Virtuelle Massen (engl. added mass)

Tensor mit

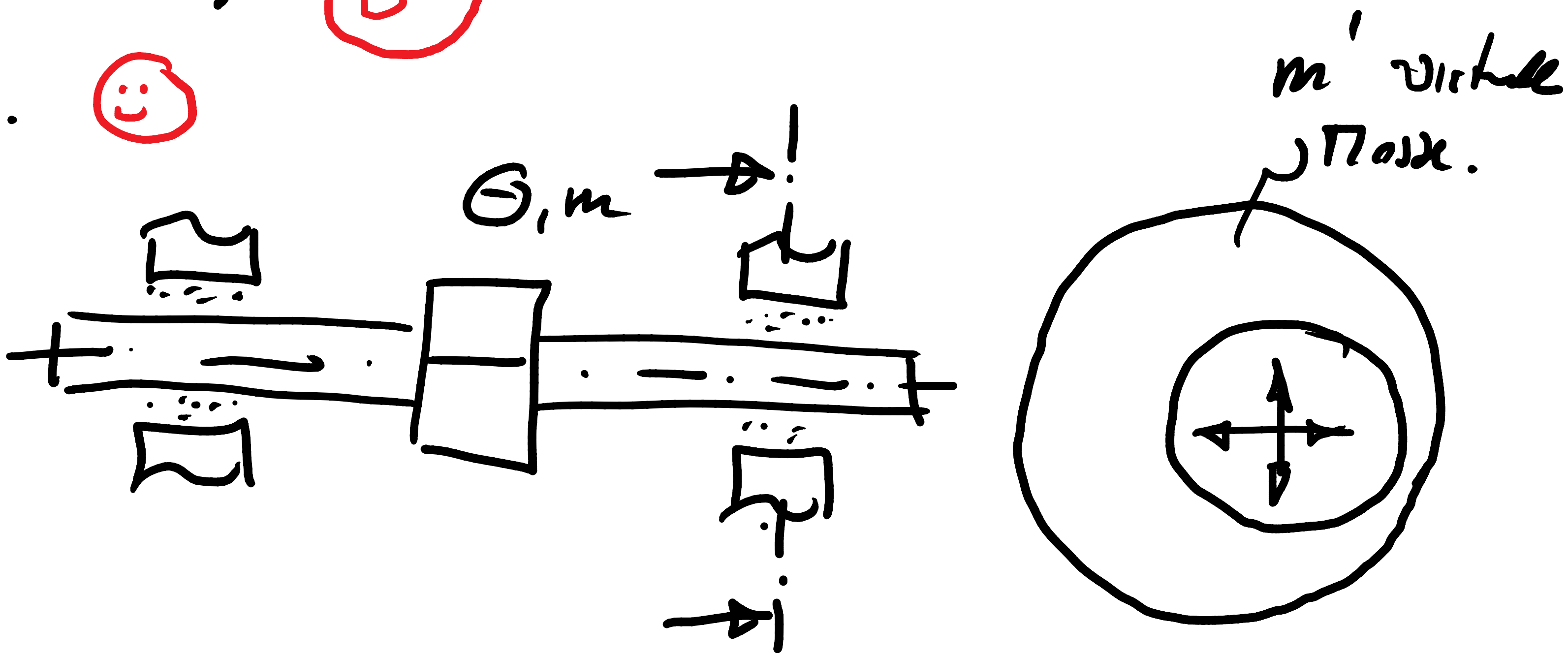
$$m_{ij} \hat{=} \begin{bmatrix} m_{11}, & m_{12}, & m_{13} & \dots & & m_{16} \\ & m_{22} & & & & \\ & & \ddots & & & \\ & \text{sym} & & \ddots & & \\ & & & & \ddots & \\ & & & & & m_{66} \end{bmatrix}$$

beschreibt die Trägheitseigenschaft d. Flüssigkeit vollständig.

Wichtig bei beschleunigten $\left(\frac{\partial}{\partial t} \neq 0\right)$, transienten Strömung.

