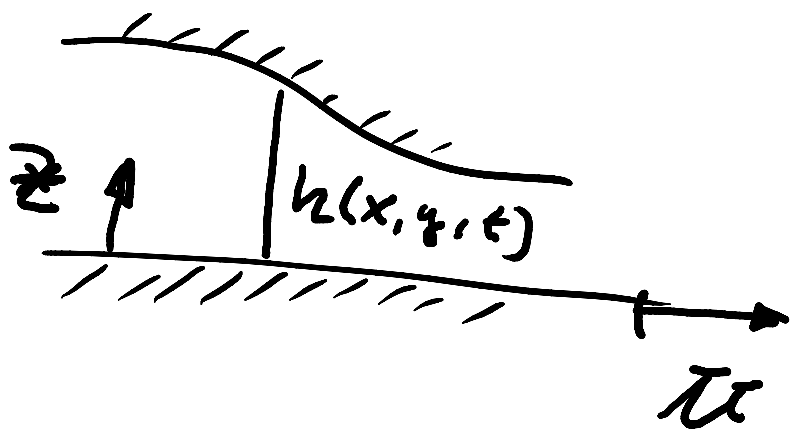
Sommerfeldzahl und Steifigkeit bei Zapfenlager

Reynoldsche Gleichung der hydrodynamischen Schmierung.

$$\vec{\dot V} = \dot V_x \vec e_x + \dot V_y \vec e_y$$



Konti:

$$\nabla \cdot \vec{\dot V} + \frac{\partial h}{\partial t} = 0$$

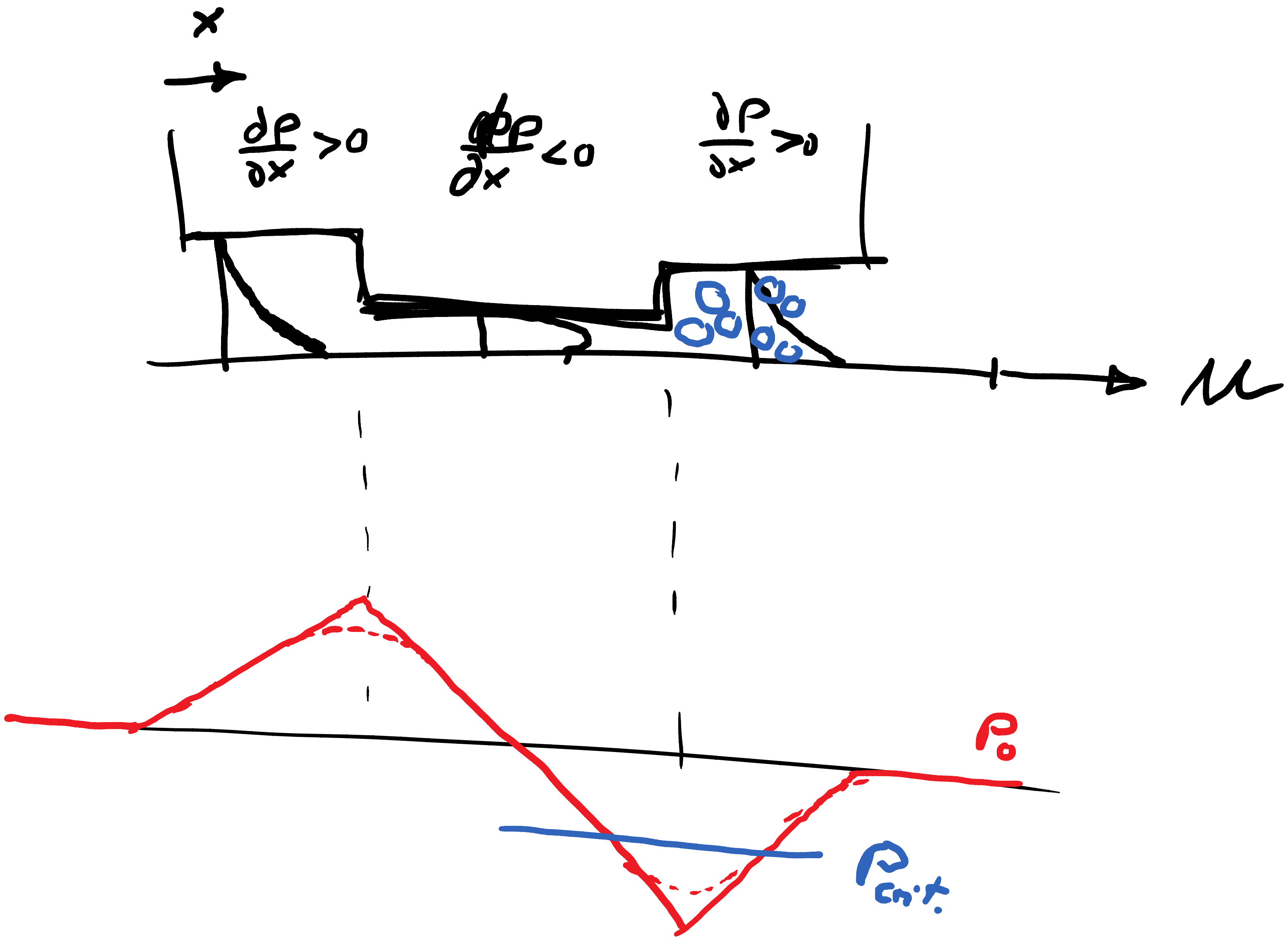
$$\Longleftrightarrow \quad \frac{\partial}{\partial x}\left(\frac{h^3}{\eta}\frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{h^3}{\eta}\frac{\partial p}{\partial y}\right) = 6\left[\frac{\partial(Uh)}{\partial x} + \frac{\partial(Vh)}{\partial y} + 2\frac{\partial h}{\partial t}\right]$$

