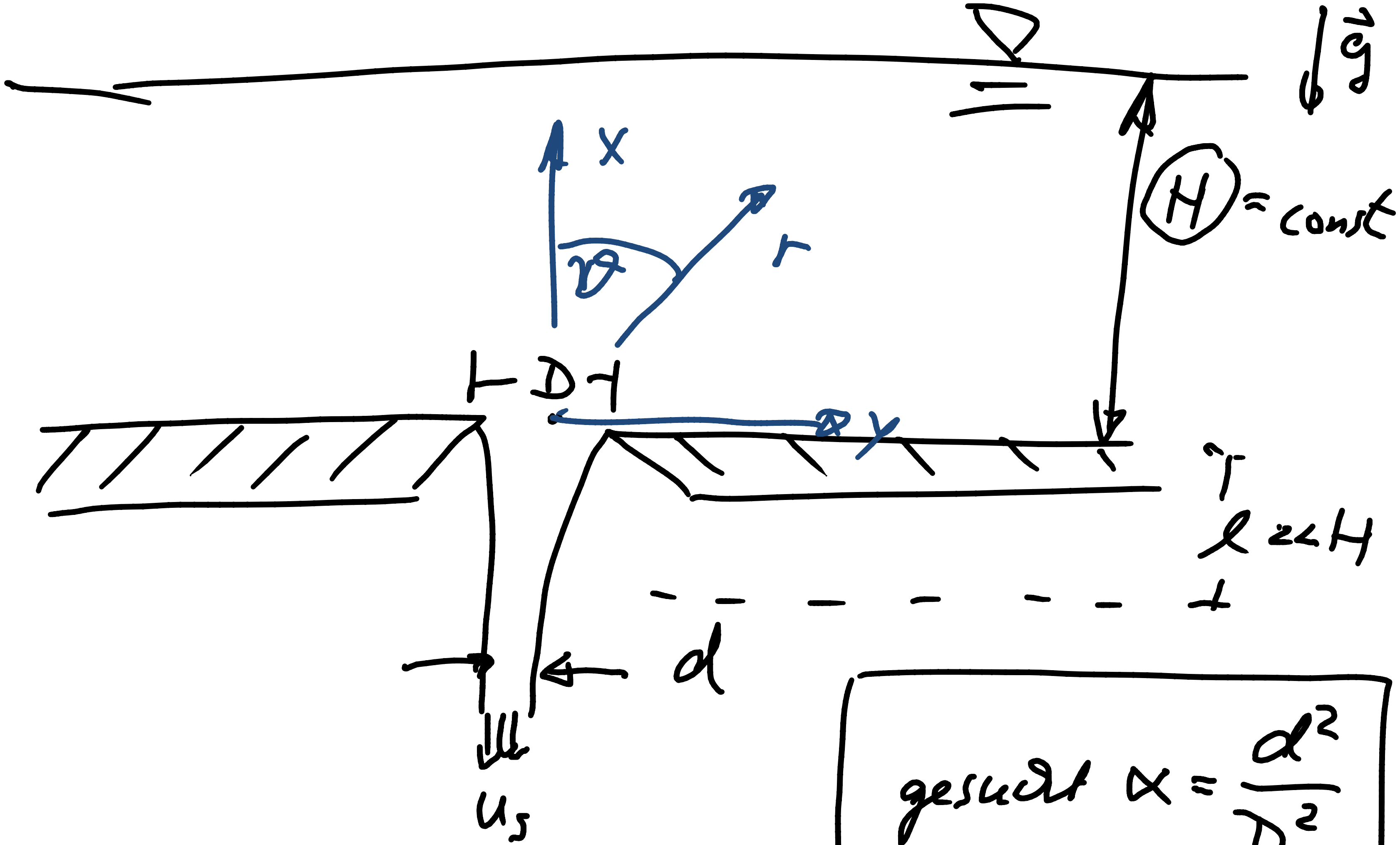
VRÜ-Blätter sind auch Klausurrelevant !



