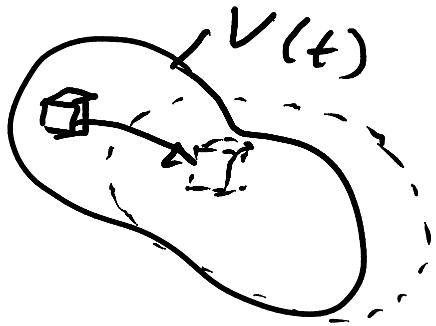
Diffusionsgesetz

TECHNISCHE UNIVERSITÄT DARMSTADT
FLUID SYSTEM TECHNIK
Biofluidmechanik

$$\frac{D}{Dt}\int_{V(t)} c\, dV = -\int_S \vec{j}\cdot\vec{n}\, dS + \int_V r\, dV$$



Erhaltungsgleichung in integraler Form (starke Formulierung).

Erhaltungsgleich in differentieller Form (schwache Formulierung, weak formulation)

Reynoldsches Transporttheorem

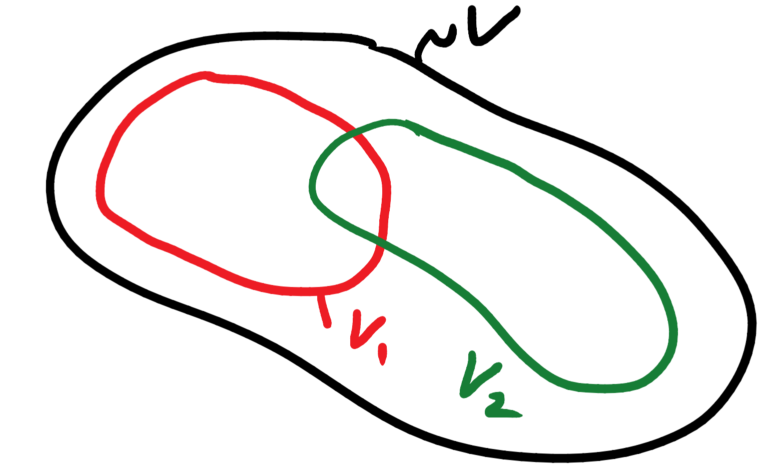
Gauß $\int_S (\ )\cdot\vec{n}\, dS = \int_V \nabla\cdot(\ )\, dV$

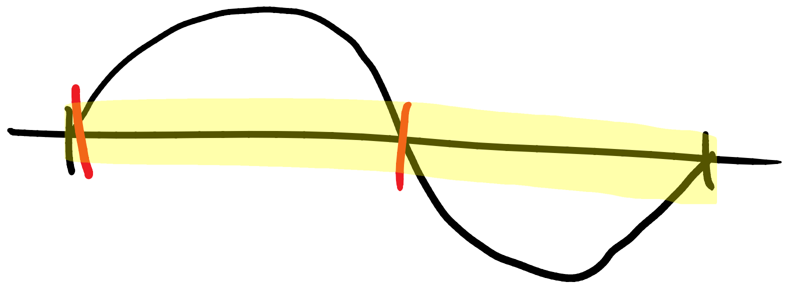
$$\int_V \left(\frac{Dc}{Dt} + c\,\underbrace{\frac{D\,dV}{Dt}\,\frac{1}{dV}}\right) dV = \int_V -\nabla\cdot\vec{j}\, dV + \int_V r\, dV$$

rel. Volumenänderungsrate eines mat. Teilchens = div $\vec{u}$

$$\int_V \underbrace{\left(\frac{Dc}{Dt} + c\,\nabla\cdot\vec{u} + \nabla\cdot\vec{j} - r\right)}_{\equiv 0.} dV \equiv 0$$