## Vielfältige Anwendung

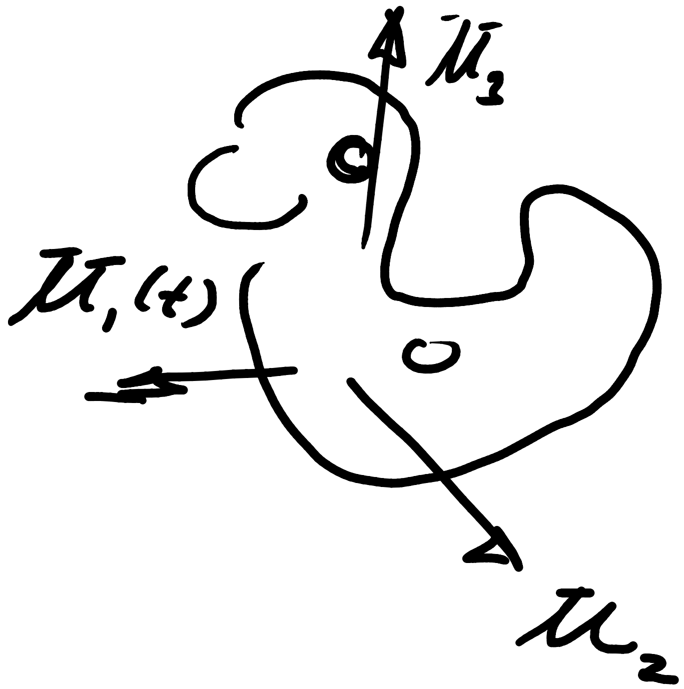
- Marine
- Offshore → regenerative Energie
- biologische Propulsion.
- Gleitlager
- Hydrotilger



Wie kann die virtuelle Masse ausgerechnet werden?

Sprich Kap. 10 (Potentialtheorie)

$$K = \int_V \frac{\varrho}{2} u_i u_i \, dV \overset{!}{=} \frac{1}{2} m'_{11} U_1^2$$



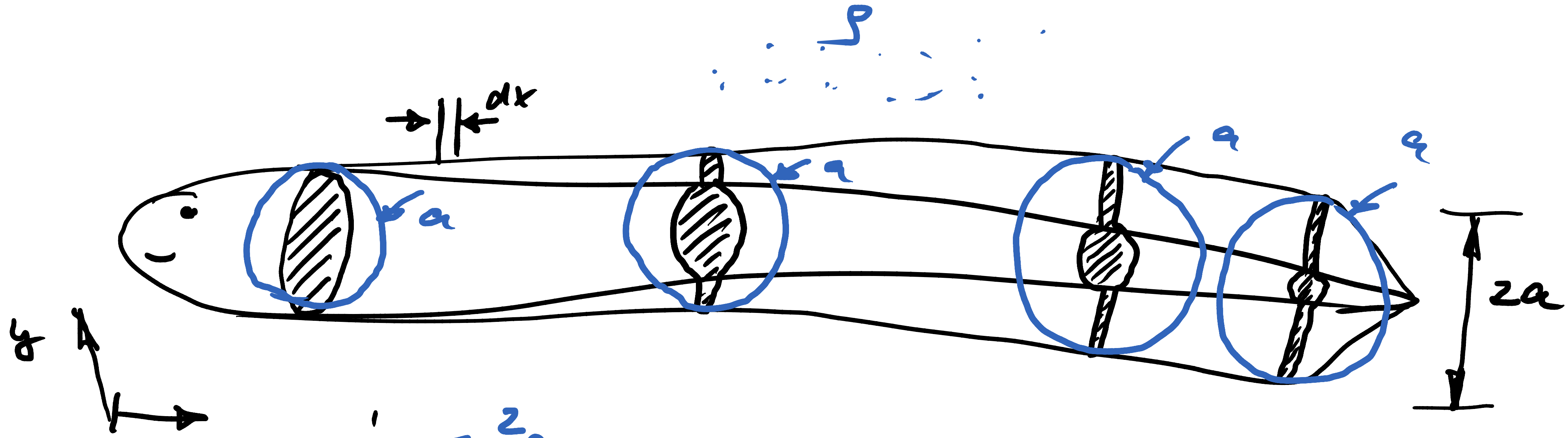

$$\Rightarrow \frac{m'_{11}}{\varrho} = \int_V \underbrace{\frac{u_i u_i}{U_1^2}}\, dV = \int_V \frac{\partial \varphi_1}{\partial x_i} \frac{\partial \varphi_1}{\partial x_i} \, dV$$

Formfunktionen,
d.h. allein von der
Gestalt des Körpers abhängig.

