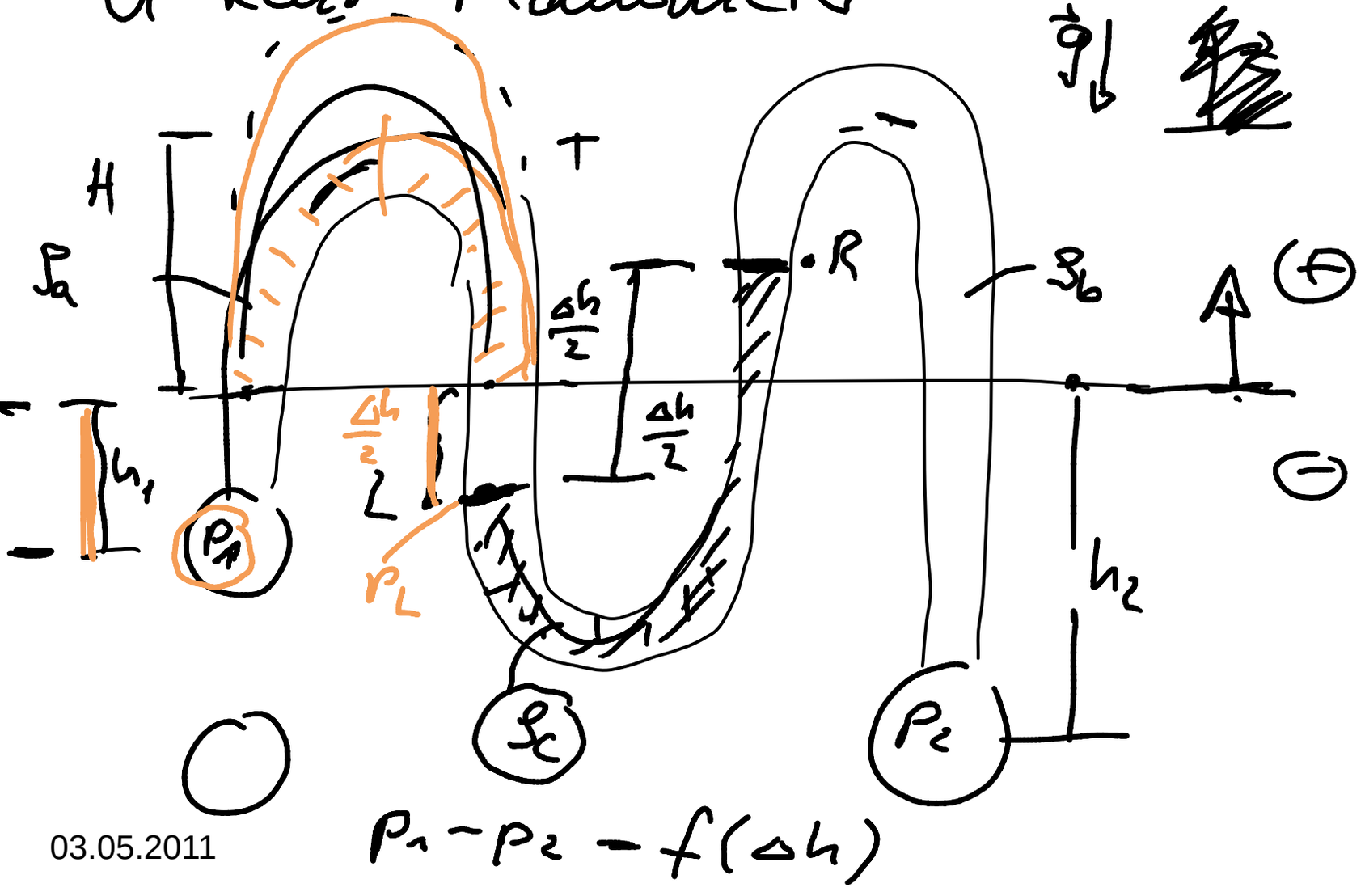
2. VRÜ Hydrostatik

U-Rohr-Manometer



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Vorrechenübung 2

$$\nabla p = \vec{f} \quad , \quad \vec{f} = -\rho g \vec{e}_z$$

$$\frac{\partial p}{\partial z}\vec{e}_z = -\rho g \vec{e}_z \quad \xrightarrow{\int dz} \quad p = -\rho g z + C$$

$$\Rightarrow \boxed{p + \rho g z = \text{const}}$$

Links: $p_L + \rho g \left(-\frac{\Delta h}{2}\right) = \text{const}$

$$p_R + \rho g \frac{\Delta h}{2} = \text{const}$$

Gleich $\Rightarrow$ setzen

$$p_L - p_R = \rho g \Delta h$$

$z \quad z$

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$$p_1 + \rho_a g (-h_1) = p_2 + \rho_a g \left(-\frac{\Delta h}{2}\right)$$

$$\Rightarrow p_2 = \rho_a g \left(\frac{\Delta h}{2} - h_1\right) + p_1$$

$$p_2 + g \rho_b (-h_2) = p_R + \rho_b g \left(\frac{\Delta h}{2}\right)$$

$$\Rightarrow p_R = p_2 - \rho_b g \left(\frac{\Delta h}{2} + h_2\right)$$

3 Gl / 3 Unk

$$\Rightarrow p_1 - p_2 = \rho_c g \Delta h - \rho_b g \left(\frac{\Delta h}{\cdot} \ldots\right)$$