Erfahrung / Axiom





$$\underbrace{\frac{Dc}{Dt} + c\,\nabla\cdot\vec{u} = -\nabla\cdot\vec{j} + r}$$

Zweite Fick'sche Gleich./Gesetz } Axiom

$$\vec{j} = -D\,\nabla c$$

Erste Fick'sche Gesetz } Materialgesetz

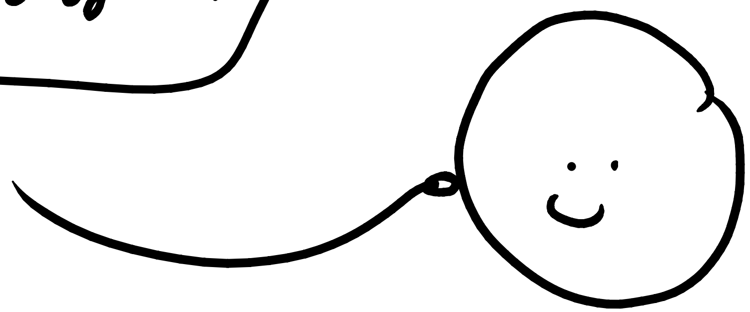
D Diffusionskoeffizient $[D] = \frac{L^2}{T}$

$$\frac{Dc}{Dt} + c\,\nabla\cdot\vec{u} = \nabla\cdot D\nabla c + r$$

<u>Spezialfall</u>

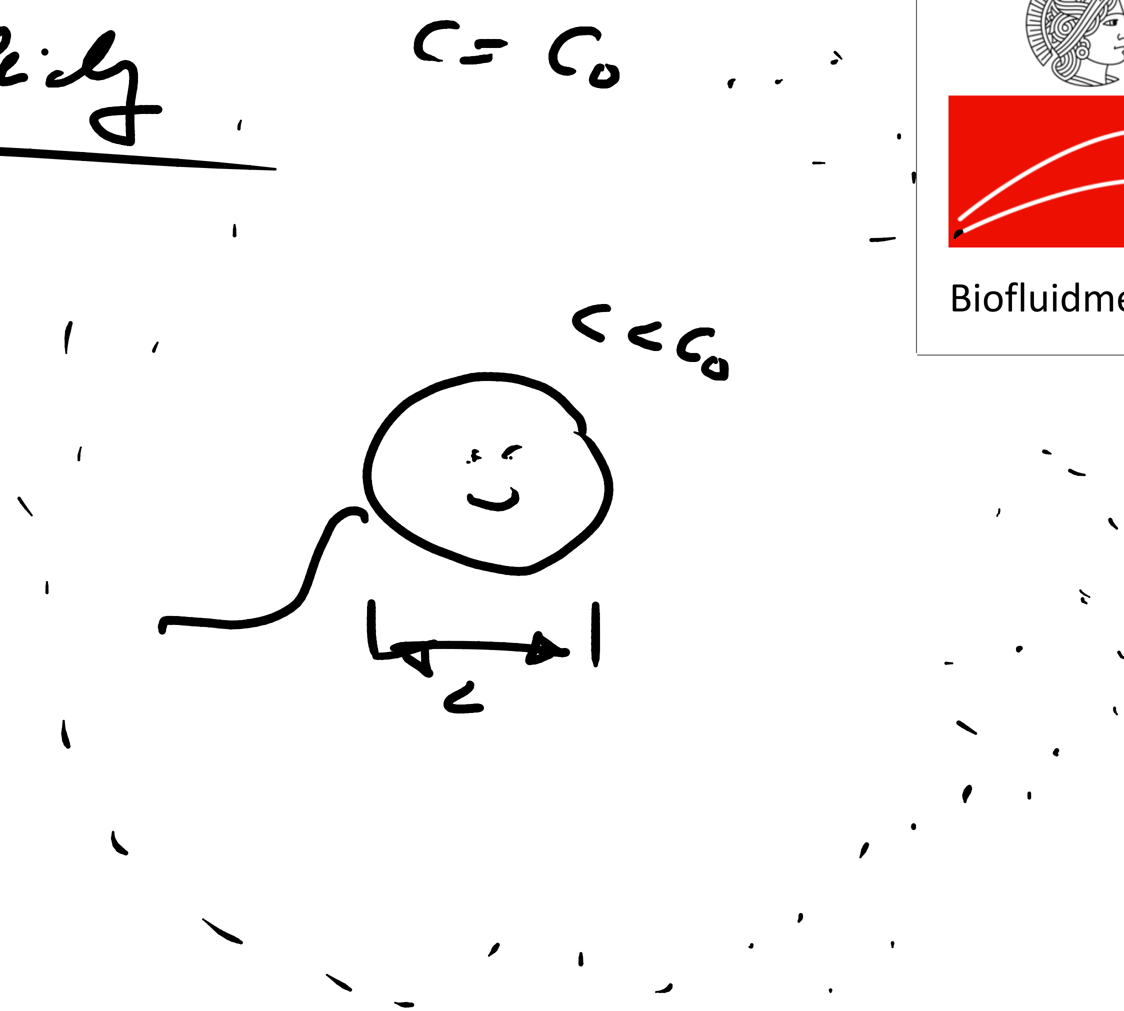
- keine Reaktion, d.h. $r \equiv 0$
- Dichtebeständige Strömung, d.h. $\nabla\cdot\vec{u} = \frac{1}{dV}\frac{D\,dV}{Dt} \equiv 0$

