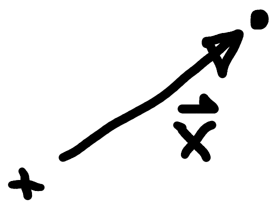
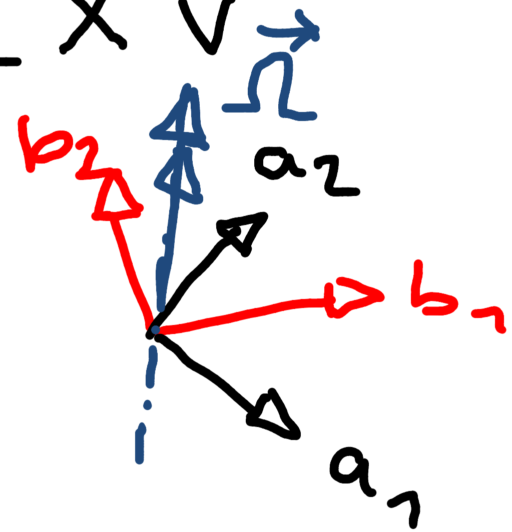
$$\frac{d_A}{dt}\vec{v} = \frac{d_B}{dt}\vec{v} + \overset{A\to B}{\vec{\Omega}} \times \vec{v}$$

$$\frac{D_I}{Dt}\vec{x} = \vec{c}$$





$$\frac{D_B}{Dt}\vec{x} = \vec{w}$$

$$\frac{D_I}{Dt}\vec{x} = \frac{D_B}{Dt}\vec{x} + \vec{\Omega} \times \vec{x}$$

$$= \vec{w} + \Omega \times \vec{x}$$

$$\frac{D_I}{Dt}\vec{c} = \frac{D_B}{Dt}\vec{c} + \vec{\Omega}\times\vec{c}$$

$$= \frac{D_B}{Dt}\left(\vec{w} + \vec{\Omega}\times\vec{x}\right) + \vec{\Omega}\times\left(\vec{w} + \vec{\Omega}\times\vec{x}\right)$$

$$= \underbrace{\frac{D_B}{Dt}\vec{w}}_{\text{Relativ-beschleunigung}} + \underbrace{\dot{\vec{\Omega}}\times\vec{x}}_{\text{Tangential}} + \underbrace{\boxed{\vec{\Omega}\times\vec{w} + \vec{\Omega}\times w}}_{\text{Coriolis}} + \underbrace{\vec{\Omega}\times\left(\vec{\Omega}\times\vec{x}\right)}_{\text{Zentrifugal}}$$

$$\vec{x} = r\vec{e}_r + z\vec{e}_z \qquad \vec{\Omega} = \Omega \vec{e}_z$$

$$\vec{\Omega} \times \vec{x} = \Omega \vec{e}_z \times r \vec{e}_r$$

$$= \Omega r \vec{e}_\varphi$$

$$\vec{\Omega} \times (\vec{\Omega} \times \vec{X}) = \Omega \vec{e}_z \times \Omega r \vec{e}_\varphi$$

$$= -\Omega^2 r \vec{e}_r$$

$$-\frac{1}{2} \nabla \left(\Omega^2 r^2\right) = -\Omega^2 r \vec{e}_r$$

$$\frac{D_I}{Dt}\vec{c} + \frac{\nabla p}{\varrho} - \vec{k} = 0 \quad \Big\| \quad \Omega = \text{const}$$

$$\frac{D_B}{Dt}\vec{w} + 2\vec{\Omega}\times\vec{w} - \nabla\left(\frac{1}{2}\Omega^2 r^2\right) + \frac{\nabla p}{\varrho} - \vec{k} = 0$$

$$\psi = -\vec{k}\cdot\vec{x} - \frac{1}{2}\Omega^2 r^2$$

$$\nabla\psi = -\vec{k} - \nabla\left(\frac{1}{2}\Omega^2 r^2\right)$$

$$\frac{D_B}{Dt}\vec{w} + 2\vec{\Omega}\times\vec{w} + \nabla\psi + \frac{\nabla p}{\varrho} = 0$$