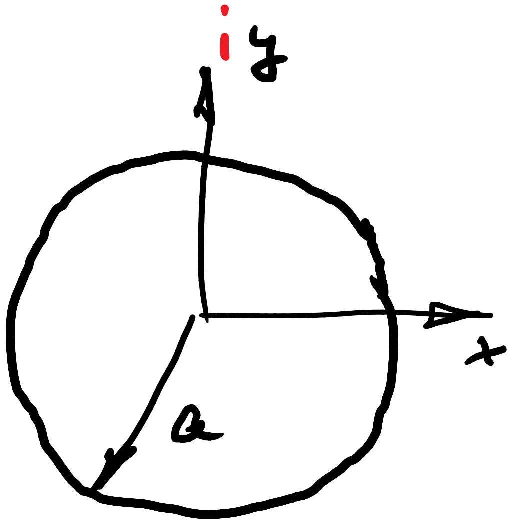
$$u_i = \frac{\partial \Phi_1}{\partial x_i} = U_1 \frac{\partial \varphi_1}{\partial x_i}$$

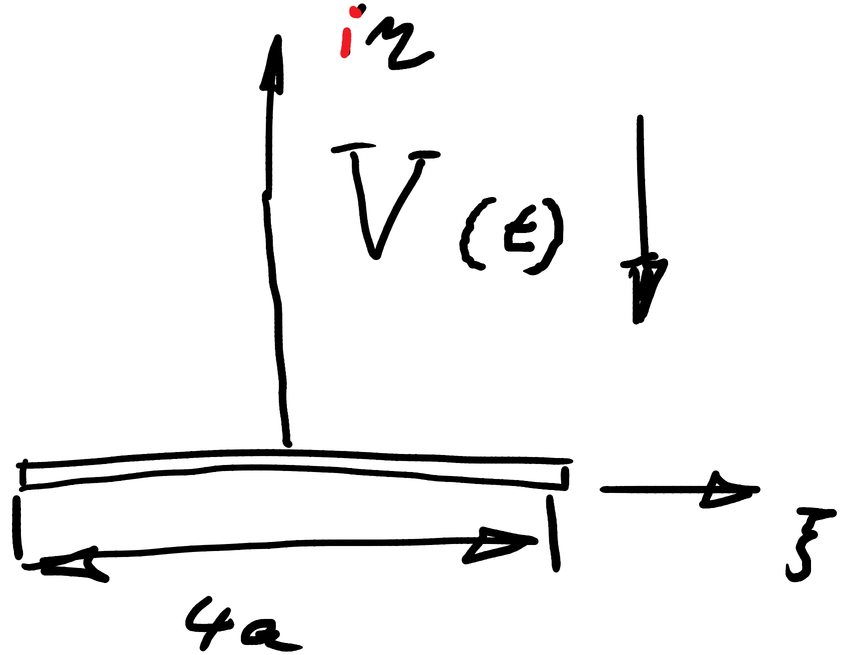
$\varphi_1$ ist ein normiertes Pot.