Reynoldsche Gleichung = Poissonsche DGL. :)
lineare

$\Delta p = k$ Poissongl.

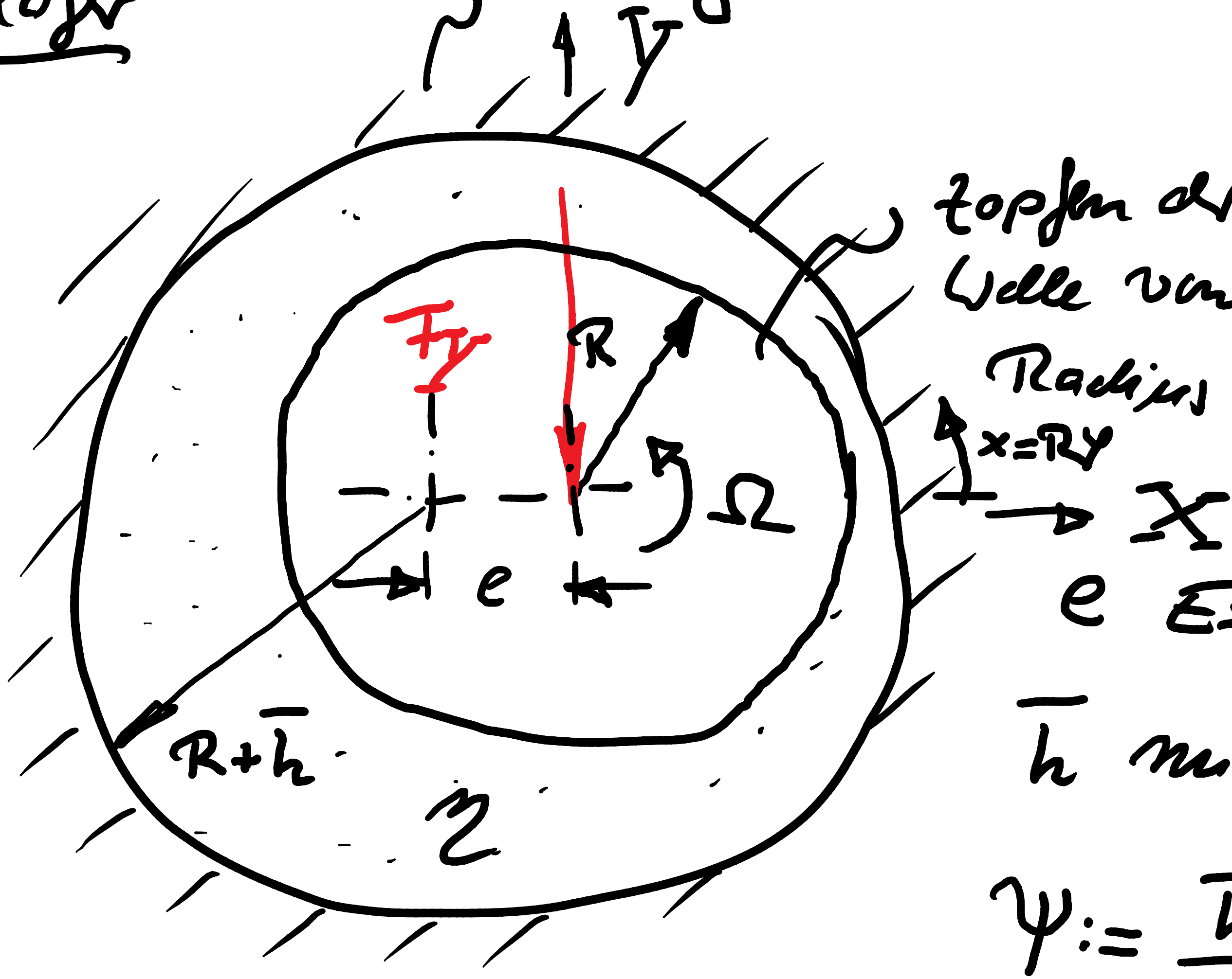
- Torsion
- Seifenhaut.
- Membran
- ⋮
- Elektrostatik

$\Delta p = 0$ Laplacegl.


x
$\frac{\partial p}{\partial x} > 0$
$\frac{\partial p}{\partial x} < 0$
$\frac{\partial p}{\partial x} > 0$
u
p_0
$p_{crit.}$