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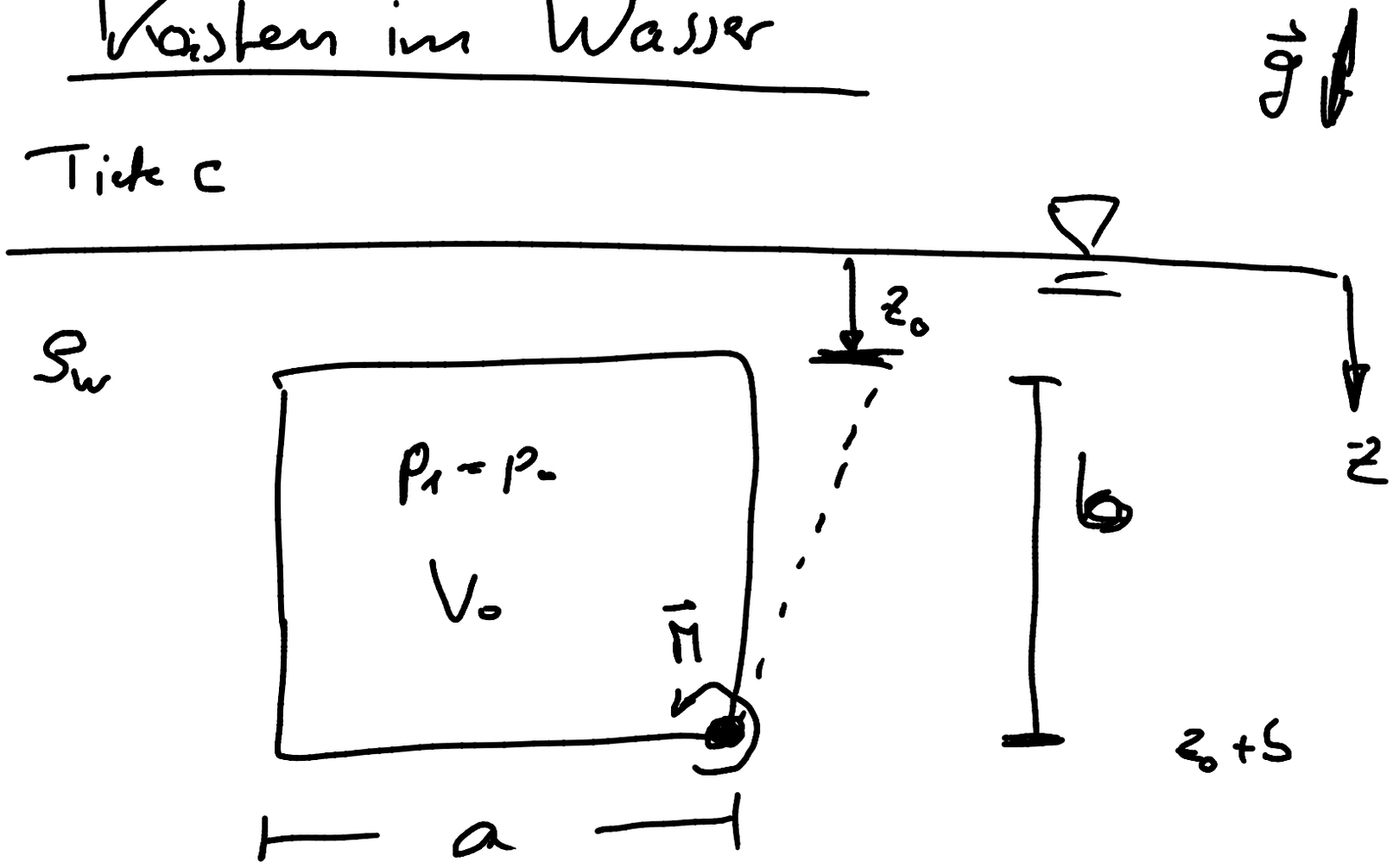
Kasten im Wasser



