TECHNISCHE UNIVERSITÄT DARMSTADT
FLUID SYSTEM TECHNIK
Technische Fluidsysteme

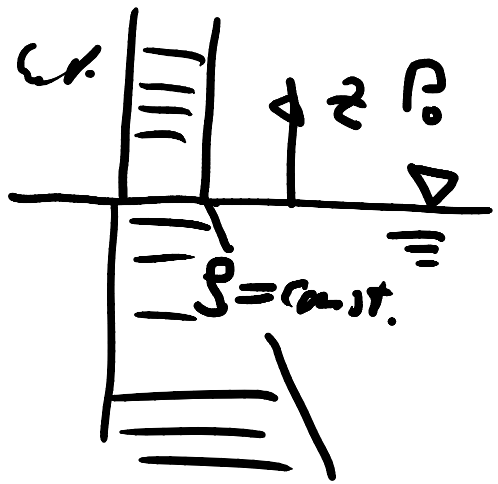
Weitere Spezialisierung

Hydrostatik $\underset{\sim}{P} \equiv 0$, da keine Relativbewegung zwischen Flüssigkeitsteilchen.

$$\varrho \frac{D\vec{u}}{Dt} \equiv 0$$

$$\Leftrightarrow \quad \underline{\underline{\nabla p = \vec{f}}} \qquad \text{Hydrostatische Grundgleichung}$$

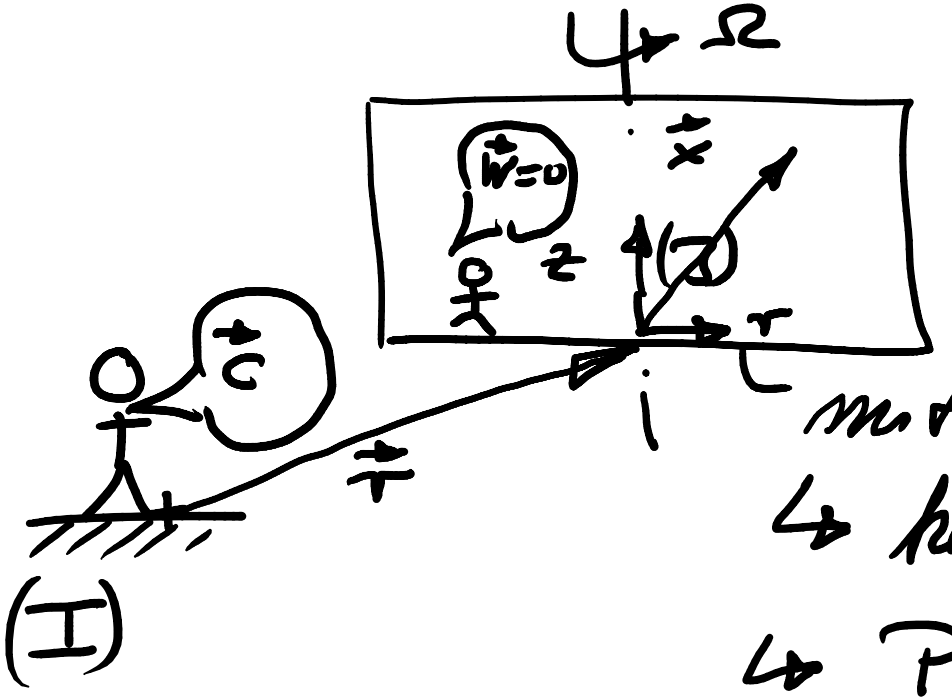
z.B. dichtebeständige, schwere Flüssigkeit



$$\vec{g} = -g\vec{e}_z$$

$$\frac{dp}{dz}\cancel{\vec{e}_z} = -g\varrho\cancel{\vec{e}_z}$$

$$p(z) = -\varrho g z + p_0$$



$$\vec c = \vec w + \vec u + \vec v$$

$$\vec v = \left[\frac{D\vec r}{Dt}\right]_I \; ; \; \vec w = \left[\frac{D\vec x}{Dt}\right]_B$$

mitbewegtes System

↳ keine Relativgeschw.