gesucht $\alpha = \frac{d^2}{D^2}$

$u_J = ?$

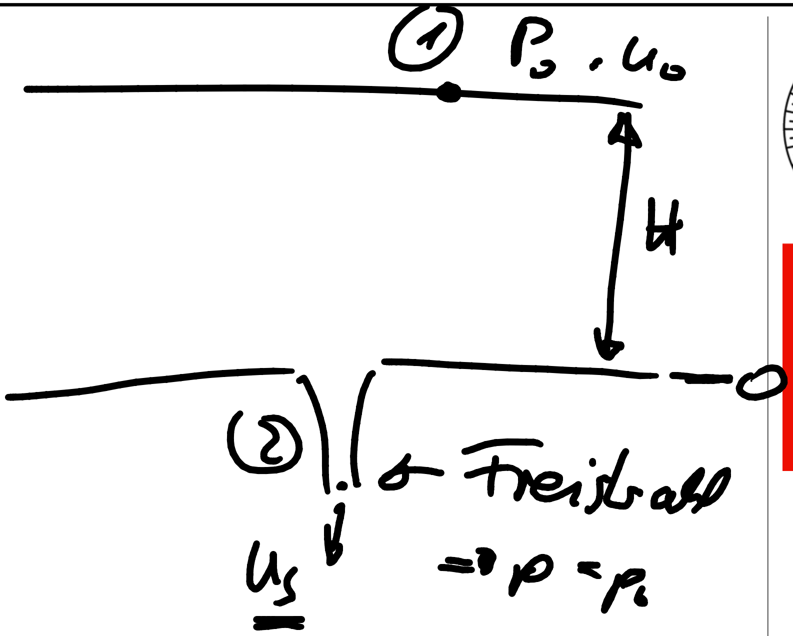
Volumenkraft $\vec{k} = -g\vec{e}_x$

$D \ll H$

$$\vec{u} = \frac{E}{4\pi}\frac{1}{r^2}\vec{e}_r$$

$E < 0$

1) u_s:



Bernoulli

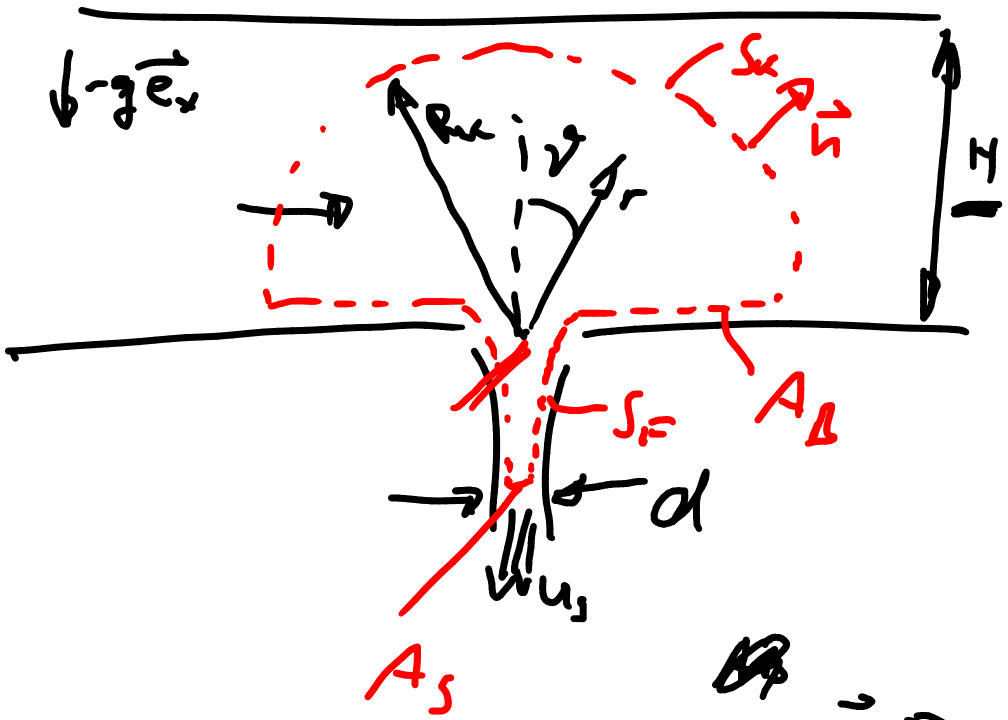
$$p_0 + \rho g H + \overset{0}{\cancel{u_0}} = p_0 + \frac{\rho}{2} u_s^2$$

$l \ll H$

$$\Rightarrow \underline{u_s} = \sqrt{2gH}$$

Torricellische Ausflussgeschw.