$$\boxed{\frac{m'_{ij}}{\rho} := \int_V \frac{\partial \varphi_i}{\partial x_k}\frac{\partial \varphi_j}{\partial x_k}\, dV}$$




$$m' = \pi a^2 \rho$$




$z = x + iy$

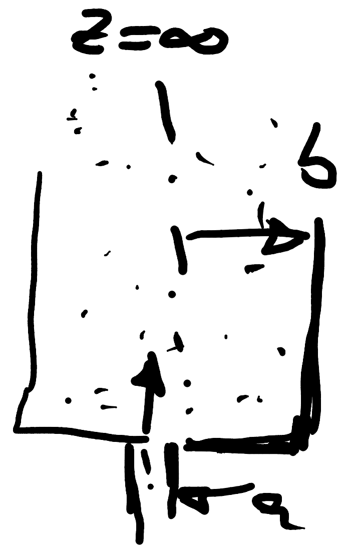
$\zeta = \xi + i\eta$

## 3 Möglichkeiten Potentialtheorie:

1.) Lösen der Feldgleichung $\Delta \Phi = 0$ unter Beachtung der Randbedingung. $\Big\}$

1.1.) Numerische Lösung

1.2.) Lösen von $\Delta \phi = 0$ über einen Separationsansatz.
→ Funktioniert nur, wenn die Ränder der Gebiete Koordinatenflächen sind.



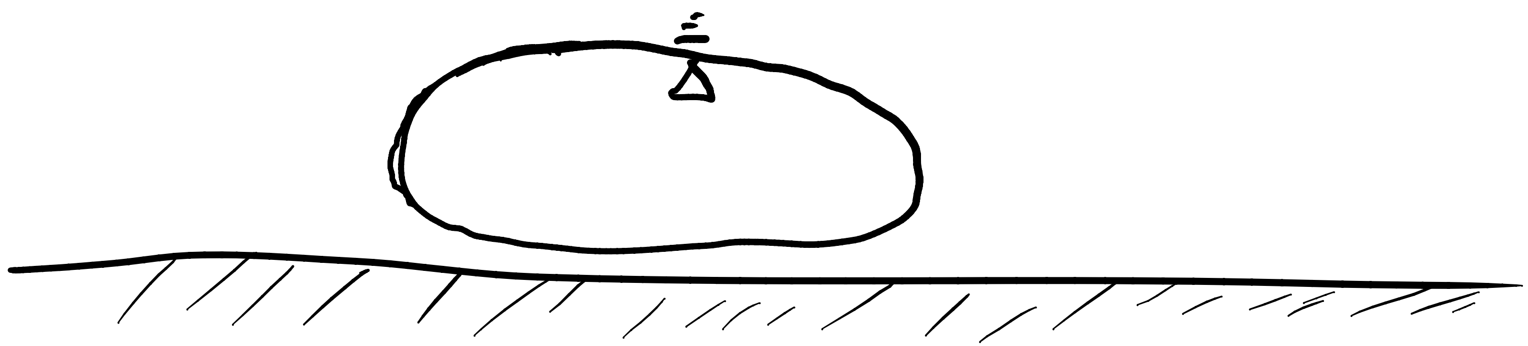

2.) Zusammensetzen (Superposition) der Grundlösungen über sog. Fundamentallösung (Singularitäten)

3.) Konforme Abbildung

TECHNISCHE UNIVERSITÄT DARMSTADT

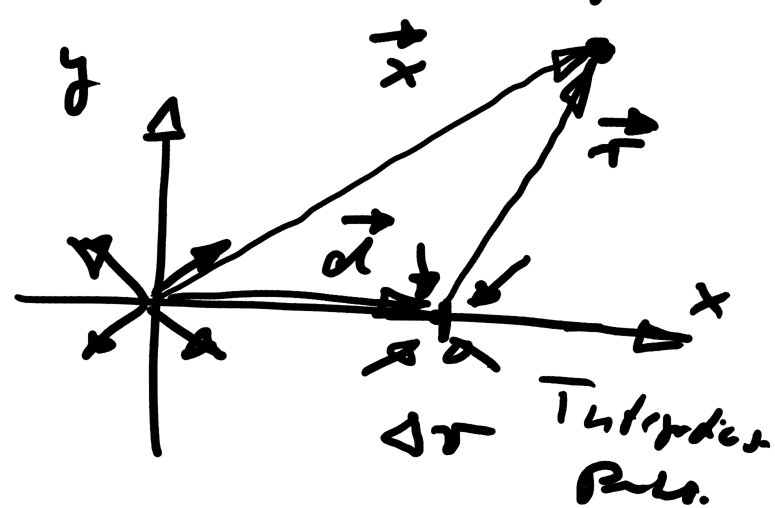
FLUID SYSTEM TECHNIK

Biofluidmechanik



$\Phi = U_i x_i$: Potential einer Parallelströmung

$\Phi_Q = \dfrac{E}{2\pi} \ln r$ Potential einer Quelle im Ursprung.