$$\boxed{\frac{Dc}{Dt} = D\,\Delta c \quad \text{Diffusionsgleichung}}$$



- homogener Diffusionskoeffiz. $D \equiv \text{const}$

Analyse der Diffusionsgleichung



$$c = c_+ c_0$$

$$t = t_+ \frac{L}{u}$$

$$\Delta = \Delta_+ \frac{1}{L^2}$$

$$\frac{u c_0}{L} \frac{Dc_+}{Dt_+} = \frac{D c_0}{L^2} \Delta_+ c_+ \quad \Big| \frac{L^2}{D c_0}$$

$$\overbrace{\frac{UL}{\mathcal{D}}\,\frac{Dc_+}{Dt_+}}^{\text{Konvektiver Transport}} = \overbrace{\Delta_+ c_+}^{\text{Diffusiver Transport}}$$

Pécletzahl für die Diffusion.

$$Pe_{\mathcal{D}} := \frac{UL}{\mathcal{D}} = \frac{UL}{\nu}\,\frac{\nu}{\mathcal{D}} = Re \times Sc \qquad \frac{\nu}{\mathcal{D}} = \text{Schmidtzahl} \sim 10^3 \text{ für wässrige Lsg.}$$

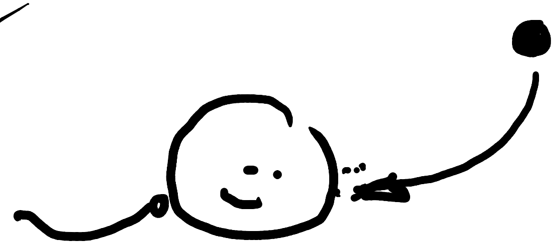
Anm.: $Pe_a := \frac{UL}{a}$; $a = \frac{\lambda}{\rho c}$ ‖ $Pe_a = \frac{UL}{a} = \frac{UL}{\nu}\,\frac{\nu}{a} = Re\,Pr$

$$Pe_a \frac{D\Theta}{Dt_+} = \Delta_+ \Theta \qquad \Theta = \frac{T - T_0}{T_w - T_0}$$

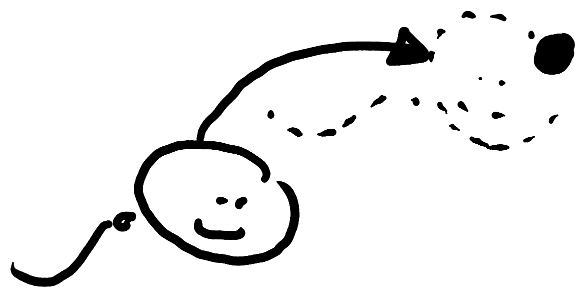
Pecletzahl $Pe_D = \frac{uL}{D} = \frac{\text{Diffusionszeit}}{\text{Konvektionszeit}}$