Zapfenlager

Bohrung Radius $R + \bar{h}$



Zapfen der Welle vom Radius R

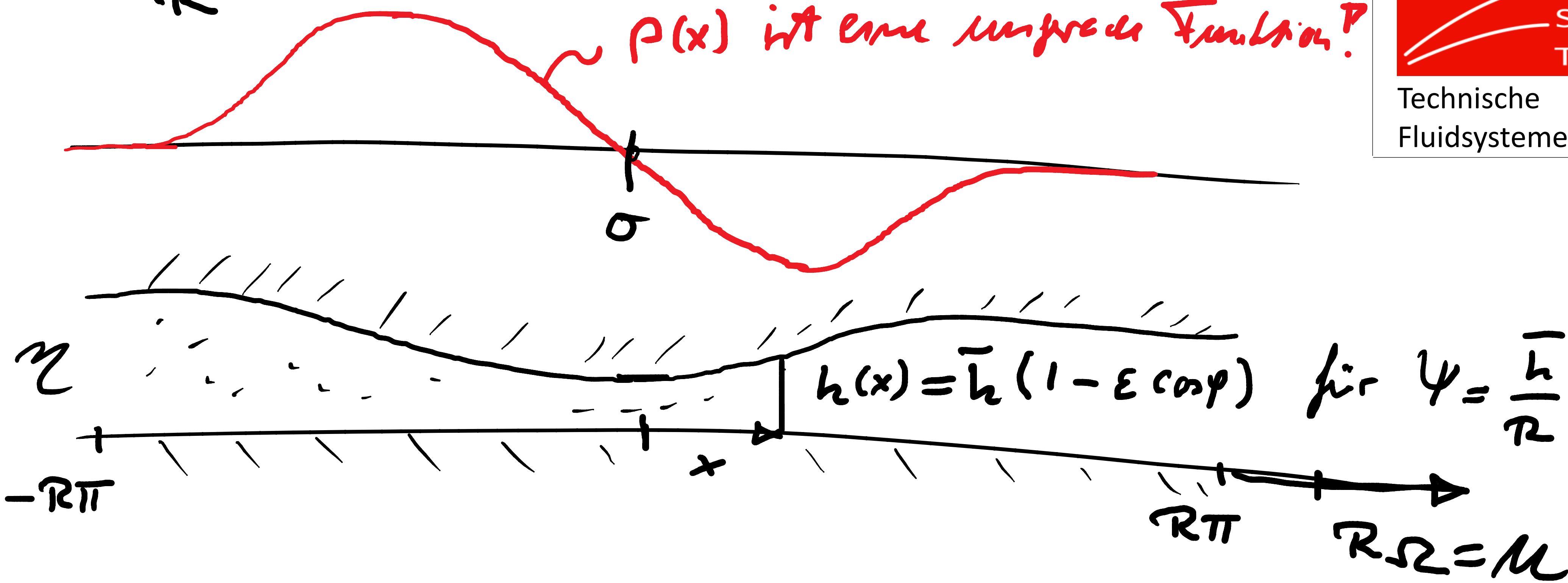
e Exzentrizität

$\bar{h}$ mittleres Lagerspiel

$$\psi := \frac{\bar{h}}{R}$$ rel. Lagerspiel

$$\psi \sim 10^{-3}$$

$\varepsilon := \frac{e}{R}$ relative Exzentrizität.

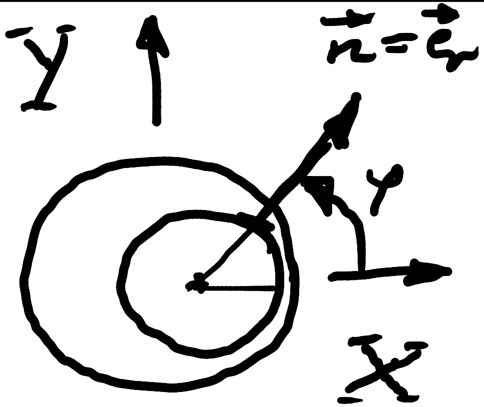


$h(x) = \bar{h}(1 - \varepsilon \cos\varphi)$ für $\psi = \frac{\bar{h}}{R} \ll 1$.

ε ist ein Betriebsparameter.

ψ ist ein konstruktiv vorgegebene Größe.

Kraft pro Tiefeneinh.



$$\vec F = F_x \vec e_x + F_y \vec e_y$$

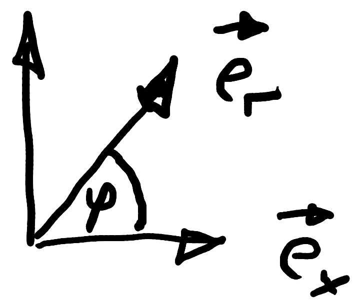
$$= \int_S d\vec F = \int_S \underbrace{\vec t\, dS}_{d\vec F} = \oint_0^{2\pi} \vec n \cdot \underset{\sim}{T}\; \underbrace{R\, d\varphi}_{dS}$$