$$\Phi = \lim_{\substack{\Delta r \to 0 \\ E \to \infty}} \left[ \frac{E}{2\pi} \ln r - \frac{E}{2\pi} \ln \sqrt{(x-\Delta r)^2 + y^2} \right]$$

$$= \underbrace{(\nabla \phi_Q) \cdot \vec e_x}_{\text{Richtung d. Dipol.}}$$

$$\vec x = \vec r + \vec a$$

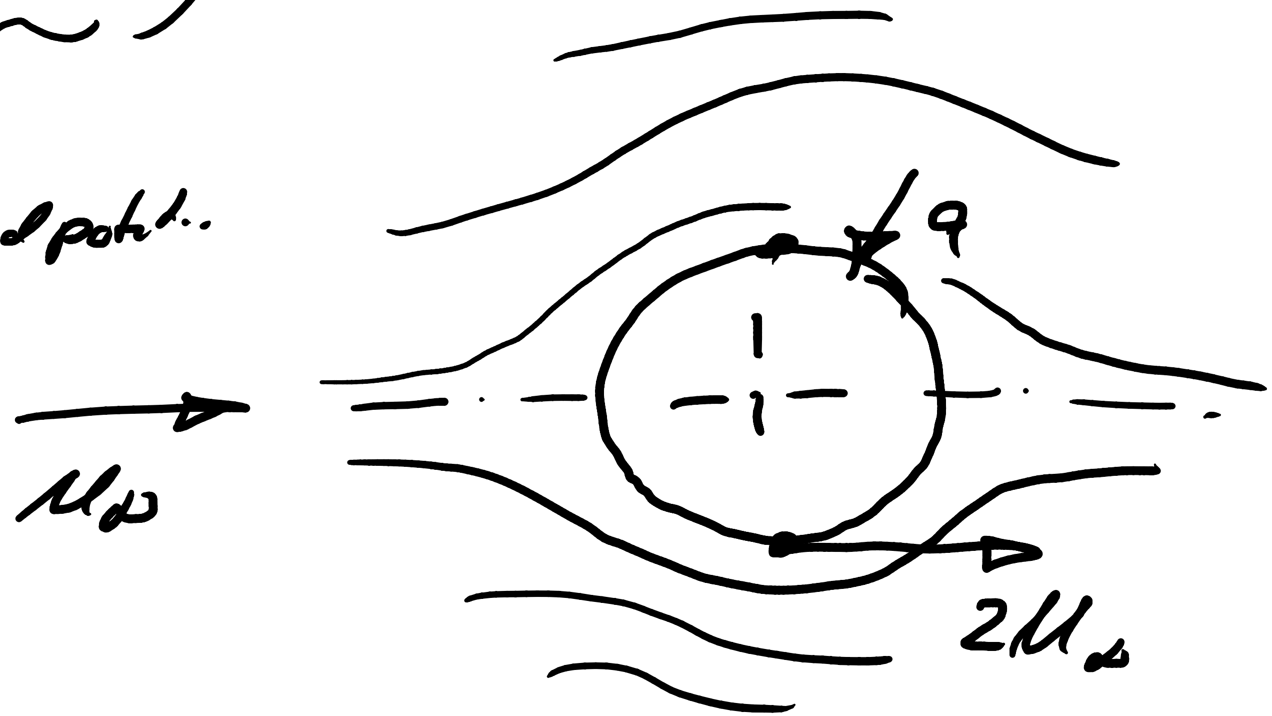
$$|\vec r| = |\vec x - \vec a| = \sqrt{(x-\Delta r)^2 + y^2}$$

Dipolpotential: zylindersymmetrisch

$$\Phi = u_\infty \Big( \underbrace{\underbrace{r\cos\varphi}_{x}}_{} + \underbrace{\frac{a^2}{r}\cos\varphi}_{} \Big)$$

