta p_v \frac{\pi}{4} d^2 = \tau_w \pi d L$$

$$\Delta p_v = \tau_w \frac{L\,4}{d}$$

$$\lambda := \frac{\Delta p_v}{\frac{\rho}{2} u^2} \frac{d}{L}$$

$$\lambda := \frac{\tau_w 4}{\frac{\rho}{2} u^2}$$

$$|\tau_w| = \eta \left|\frac{du}{dy}\right|$$

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FLUID SYSTEM TECHNIK

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Sommersemester 2010
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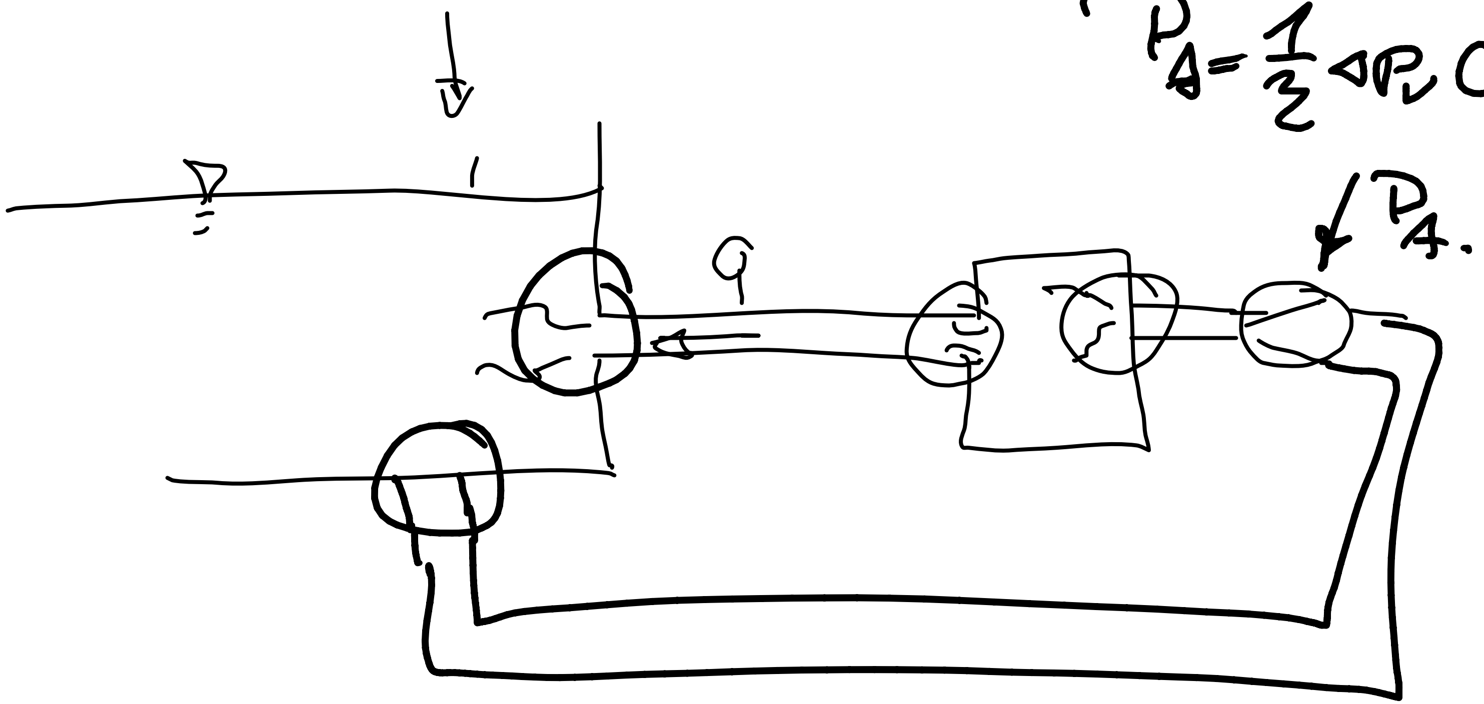
$$P_A = \frac{1}{2}\Delta p_v\, Q.$$



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Sommersemester 2010
Strömungslehre für Mechatroniker
Vorlesung 9