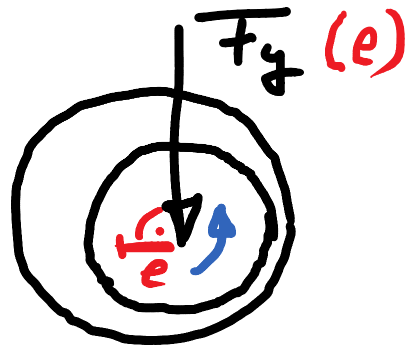
$$= \int_0^{2\pi} \vec e_r \cdot \Big(-p\, \vec e_r \vec e_r + \underbrace{2\eta \frac{\partial u}{\partial r}}_{\sigma} \vec e_r \vec e_r + \eta \frac{\partial u}{\partial z} \vec e_r \vec e_z + \dots\Big) R\, d\varphi$$

$$= \int_0^{2\pi} \underline{\underline{-p\, \vec e_r}} + \underbrace{\eta \frac{\partial u}{\partial z}}_{\sigma} \vec e_z \; R\, d\varphi \quad \Bigg| \quad \begin{aligned} &\cdot \vec e_x \rightsquigarrow F_x \\ &\cdot \vec e_y \rightsquigarrow F_y \end{aligned}$$

$$F_x = \int_0^{2\pi} \underbrace{-p(\varphi)}_{\text{ungerade}} \underbrace{\vec{e}_r \cdot \vec{e}_x}_{\cos\varphi \text{ gerade}} R\, d\varphi = 0$$

ungerade.