Parallelströmung Dipolpotential

x-Ri:



Komplexes Potential

$$F(z) = \Phi + i\Psi$$

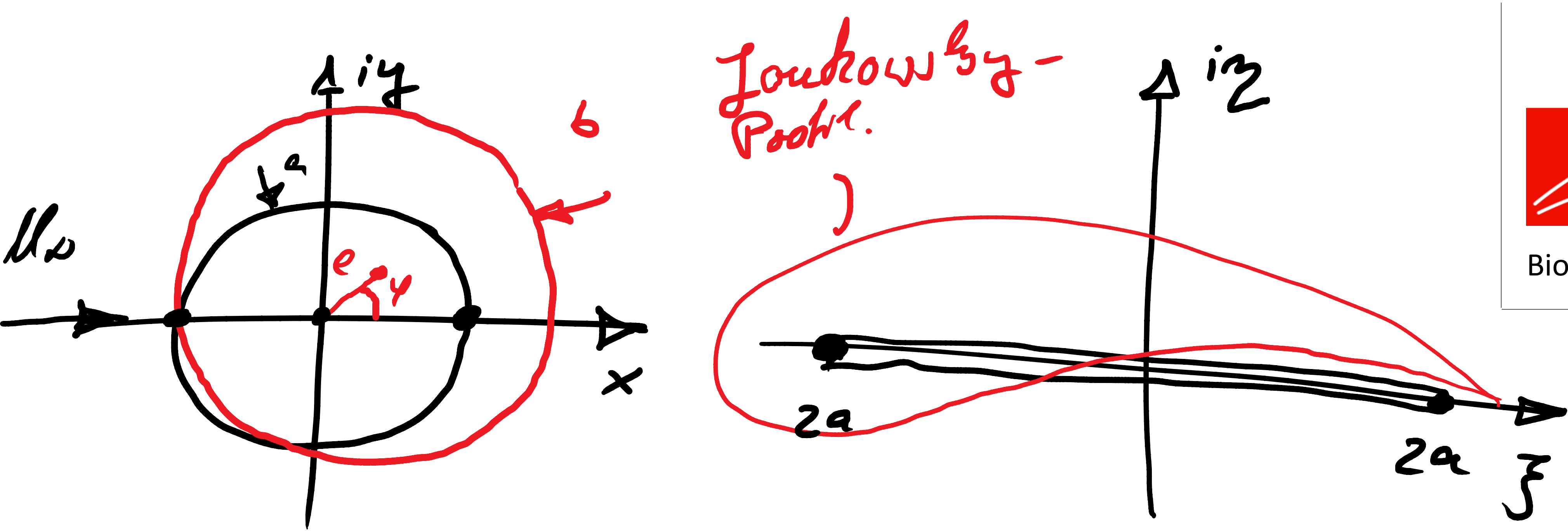
reelles Potential

Stromfunktion

Für die Zylinderumströmung

$$F(z) = u_\infty \left( z + \frac{a^2}{z} \right)$$