2) $\frac{d^2}{D^2}$



$A_B: \vec{u}\cdot\vec{n}=0$

$S_F: \vec{u}\cdot\vec{n}=0$

Impulssatz:

$$\oint_S \rho\vec{u}(\vec{u}\cdot\vec{n})\,dS = \int_V \underline{\rho\vec{k}\,dV + \int\vec{t}\,dS}$$

$$\underbrace{\oint \rho\vec{u}\cdot\vec{e}_x(\vec{u}\cdot\vec{n})\,dS}_{LS} = \underbrace{\int_V \rho\vec{k}\cdot\vec{e}_x\,dV + \oint t\,dS}_{RS} \quad ①$$

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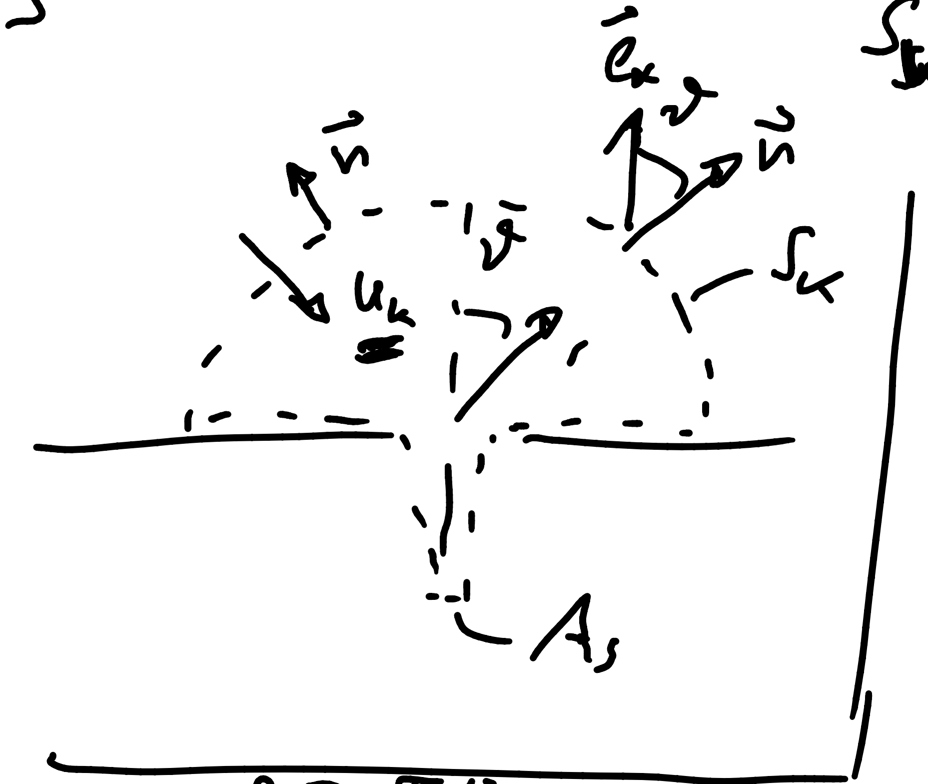
$$\oint_S \varrho\, \vec{u}\cdot\vec{e}_x\,(\vec{u}\cdot\vec{n})\, dS = \int_{S_K} \varrho\, \vec{u}\cdot\vec{n} \dots dS + \int_{A_S} \varrho\, \vec{u} \dots dS$$