↳ $\underset{\sim}{P} \equiv 0$

↳ $\rho \frac{D\vec w}{Dt} \equiv 0$

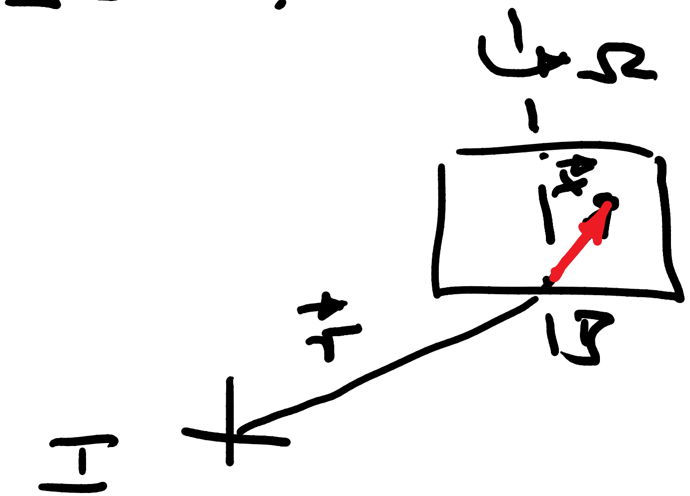
$\vec w$ Relativgeschw.

$\vec u$ Umfangsgeschw. $\vec u = \vec \Omega \times \vec x$.

$\vec v$ Führungsgeschw.

$\vec c$ Absolutgeschw.

$$\vec{c} = \left[\frac{D\vec{x}}{Dt}\right]_B + \cancel{\left[\frac{D\vec{r}}{Dt}\right]_I} + \vec{\Omega} \times \vec{x}$$



B : beschleunigtes System

I : Inertialsystem

$$\vec{c} = \left[\frac{D}{Dt}(\vec{x} + \vec{r})\right]_I = \left[\frac{D\vec{x}}{Dt}\right]_I + \cancel{\left[\frac{D\vec{r}}{Dt}\right]_I}$$

$$\boxed{\left[\frac{D\vec{x}}{Dt}\right]_I = \left[\frac{D\vec{x}}{Dt}\right]_B + \underline{\underline{\vec{\Omega} \times \vec{x}}}}$$

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ganz allgemein für einen beliebigen Vektor $\vec{b}$ z.B. $\vec{I} = \int \rho \vec{c}\, dV$

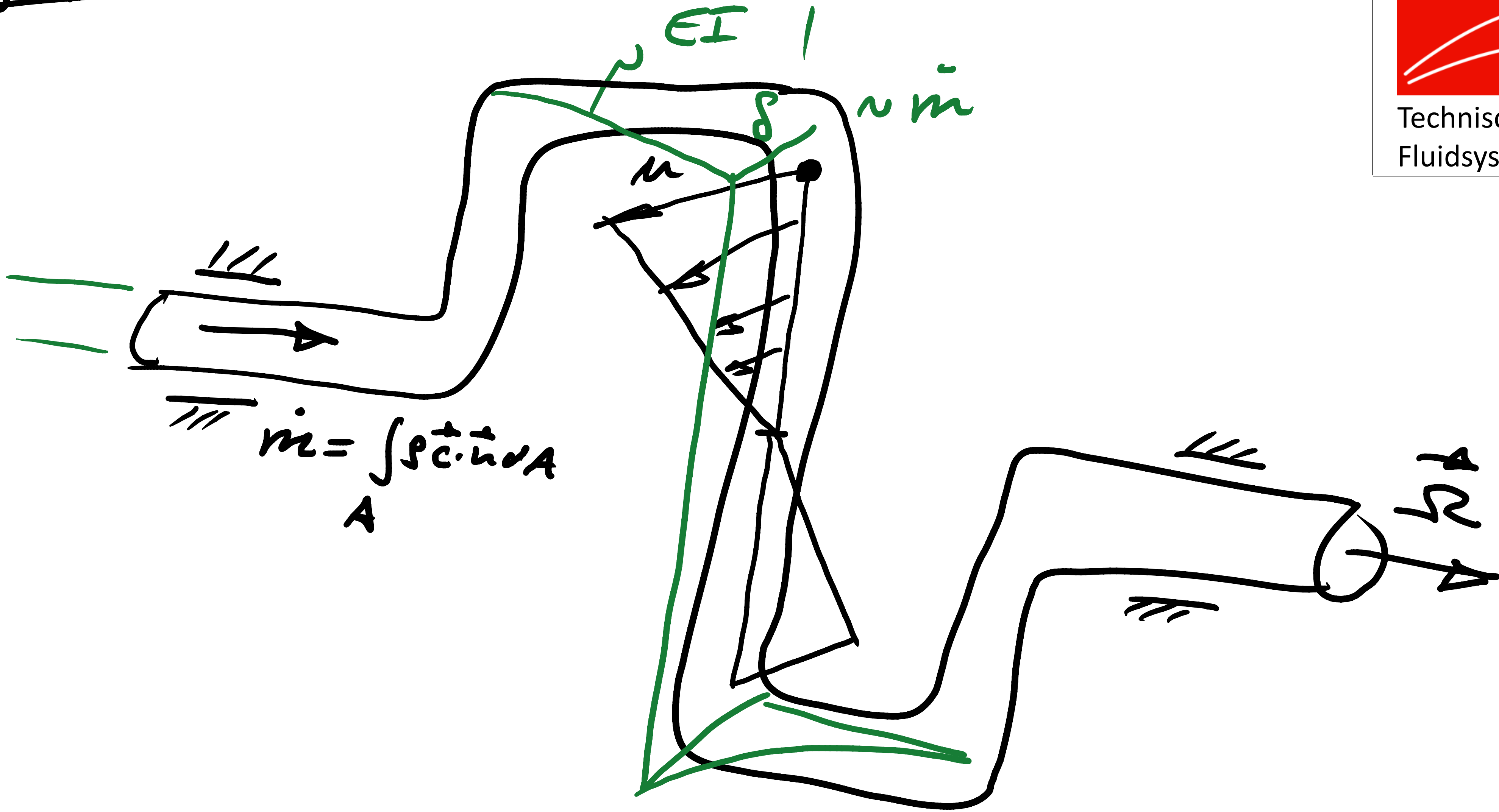
$$\vec{D} = \int \rho \vec{x} \times \vec{c}\, dV$$