$$z = x + iy = r(\cos\varphi + i\sin\varphi) = r\exp(i\varphi)$$



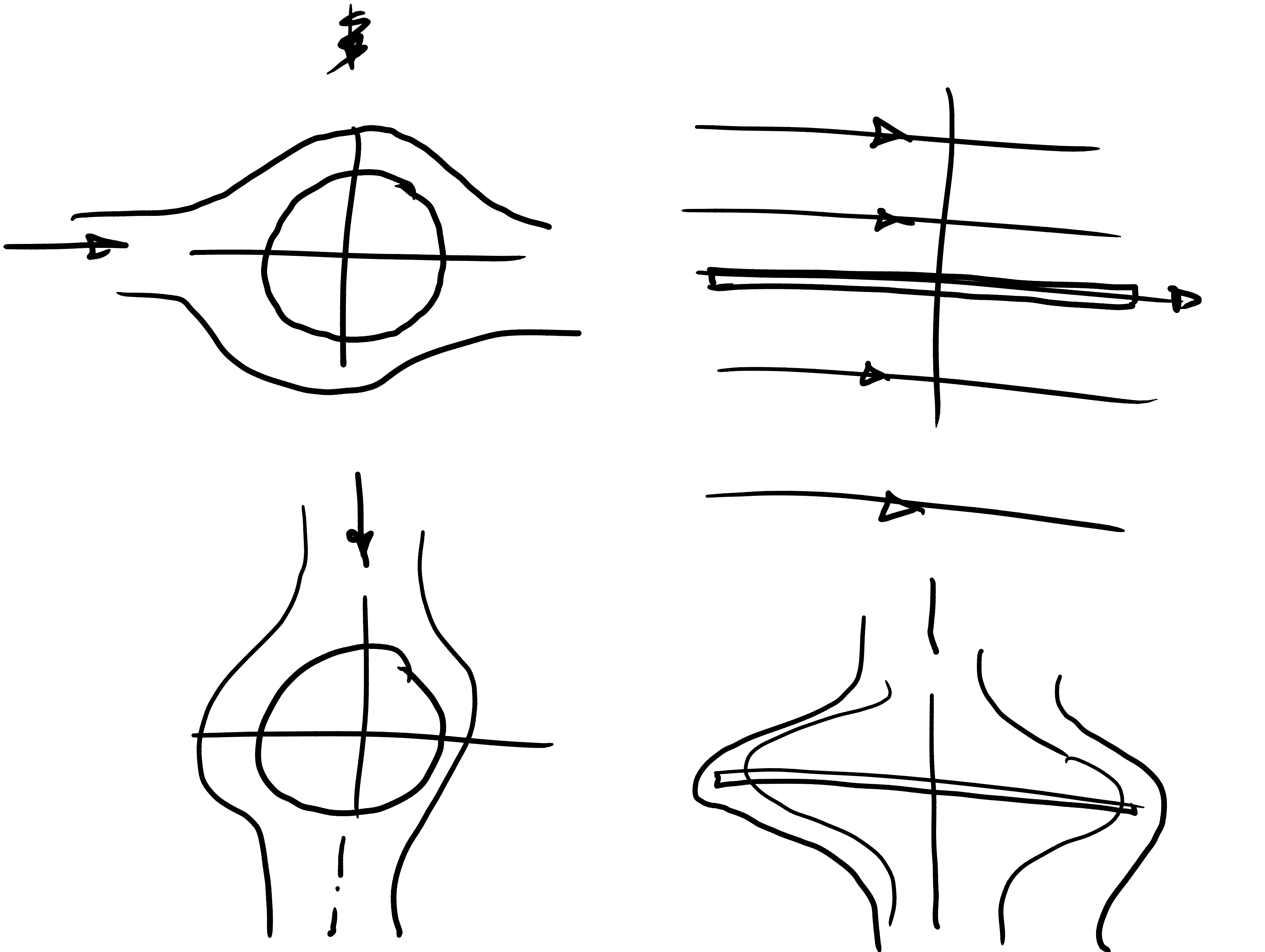

$$\zeta = f(z) = \underbrace{z + \frac{a^2}{z}}_{\downarrow} \quad \text{Joukowskysche Abbild.}$$