$$S_K:\ dS = r^2 \sin(\vartheta)\, d\vartheta\, d\varphi$$

$$\vec{u}\cdot\vec{n} = -u_K = \frac{E}{4\pi r^2} \qquad \underline{E<0}$$

$$\Rightarrow \int_{\varphi=0}^{2\pi}\int_{\vartheta=0}^{\pi/2} \varrho\, \underbrace{\frac{E}{4\pi \boxed{r^2}}}_{\vec{u}\cdot\vec{n}}\; \underbrace{\frac{E}{4\pi r^2}\overbrace{\cos(\vartheta)}^{\vec{u}\cdot\vec{e}_x}}_{(\vec{u}\cdot\vec{e}_x)}\; \underbrace{r^2 \sin(\vartheta)\, d\vartheta\, d\varphi}_{dS}$$

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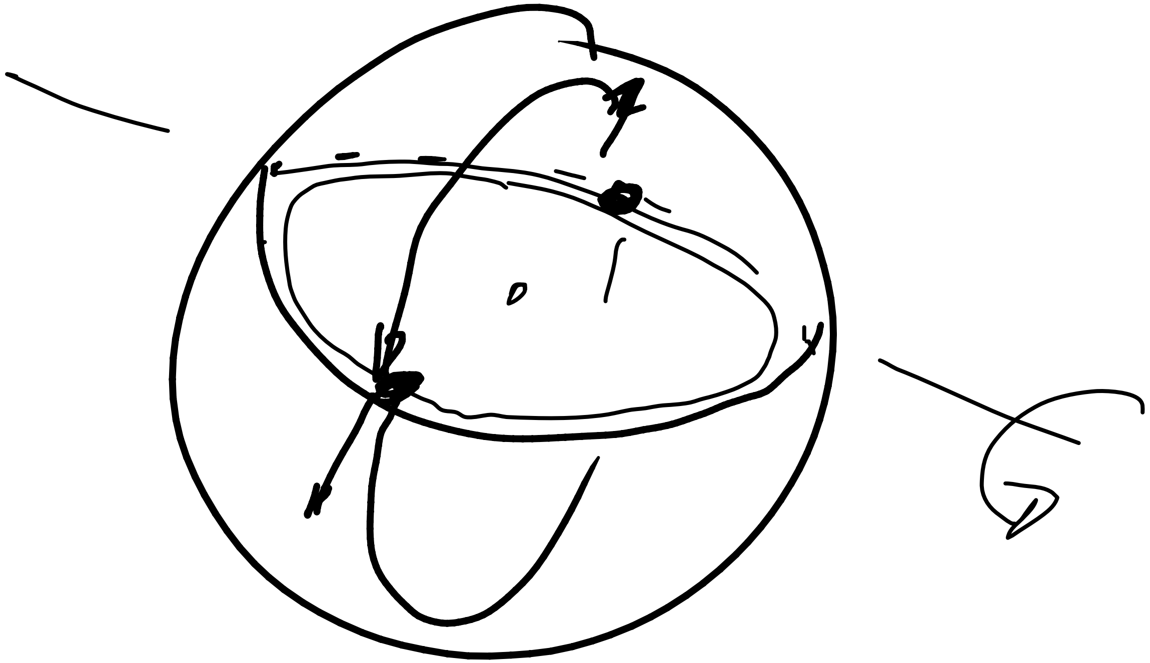

0

$$\Rightarrow \int_{\varphi=0}^{2\pi} \int_{\vartheta=0}^{\pi/2} \rho \frac{E^2}{(4\pi)^2 r^4} \underbrace{\cos(\vartheta) r^2 \sin(\vartheta) d\vartheta d\varphi}_{\frac{1}{2}\sin^2(\vartheta)}$$