Diffusionszeit $t_D = L^2/D$

$t_D < t_K$



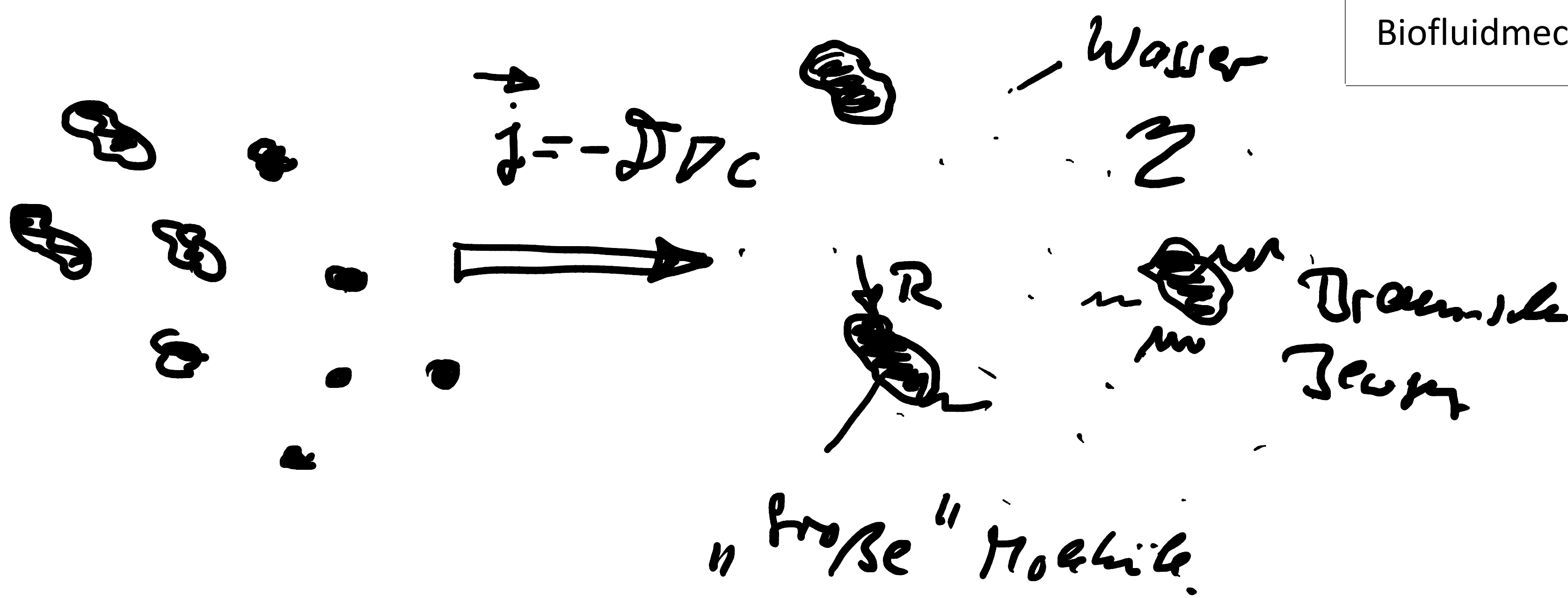
Konvektionszeit $t_K = \frac{L}{u}$



$$Pe = Re * Sc = \frac{10\,\cancel{10^{-6}} * 30*10^{-6}}{\cancel{10^{-6}}} * \frac{\nu}{D}$$

$$= 3*10^{-4}\,\frac{\nu}{D} \ll 1$$

Diffusionskoeffizient in einer wässrigen Lösung



$$D = D(\eta, T k, R, \text{Form})$$

D: $\frac{L^2}{T}$

η: $L^{-1} M T^{-1}$

$T k$: $FL = ML^2T^{-2}$

R: L

Form: dimensionslos

$Re <<<<< 1$

	$D\eta$	kT	R	$\cancel{\eta}$	Gestalt
L	0	1	1	-2	0
F	1	1		1	0
~~T~~	~~1~~ 0			~~1~~	~~0~~

Jede physikalische Zusammenhang muß invariant gegen Änderung des Basiseinheitensystems sein!