$$\vec{e}_s = \vec{w}/|\vec{w}|$$

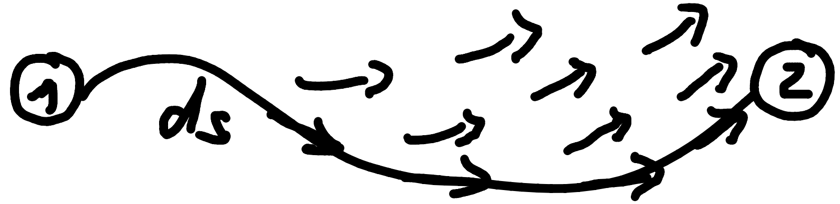
$$\nabla p = \frac{\partial p}{\partial x_1}\vec{e}_1 + \frac{\partial p}{\partial x_2}\vec{e}_2 + \frac{\partial p}{\partial x_3}\vec{e}_3$$

$$\vec{e}_1 \cdot \nabla p = \frac{\partial p}{\partial x_1}$$

$$\vec{e}_s \cdot \nabla p = \frac{\partial p}{\partial s}$$

$$\underbrace{\frac{\partial}{\partial t}\vec{w} + \vec{w}\cdot\nabla\vec{w}}_{\frac{D}{Dt}\vec{w}} + 2\vec{\Omega}\times\vec{w} + \nabla\psi + \frac{\nabla p}{\varrho} = 0 \quad \Big|\cdot\vec{e}_s$$

$$\vec{e}_s\cdot\frac{\partial}{\partial t}\vec{w} + \vec{e}_s\cdot\nabla\left(\frac{1}{2}w^2\right) + \cancel{\vec{e}_s\cdot\left(2\vec{\Omega}\times\vec{w}\right)} + \frac{d\psi}{ds} + \frac{1}{\varrho}\frac{dp}{ds} = 0$$



$$\frac{\partial}{\partial t}\int_1^2 w\,ds + \int_1^2 \frac{\partial}{\partial s}\left(\frac{1}{2}w^2\right)ds + \int_1^2 \frac{\partial \psi}{\partial s}\,ds + \int_1^2 \frac{1}{\rho}\frac{\partial p}{\partial s}\,ds = 0$$

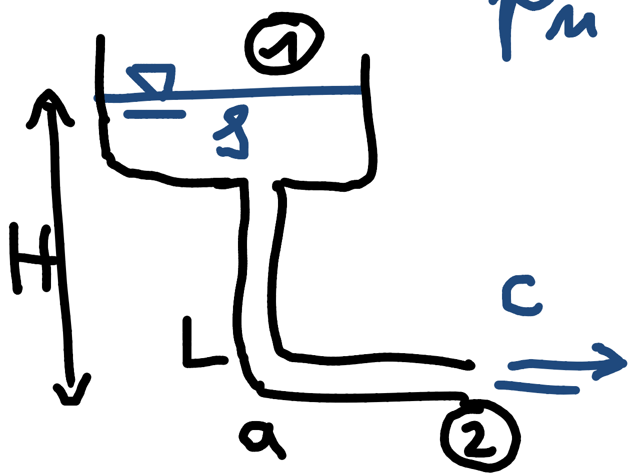
$$\boxed{\frac{\partial}{\partial t}\int_1^2 w\,ds + \frac{1}{2}w_2^2 + \psi_2 + \frac{1}{\rho}p_2 = \frac{1}{2}w_1^2 + \psi_1 + \frac{1}{\rho}p_1}$$

$$\frac{\partial}{\partial t}\int_1^2 w\,ds + \frac{1}{2}w_2^2 - \vec{k}\cdot\vec{x}_2 - \frac{1}{2}\Omega^2 r_2^2 + p_2/\rho =$$

$$\frac{1}{2}w_1^2 - \vec{k}\cdot\vec{x}_1 - \frac{1}{2}\Omega^2 r_1^2 + p_1/\rho$$

$$\vec{k} = \vec{g}$$

$$\frac{\partial}{\partial t}\int w\, ds + \frac{1}{2}w_2^2 + g h_2 - \frac{1}{2}(\Omega r_2)^2 + p_2/\rho = \underbrace{\frac{1}{2}w_1^2 + g h_1 - \frac{1}{2}(\Omega r_1)^2 + p_1/\rho}_{\alpha}$$



p_u $\quad \downarrow g$

$$\dot{c} L + \frac{c^2}{2} + \cancel{\frac{p_u}{\rho}} = gH + \cancel{\frac{p_u}{\rho}}$$