Integral 354 Bronstein

$$\vdots$$

$$\Rightarrow \pi \rho \frac{E^2}{(4\pi)^2} \underline{\frac{1}{R_k^2}}$$

$$\left(\iint_{S_k} \rho \ldots\ldots dS \right)$$

A_s : $dS = r\,dr\,d\varphi$

$$\vec{u} = -u_s \vec{e}_x$$

$$\vec{u} \cdot \vec{n} = u_s$$

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$$\Rightarrow \iint_{A_S} \rho \vec{u}\cdot\vec{e}_x (\vec{u}\cdot\vec{n})\, dS = \int_{\varphi=0}^{2\pi}\int_{r=0}^{d/2} \overbrace{-\rho u_S^2}^{\vec{u}\cdot e_x(\vec{u}\cdot\vec{n})}\, \underbrace{r\,dr\,d\varphi}_{dS}$$

$$= -\rho\pi u_S^2 \frac{d^2}{4}$$

$$+ \iiint_V \rho\, \underset{\substack{\downarrow \\ -g\vec{e}_x}}{\vec{g}}\cdot\vec{e}_x\, dV = -\rho g \iiint_V dV = -\rho g \underset{\substack{\downarrow \\ \frac{1}{2}\frac{4}{3}\pi \underline{r}^3}}{V}$$

$$\Rightarrow -\frac{2}{3}\rho g \pi \underline{R_K}^3$$

L.S ✓

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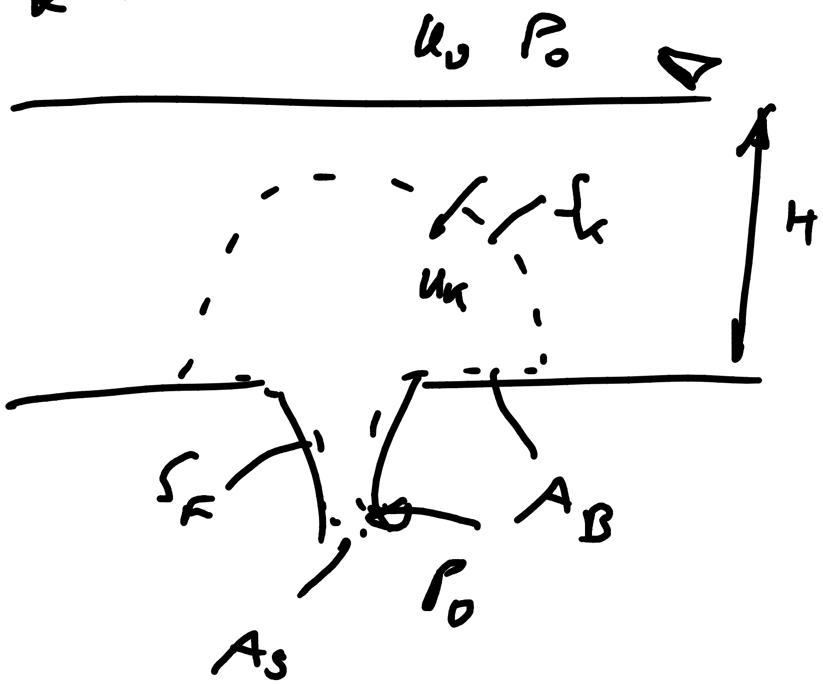
R.S $\oint_S \vec{t}\cdot\vec{e}_x \, dS$