Bridgman Postulat

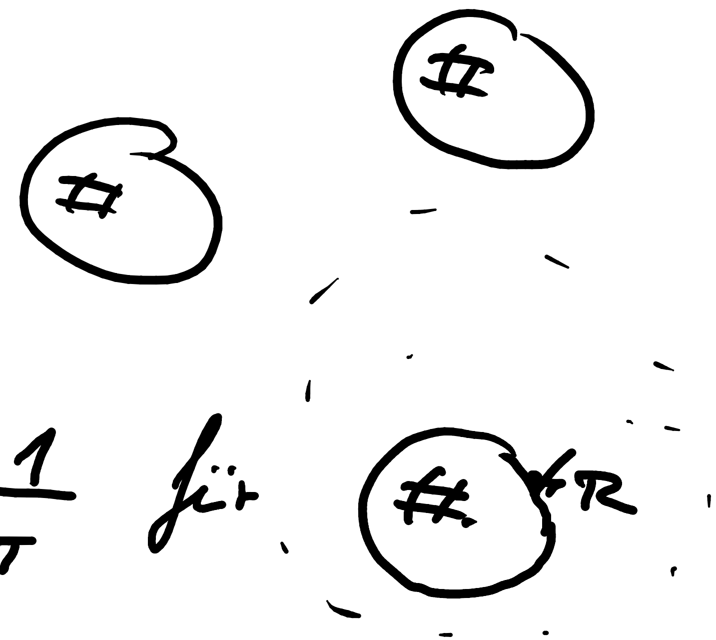
$$[D] = \frac{L^2}{T} \qquad [kT] = FL \qquad [\eta] = \frac{F}{L^2}T$$

$$D\eta = fn(kT,\ R,\ \cancel{\eta})$$

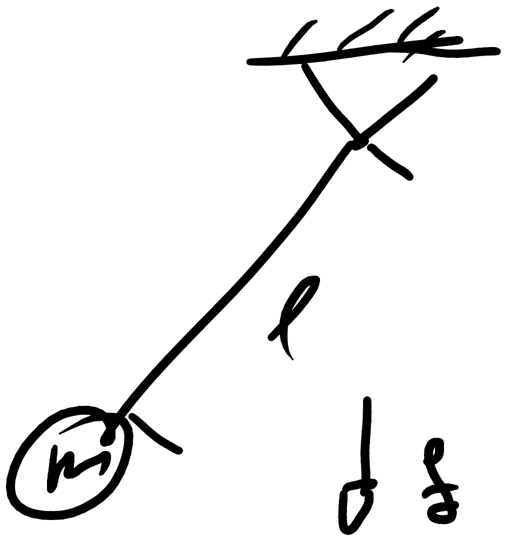
$1\,\mathrm{mPa\,sec} = 10^3\,\mathrm{Pa\,\mu sec}$

	$D\eta/kT$	$\cancel{kT}$	R
L	$\cancel{0}\ -1$	$\cancel{1}$	1
$\cancel{F}$	$\cancel{0}$	$\cancel{1}$	$\cancel{0}$

$$\frac{D\eta R}{kT} = \text{const (Gestalt)} = \frac{1}{6\pi} \quad \text{für}$$



$$\Longleftrightarrow \quad D = \frac{1}{6\pi}\,\frac{kT}{\eta R} \quad \text{Einsteinsche Gleichung.}$$



$$10\,kg = 10^4\,g$$

$$\tau = \tau(l, \cancel{m}, g)$$

τ = 1 sec, l = 1 m, g = $9.81 \frac{m}{sec^2}$

	τ	l	$\cancel{m}$	g
L		1		1
~~M~~			$\cancel{1}$	
T	1			-2

$$Sc = \frac{\nu}{D} = \frac{\eta}{\rho D} = \frac{\eta^2}{\rho}\,\frac{R\,6\pi}{kT}$$

$$= \frac{\text{viskose Kraft}}{\text{Diffusionskraft}} = \frac{\eta^2/\rho}{kT/R}\,6\pi \sim 10^3.$$