$\vdots$

$$\underbrace{\left[\frac{D\vec{b}}{Dt}\right]_I = \left[\frac{D\vec{b}}{Dt}\right]_B + \vec{\Omega} \times \vec{x}}$$

Prof. Dr.-Ing. Peter Pelz

Beispiel Coriolisdurchflussmesser



$$\dot m = \int_A \rho \vec c \cdot \vec n \, dA$$

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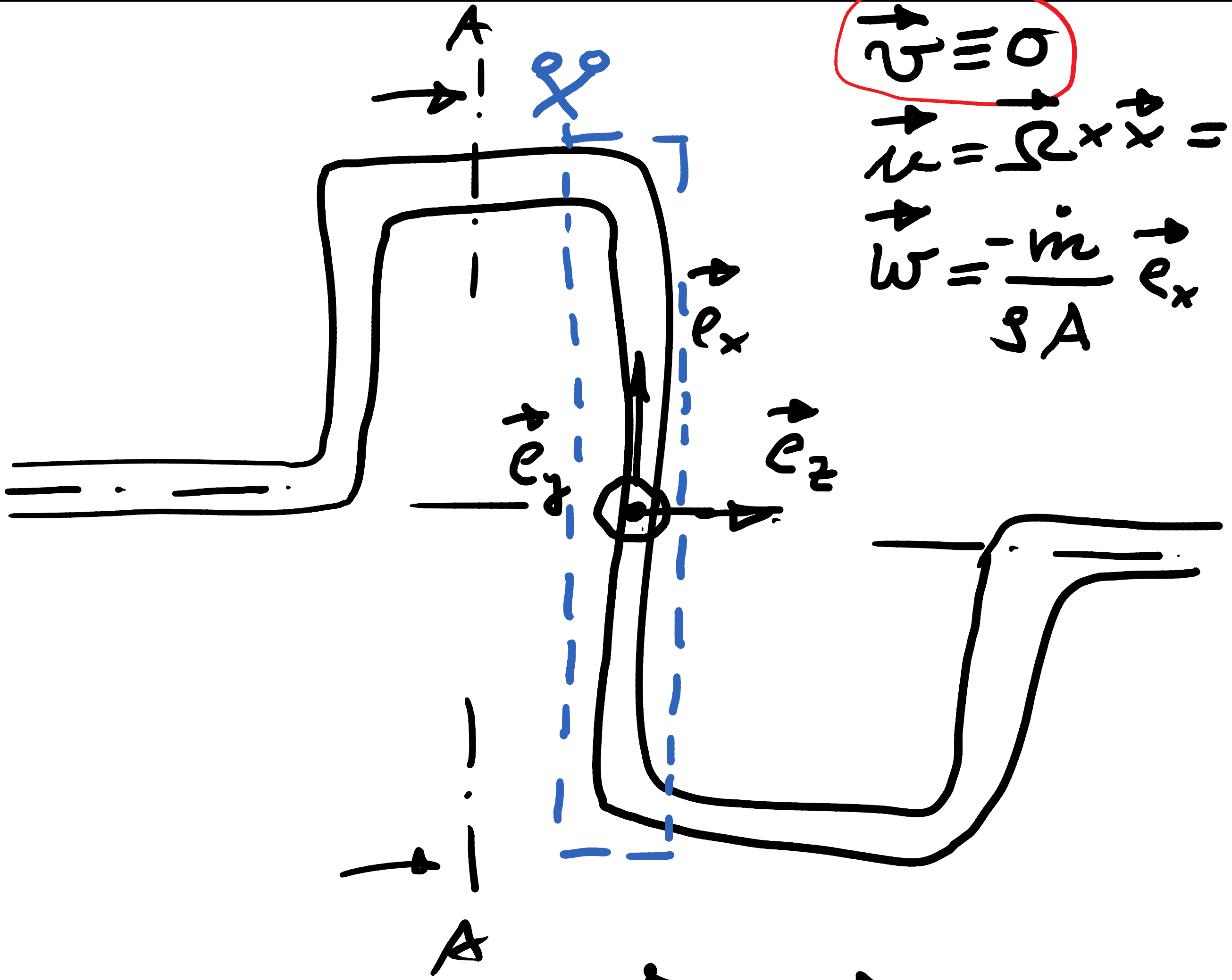
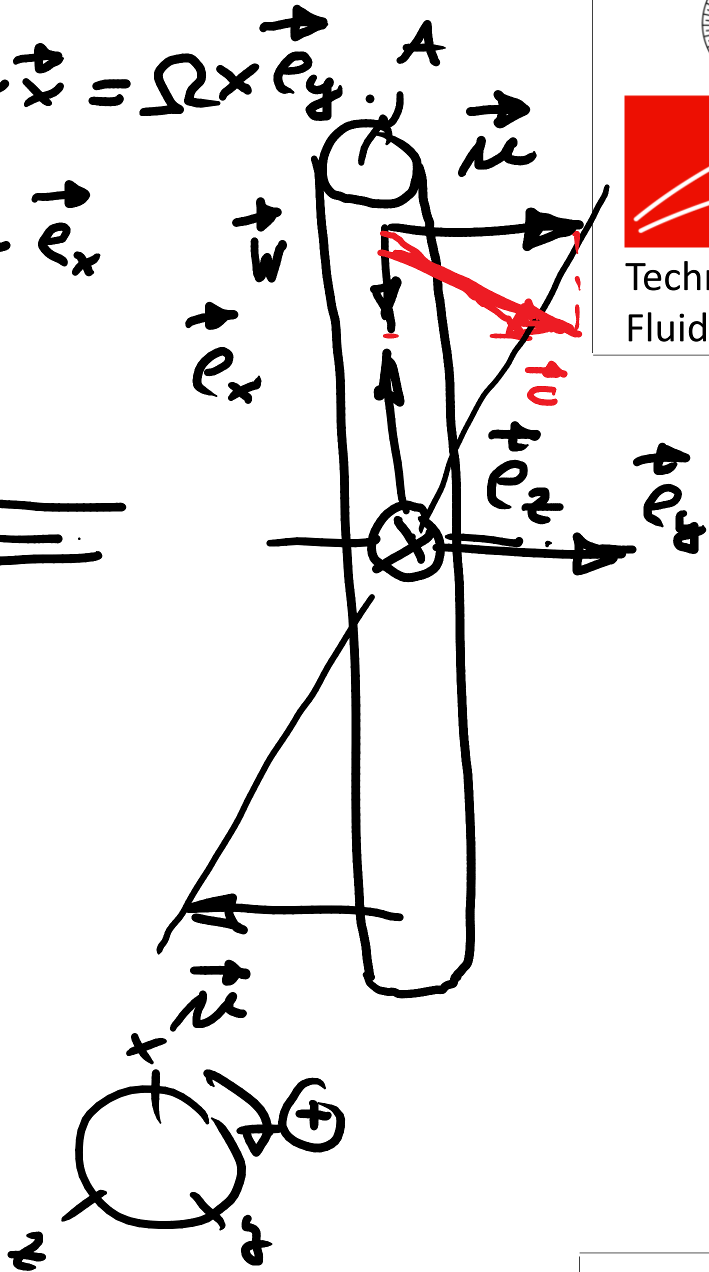
$\vec{v} \equiv 0$