$\vec{t} = -p\vec{n}$



$\underline{p_K} = ?$ $p_B = ?$

$p_{SF} = p_0$; $p_{AS} = p_0$

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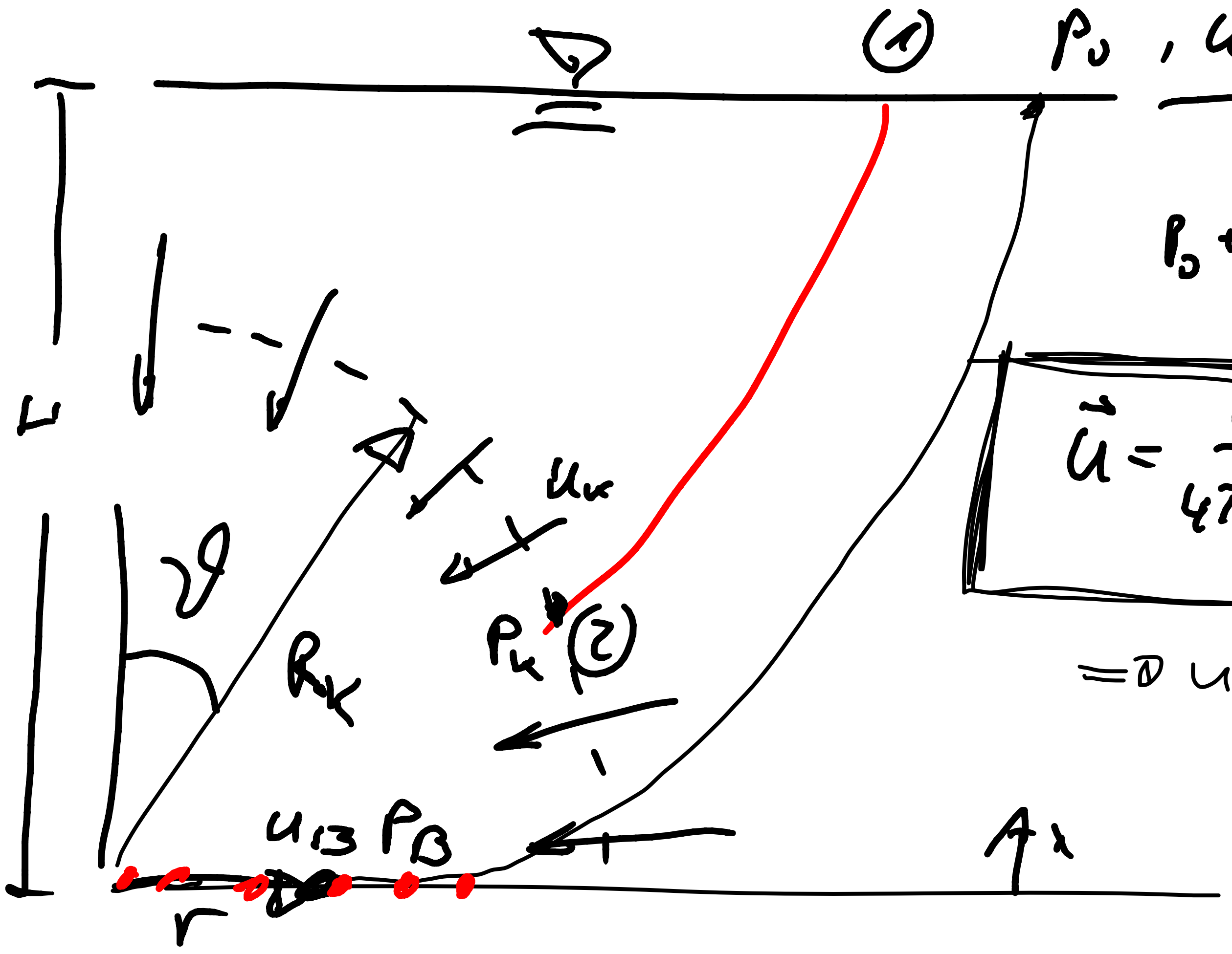

$$p_0 + \rho g H = p_B + \frac{\rho}{2} u_B^2$$

$$\boxed{\vec{u} = \frac{E}{4\pi}\frac{1}{r^2}\vec{e}_r}$$

$$\Rightarrow u_B = \frac{E}{4\pi}\frac{1}{r^2} \quad \Big\} \Rightarrow \boxed{p_B}$$

$$p_0 + \cancel{\rho}\,\rho g H = p_K + \frac{\rho}{2} u_K^2 + \rho g R_K \cos(\vartheta)$$

$$\downarrow$$

$$u_K = \frac{E}{4\pi}\frac{1}{R_K^2} \Rightarrow \boxed{p_K}$$

$p_0 \equiv 0$

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$$S_k: \; -\iint_{S_k} \underset{\uparrow \; ✓}{p_k} \; \underbrace{\vec{n}\cdot\vec{e}_x}_{\cos(\vartheta)} \; \underset{\uparrow \; ✓}{dS}$$