$$2 \dot{c} L = 2gH - c^2$$

$c(t=0)=0$

$$2L \int_0^c \frac{d\bar{c}}{2gH - \bar{c}^2} = \int_0^t d\bar{t}$$

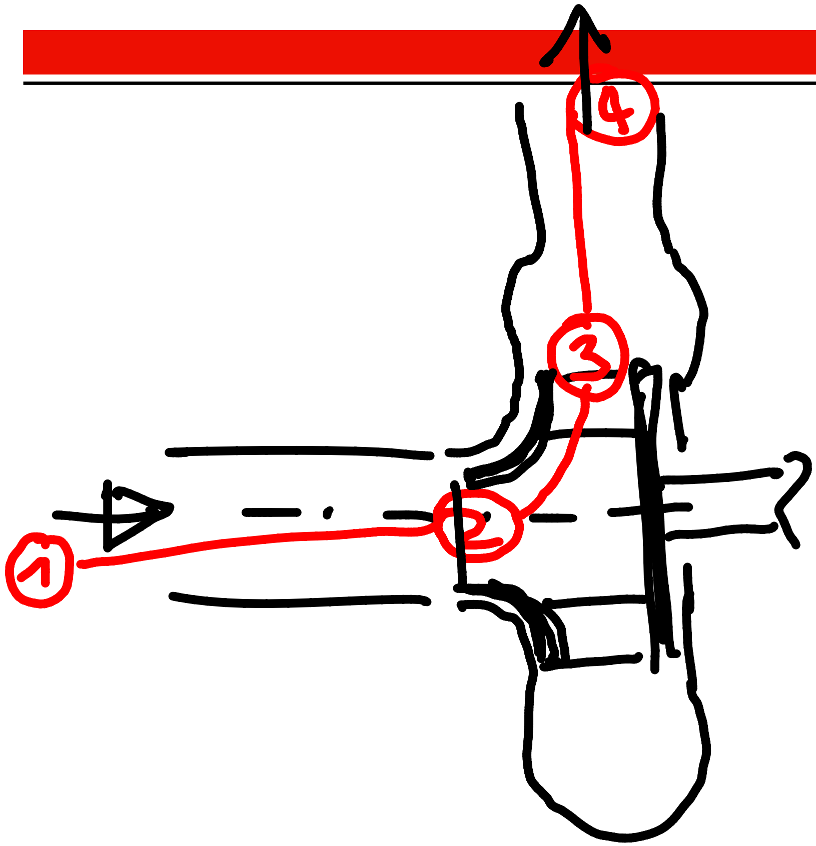
$$2L \frac{1}{\sqrt{2gH}} \operatorname{arctanh}\left(\frac{c}{\sqrt{2gH}}\right) = t$$