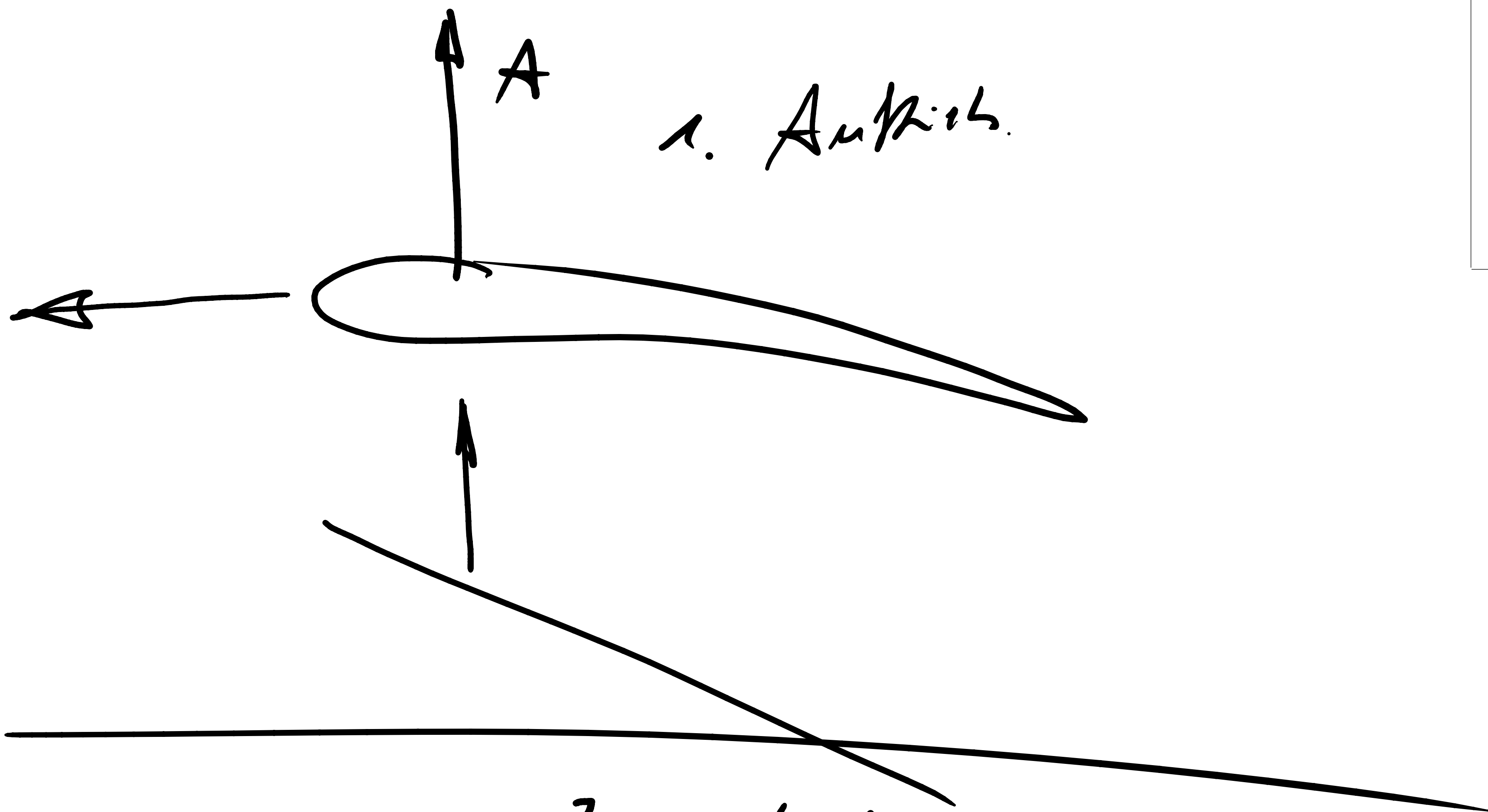




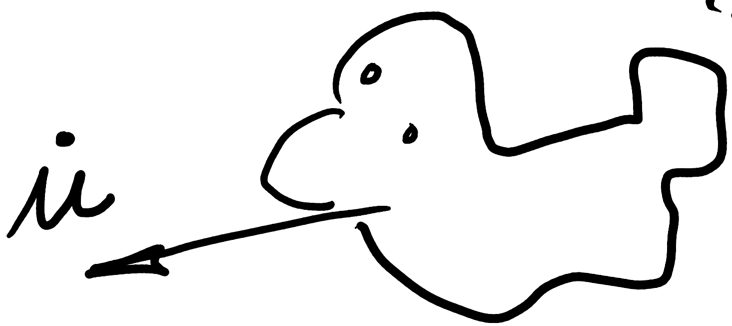
$$F_y = \int_0^{2\pi} -p(\varphi) \underbrace{\vec{e}_r \cdot \vec{e}_y}_{\sin\varphi} R\, d\varphi \neq 0 \rightarrow \text{haftend.}$$