1) Welche resultierende Kraft F_{Res} wirkt auf die Tür

2) Welches Moment ist zum Öffnen der Tür notwendig

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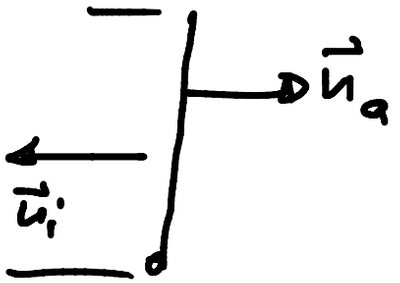
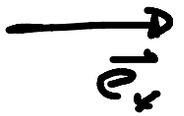
$$\vec{F} = -\int p\vec{n}\, dS$$



Innenseite: $\vec{n} = -\vec{e}_x$



$$F_{xi} = \vec{F}\cdot\vec{e}_x = -\int p\,\underbrace{\vec{n}\cdot\vec{e}_x}_{-1}\, dS$$

$$\Rightarrow F_{xi} = \int p\, dS \qquad dS = dz\, dc$$

$$F_{xi} = \int_0^c \int_{z_0}^{z_0+b} p_1\, dz\, dc = p_1 c z \Big|_{z_0}^{z_0+b}$$

$$\Rightarrow F_{xi} = p_1 bc = pA$$

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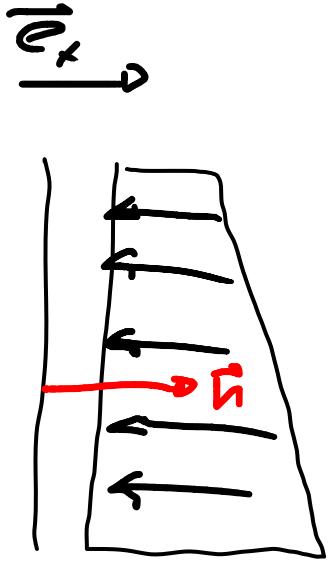

$$p_R = p_0 + \rho_w g z$$

$$F_{xa} = \vec F \cdot \vec e_x = -\int_{z_0}^{z_0+b} \underbrace{(p_0 + \rho_w g z)}_{p_R} \vec e_x \, dS$$

$$dS = dz\, dc \qquad \vec n \cdot \vec e_x = 1$$

$$F_{xa} = -\left(p_0 c z \Big|_{z_0}^{z_0+b} + \rho_w g c \underbrace{\int_{z_0}^{z_0+b} z \, dz}_{\text{Flächenträgheitsmoment 1. O.}} \right)$$

$$\boxed{F_{xa} = -\left(p_0 + \rho_w g c \left(z_0 + \frac{b}{2}\right)\right) b c}$$

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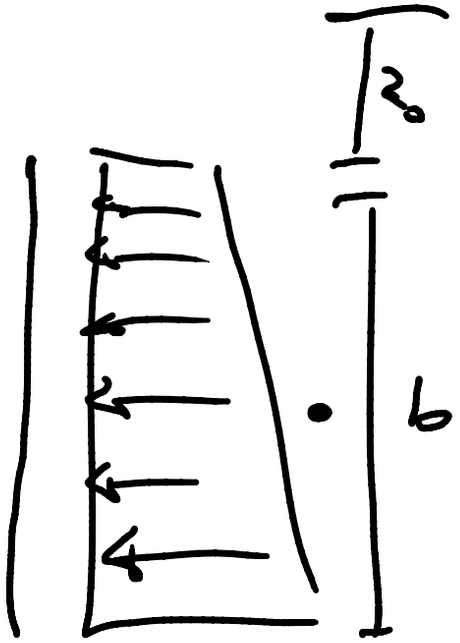
$$\overline{F_R} = \overline{2F} = F_{xi} + F_{xa}$$

$$= p_1 bc - \underbrace{bc\left(p_0 + \rho_w g\left(z_s + \frac{b}{2}\right)\right)}_{-\int p\vec{n}\,dS \;=\; F_{xa}}$$

$$= -A \cdot \vec{n} \cdot \vec{e}_x \, p_s$$

$$F_x = A_{proj} \cdot p_s$$



$$p_s = \left(p_0 + \rho g\left(z_0 + \frac{b}{2}\right)\right)$$

$$A_{proj} = bc$$

$$\Rightarrow F_{xa} = -\left(p_0 + \rho g\left(z_0 + \frac{b}{2}\right)\right) bc$$