$$\Rightarrow \int_0^{2\pi}\int_0^{\pi/2} \Big(\overbrace{\varrho g H - \frac{\varrho}{2}\frac{E^2}{(4\pi)^2}\frac{1}{R_k^4} - \underbrace{\varrho g R_k \cos(\vartheta)}_{\varrho g H}}^{p_k} \; \underbrace{\vec{e}_r\cdot\vec{e}_x}_{\cos(\vartheta)} \cdot \ldots$$

$$\ldots \cdot \underbrace{R_k^2 \sin(\vartheta)\,d\vartheta\,d\varphi}_{dS}$$

$$\Rightarrow$$

$$\Rightarrow -\pi\left(\varrho g H - \frac{\varrho}{2}\frac{E^2}{(4\pi)^2}\frac{1}{R_k^4}\right)R_k^2 + \frac{2}{3}\pi\varrho g R_k^3$$

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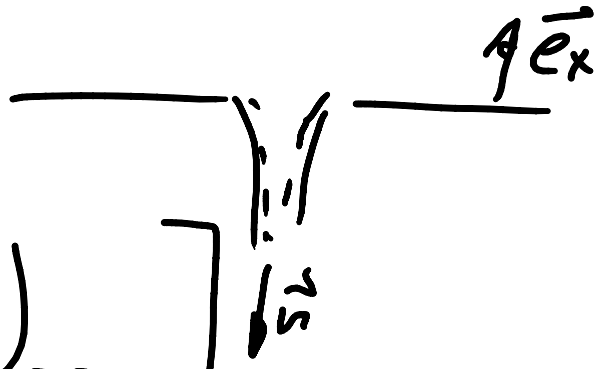
$$\int_{A_B} p_B\, \vec{n}\cdot\vec{e}_x \, dS$$

$$dS = r\,dr\,d\varphi$$

$$\vec{n}\cdot\vec{e}_x = -\vec{e}_x\cdot e_x = -1$$

$\vdots$



$$= 2\pi\left[\left(\rho g H\left(\frac{R_K^2}{2}-\frac{D^2}{2}\right)\dots\right]\right.$$