$\vec{u} = \vec{\Omega} \times \vec{x} = \Omega x \vec{e}_y$

$\vec{w} = \frac{-\dot{m}}{\varrho A} \vec{e}_x$





$\vec{\Omega} = \Omega \vec{e}_z$

$\vec{x} = x \vec{e}_x$

$\vec{\Omega} \times \vec{x} = \Omega x \vec{e}_y$.

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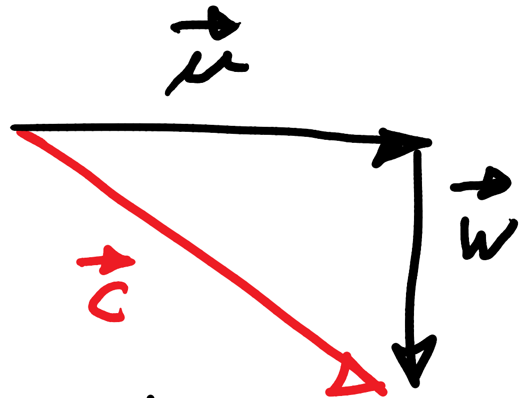
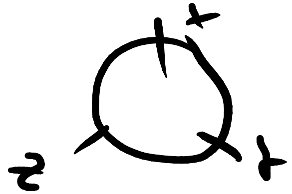
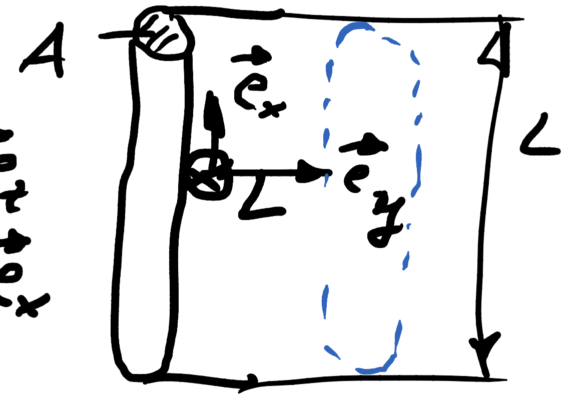
$$\left[\frac{D\vec{I}}{Dt}\right]_I = \vec{F}$$



$$\vec{I} = \int_V \rho\,\vec{c}\,dV = \int_V \rho(\vec{u}+\vec{w})\,dV$$

Geschwindigkeitsdreieck.





$\vec{e}_z$

$$\left[\frac{D\vec{I}}{Dt}\right]_I = \underbrace{\left[\frac{D\vec{I}}{Dt}\right]_{II}}_{\equiv 0} + \underbrace{\vec{\Omega}\times\vec{I}}_{\text{Coriolis}} = \vec{F} = \Omega\,\dot{m}\,L\,\vec{e}_z\times\vec{e}_x = -\Omega\,\dot{m}\,L\,\vec{e}_y \qquad \dot{m} = w\rho A$$

$$= \underbrace{\vec{\Omega}\times\int_V \rho(\vec{\Omega}\times\vec{x})\,dV}_{\equiv 0} + \underbrace{\vec{\Omega}\times\int_V \rho\,\vec{w}\,dV}_{\text{Coriolis}} = \vec{\Omega}\times\rho\,\vec{w}\,A\,L$$

$$\varrho\left[\frac{D\vec{c}}{Dt}\right]_I = \nabla\cdot\underset{\sim}{T} + \vec{f}$$

$$\varrho\left[\frac{D\vec{c}}{Dt}\right]_B + \varrho\,\vec{\Omega}\times\vec{c} = \nabla\cdot\underset{\sim}{T} + \vec{f}$$

$$\vec{c} = \vec{w} + \vec{v} + \vec{\Omega}\times\vec{x}$$

Scheinkräfte

$$\varrho\left[\frac{D\vec{w}}{Dt}\right]_B = \nabla\cdot\underset{\sim}{T} + \vec{f} \underbrace{- \varrho\left(\vec{a} + 2\,\vec{\Omega}\times\vec{w} + \left(\vec{\Omega}\times(\vec{\Omega}\times\vec{x})\right) + \varrho\,\dot{\vec{\Omega}}\times\vec{x}\right)}_{\text{Scheinkräfte}}$$

$$\vec{a} = \dot{\vec{v}}$$

$\nabla p = \vec{f}$. Hydrostatische Grundgleich.