$$c = \sqrt{2gH}\,\tanh\left(\frac{\sqrt{2gH}\,t}{2L}\right)$$

$$c_{t\to\infty} = \underline{\underline{\sqrt{2gH}}}$$

Torricelli-Geschwindigkeit

Kreiselpumpe

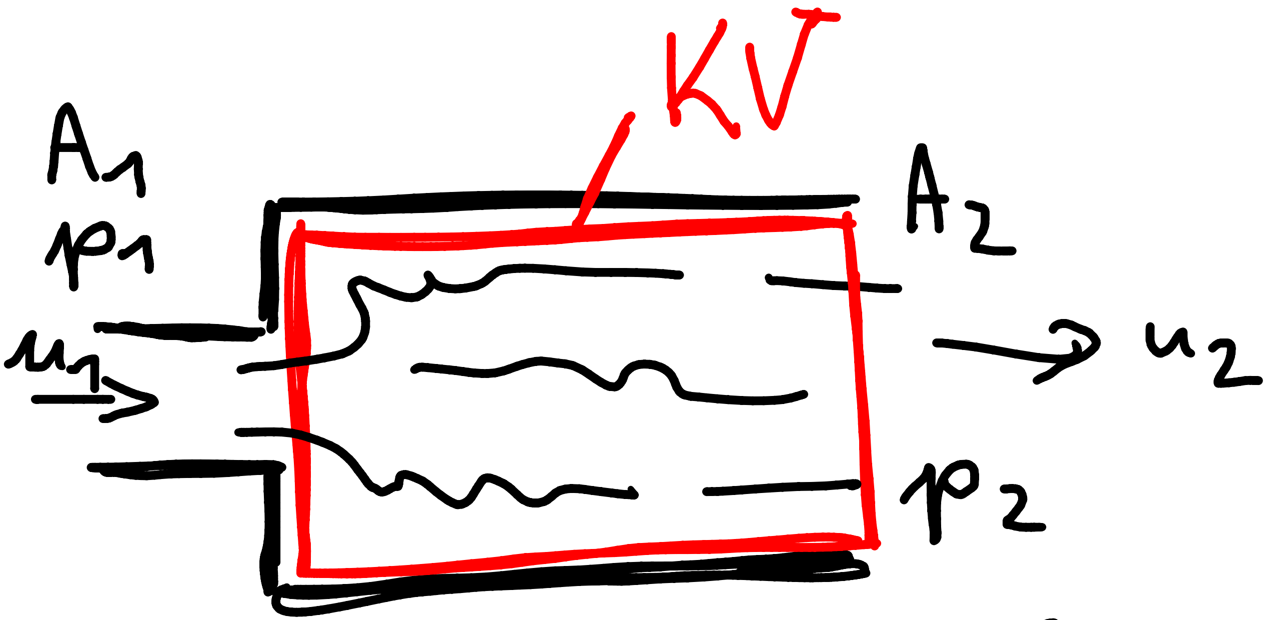


1 – 2 stehender Bernoulli

2 – 3 rotierender Bernoulli

3 – 4 stehender Bernoulli

CARNOTSCHER STOSSVERLUST



$$\rho Q (u_2 - u_1) = (p_1 - p_2) A_2 \quad \text{Impulssatz}$$

$$\rho u_1^2 \frac{A_1}{A_2} \left(\frac{A_1}{A_2} - 1 \right) = p_1 - p_2$$

$$\underbrace{p_2 - p_1 = \varrho u_1^2 \frac{A_1}{A_2}\left(1 - \frac{A_1}{A_2}\right)}_{\Delta p_{real}}$$

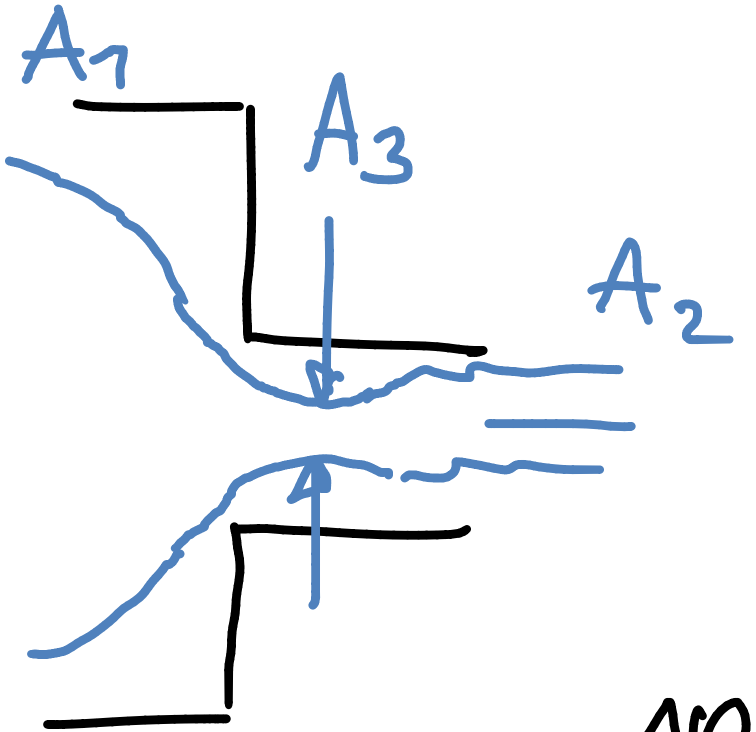
$$p_1 + \frac{\varrho}{2} u_1^2 = p_2 + \frac{\varrho}{2} u_2^2$$

$$\underline{p_2 - p_1} = \frac{\varrho}{2}\left(u_1^2 - u_2^2\right) = \underbrace{\frac{\varrho}{2} u_1^2 \left(1 - \frac{A_1^2}{A_2^2}\right)}_{\Delta p_{ideal}}$$

$$\underline{\underline{\Delta p_{vc}}} = \Delta p_{id} - \Delta p_{re}$$

$$= \underline{\underline{\frac{\varrho}{2} u_1^2}} \, \underline{\underline{\left(1 - \frac{A_1}{A_2}\right)^2}} = \underline{\underline{\frac{\varrho}{2} \left(u_1 - u_2\right)^2}}$$

ideal schlechte Düse



$$A_3 = \alpha A_2$$

für kreisrunde Bohrung

$$\alpha = 0.58$$

$$\boxed{\Delta p_{vc} = \frac{8}{2} u_2^2 \left(\frac{1}{\alpha} - 1 \right)^2}$$