▷

$$F(z) = u_\infty\left(z + \frac{a^2}{z}\right)$$

$$F(\zeta) = u_\infty \zeta$$

◁



Newmann: Marine Hydrodynamics

TECHNISCHE UNIVERSITÄT DARMSTADT

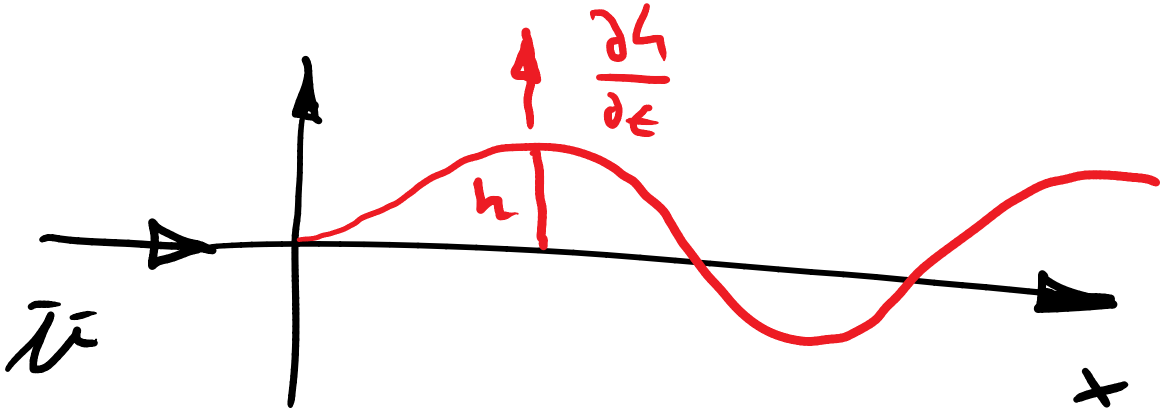
FLUID SYSTEM TECHNIK

Biofluidmechanik

Optimierungsaufgabe $\eta_{Fr} := \dfrac{\overline{F U}}{\overline{\dot W}} = 1 - \dfrac{\overline{\dot K}}{\overline{\dot W}}$

$\overline{\dot W} \mathrel{\hat=} \overline{P_{dr}}$ Wellenleistung oder schnellich technisch bekannt.

$$\dot W = \int_0^l \frac{\partial h}{\partial t} F_y \, dx$$




$$F_y = \frac{D}{Dt}\Big(\underbrace{\rho A(x)}_{m'} w(x,t)\Big)$$

$m' = \rho \pi a^2$

Oseensche Linearisierung

$$w(x,t) = \frac{\partial h}{\partial t} + \overbrace{U \frac{\partial h}{\partial x}} + O(\varepsilon^2).$$

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + w\frac{\partial}{\partial y} + (U+u)\frac{\partial}{\partial x} \approx \frac{\partial}{\partial t} + \bar{U}\frac{\partial}{\partial x} + O(\varepsilon^2)$$

$$\dot{W} = \rho \int_0^l \underline{\underline{\frac{\partial h}{\partial t}}} \left(\frac{\partial}{\partial t} + \bar{U}\frac{\partial}{\partial x}\right)\left\{w(x,t)\,A(x)\right\} dx =$$

$$= \rho \int_0^l \left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right)\left\{\frac{\partial h}{\partial t}\, w(x,t)\,A(x)\right\} dx +$$

$$-\rho \int_0^l \underbrace{\left(\frac{\partial^2 h}{\partial t^2} + U\frac{\partial^2 h}{\partial x\,\partial t}\right)}_{\frac{\partial w}{\partial t},\ \text{da}\ w = \frac{\partial h}{\partial t} + U\frac{\partial h}{\partial x}.} w(x,t)\,A(x)\,dx$$

$$\nabla \quad \phi \frac{\partial \phi}{\partial t} = \frac{1}{2} \frac{\partial}{\partial t}(\phi^2)$$

vgl. Helmholtz Bewegung:

$$\Delta$$

$$\Leftrightarrow \quad w \frac{\partial w}{\partial t} = \frac{1}{2} \frac{\partial}{\partial t}(w^2).$$

Alle Terme zusammenfassen mit $\frac{\partial}{\partial t}$

$$\dot{W} = \rho \frac{\partial}{\partial t} \left\{ \int_0^l \frac{\partial h}{\partial t} w A \, dx - \frac{1}{2} \int_0^l w^2 A \, dx \right\} + \quad \{ \equiv 0$$

$$+ \rho \mathcal{U} \left[ \frac{\partial h}{\partial t} w(x,t) A \right]_0^l = \rho \mathcal{U} \left. \frac{\partial h}{\partial t} \right|_l w(l,t)$$

Einfacher Zusammenhang

$$\dot{\overline{W}} = \varrho U A(\ell) \, \overline{\left.\frac{\partial h}{\partial t}\right|_{\ell} w(\ell,t)}$$