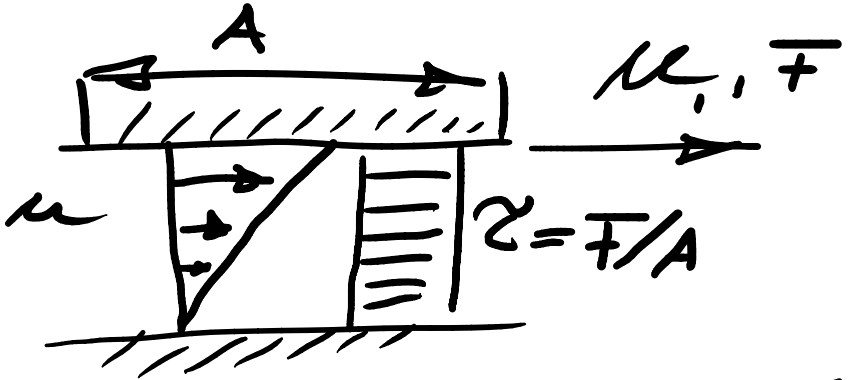
$\vec{f} = -\nabla \psi^*$ ψ^* ist das Potential der Volumenkraft.

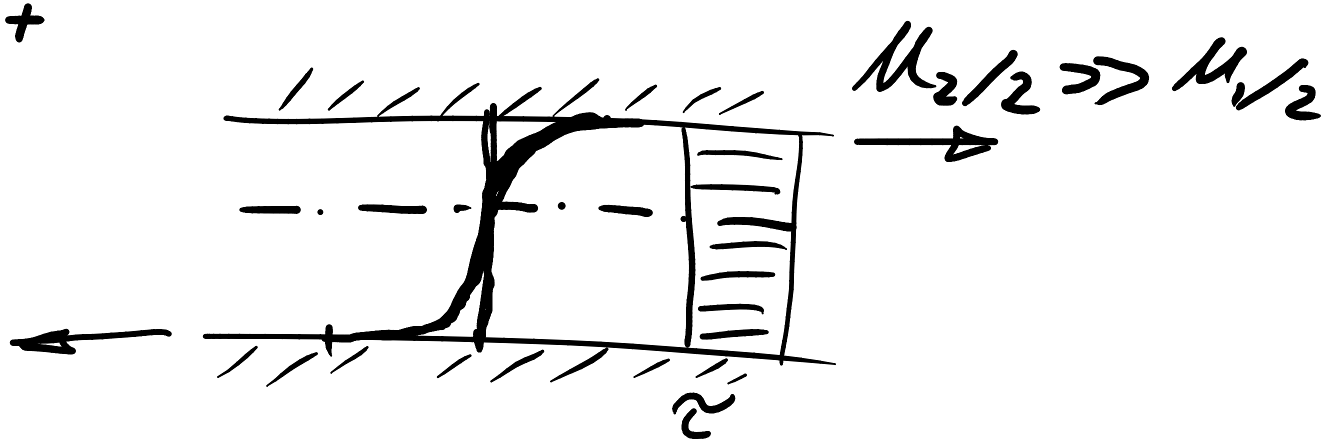
z.B.
$\psi^* = \rho g z + const.$ Potential der Schwere

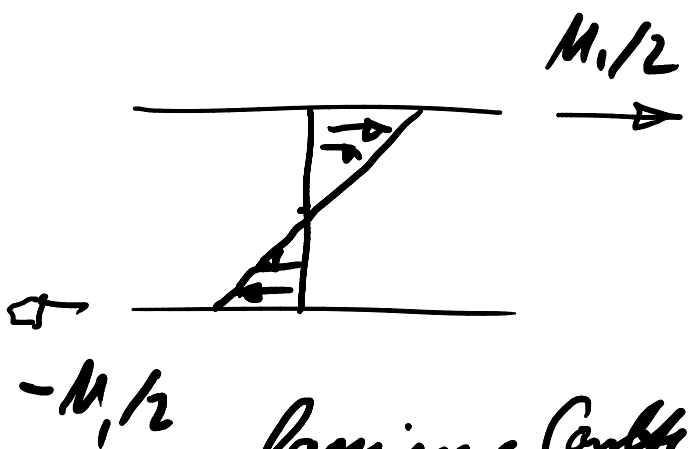
$\psi^* = \frac{1}{2}\rho \Omega^2 r^2 + const$ Potential der Zentrifugalkraft

$\Rightarrow \underline{p + \psi^* = const}$

Geschwindigkeitsverteilung im Spalten infolge geschleppter Wand und infolge Druckgradient.