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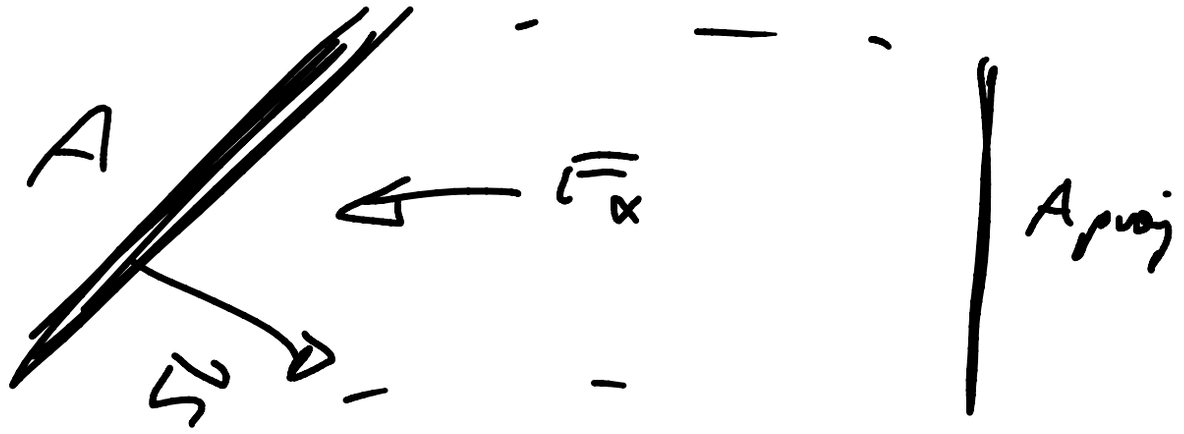

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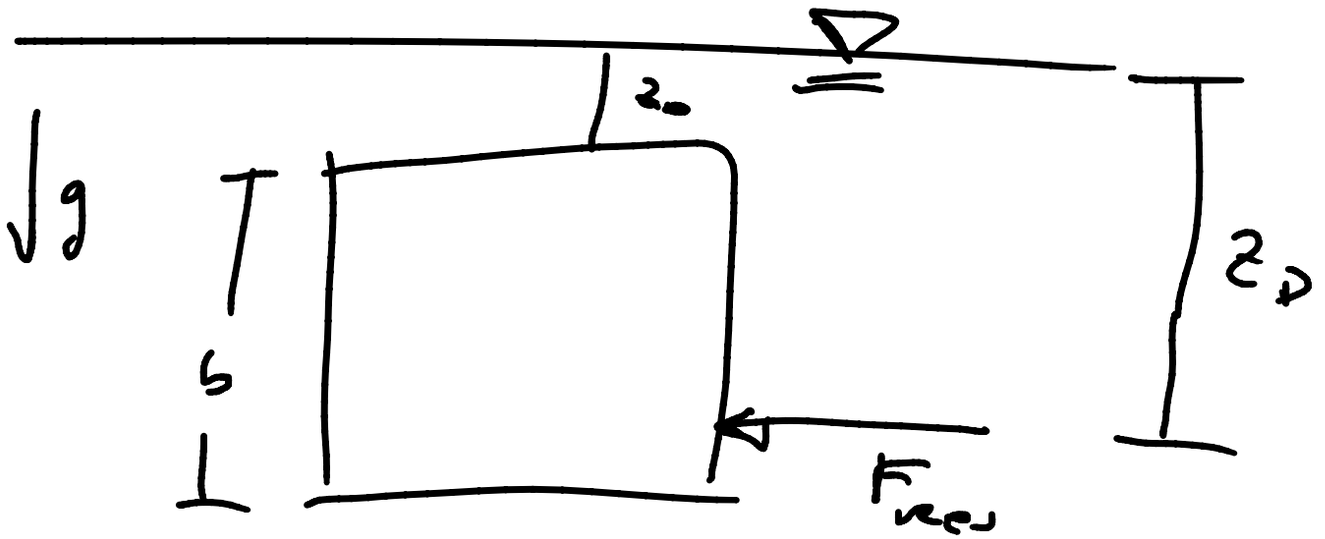

$$F_{res}\, z_D = \int_S p\, z\, dS \qquad p = p_3 + \rho g z$$

$$\Rightarrow F_{res}\, z_D = \int_{z_0}^{z_0+s} p_1\, c\, z\, dz - \int_{z_0}^{z_0+s} (p_3 + \rho g z)\, z\, c\, dz$$

Innendruck — Aussendruck

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$$\Rightarrow F_{Res}\, z_D = \rho_1 c \int_{z_0}^{z_0+b} z\, dz - c \int_{z_0}^{z_0+b} \rho_0 z\, dz$$

$$- \rho_w g c \int_{z_0}^{z_0+b} z^2\, dz$$

z_0 — Flächenträgheitsmoment 2.0.

$$z_D = \frac{(\rho_1 - \rho_0) c b (z_0 2 + b)}{2 F_{Res}} - \frac{\rho_w g c b (3 z_0^2 + 3 z_0 b + b^2)}{3 F_{Res}}$$

$$F_{Res} = f(\rho(z_0))$$

$$F_{Res}(z_0 + b - z_D) + M = 0 \Rightarrow M$$

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