R.S ✓

L.S = R.S

$$U_s^2\,\frac{d^2}{4} = g H \frac{D^2}{4} + 2\frac{E^2}{(4\pi)^2 D^2}$$

$$E = ?$$

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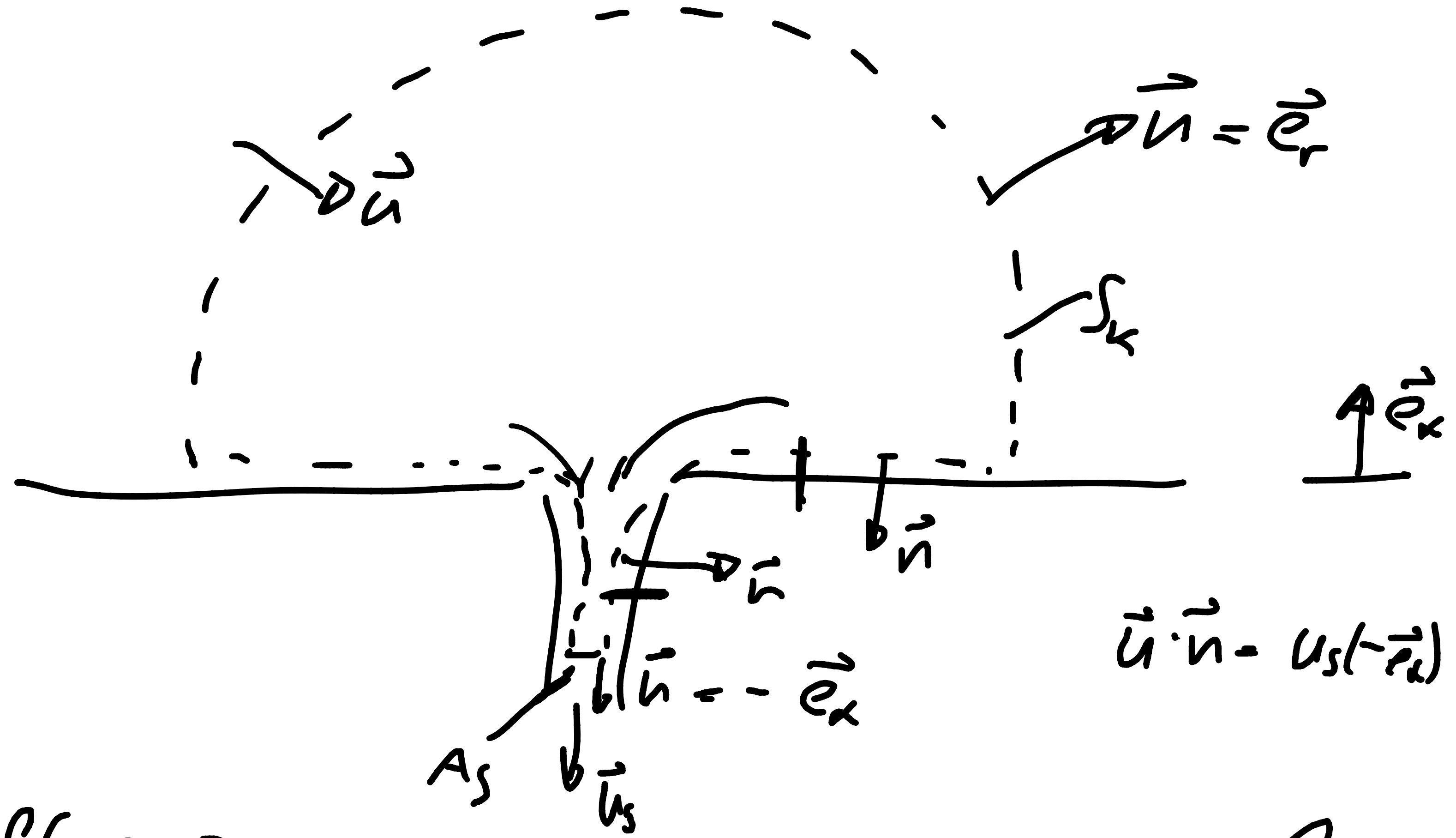

$$\vec u \cdot \vec n = u_s(-\vec e_x)\cdot(-\vec e_x)$$

$$\oint \vec u \cdot \vec n \, dS = 0$$

$E < 0$

$$\Rightarrow \underbrace{\int_0^{2\pi}\int_0^{\pi/2} \overbrace{\frac{E}{4\pi}\frac{1}{R_k^2}}^{\vec u\cdot\vec n} \underbrace{R_k^2 \sin(\vartheta)\, d\vartheta\, d\varphi}_{dS}}_{S_n} + \overbrace{\int_2^{2\pi}\int_0^{d/2} \overset{\vec u\cdot\vec n}{u_s}\, \underbrace{r\, dr\, d\varphi}_{dS}}^{A_s} = 0$$

$$\Rightarrow \quad \underline{E} = \frac{\pi d^2}{2} u_s \qquad A = \frac{E}{4\pi}$$

$$A = u_s \frac{d^2}{8}$$

(I) $u_s = \sqrt{8gH}$ **<u>Bernoulli</u>**

(II) $\underline{u_s^2} \frac{d^2}{4} = gH \frac{D^2}{4} + 2 \frac{E^2}{(4\pi)^2 D^2}$ **<u>Impuls</u>**

(III) $E = \frac{\pi d^2}{2} \underline{u_s}$ **<u>Konti</u>**

I..III Ineinander einsetzen $\Rightarrow$

$$2gH\frac{d^2}{4} = gH\frac{D^2}{4} + 2\frac{2gH\pi^2 d^4}{4(4\pi)^2 D^2} \quad \Big| \frac{1}{D^2 gH}$$

$$\alpha = \frac{d^2}{D^2}$$

$$\vdots$$

$$\frac{1}{2}\alpha = \frac{1}{4} + \frac{1}{16}\alpha^2 \xRightarrow{P.Q} \alpha_{1,2} = 4 \pm \sqrt{12}$$

$$\alpha = 4 - \sqrt{12} = \underline{\underline{0.536}}$$

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