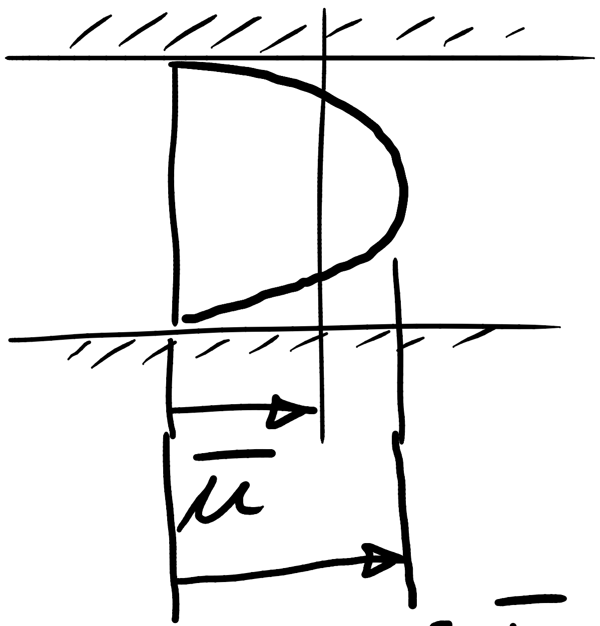




laminare Couette-Strömung

turbulente Couette-Strömung

2D



$u_{max} = \frac{3}{2}\bar{u}$

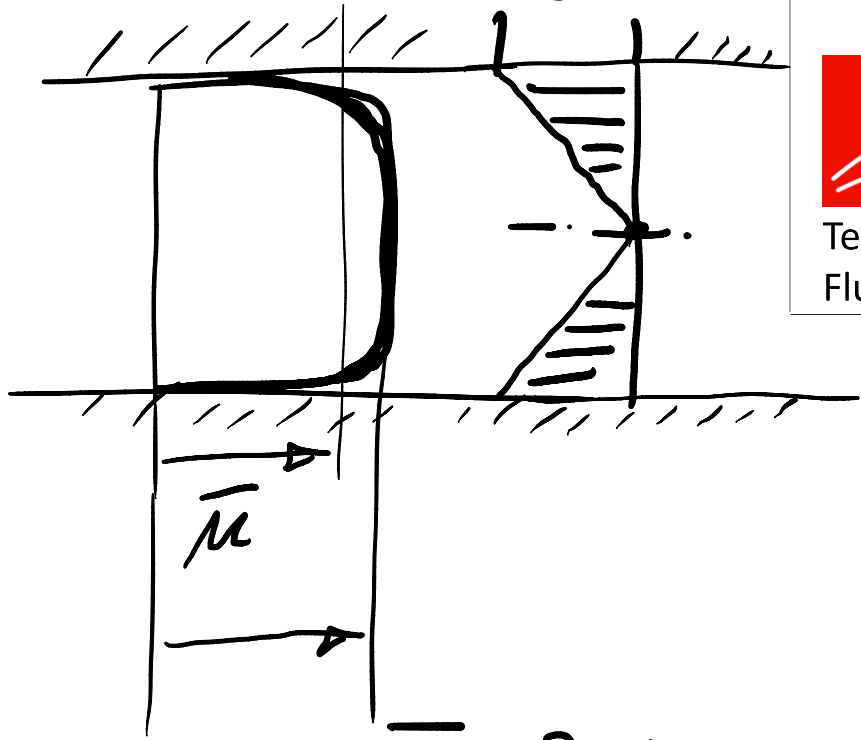
Anm. im Rohr ist

$u_{max} = 2\bar{u}$

Poiseuille-Strömung

od. Druckstrom.

τ_w



$u_{max} = \bar{u} + \frac{3}{2}\frac{1}{\varkappa}u_*$

$\varkappa = 0.4$ Kármánsche Konstante

$u_* = \sqrt{\tau_w/\rho}$ Schubspannungsgeschw.