gerade

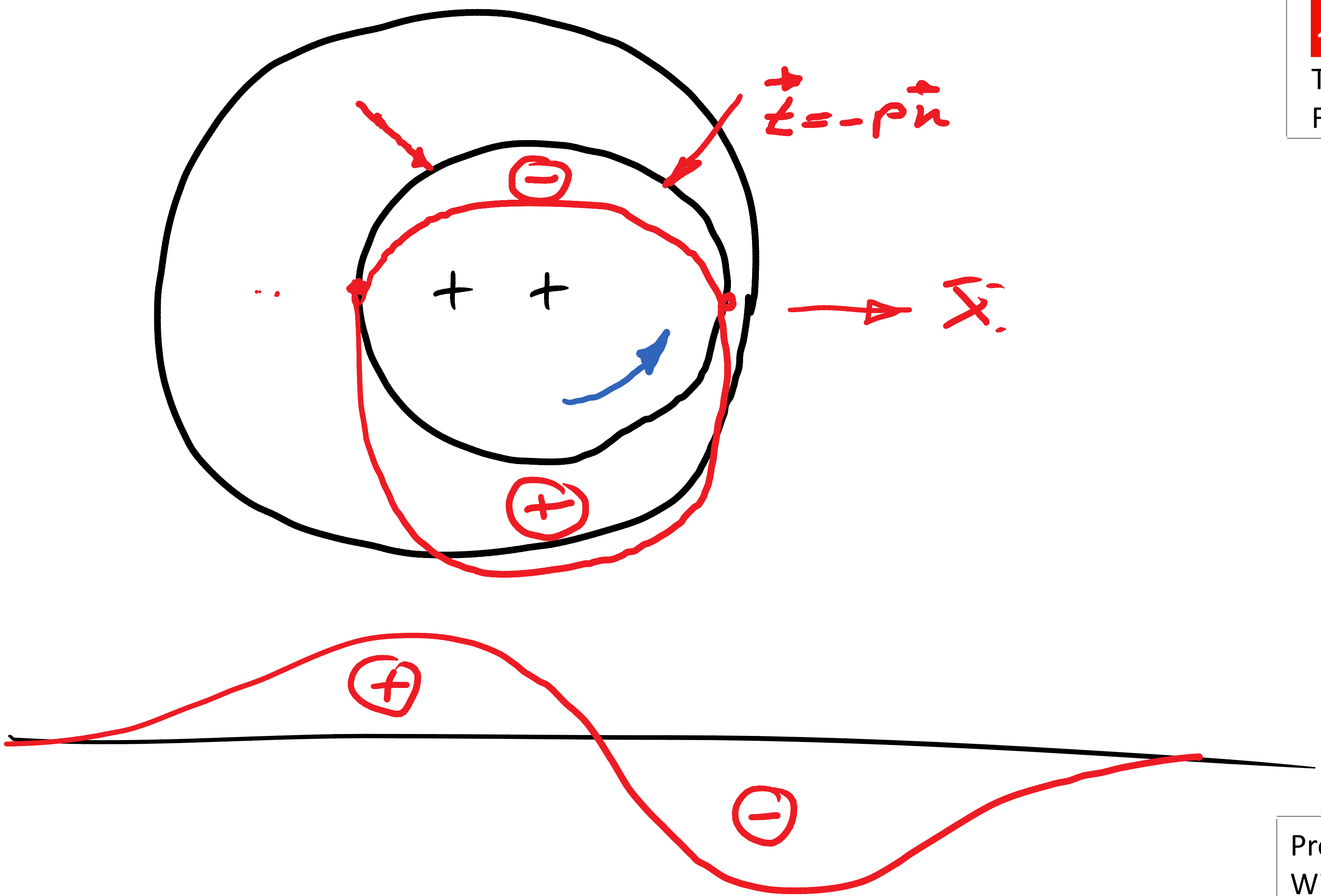
1. Auftrieb.

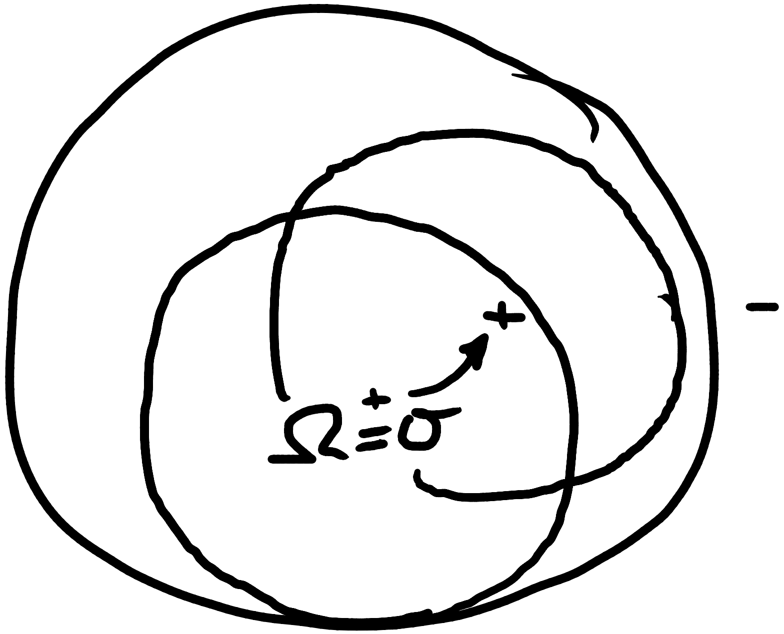


2. virtuelle Masse.



3. Gleitlager

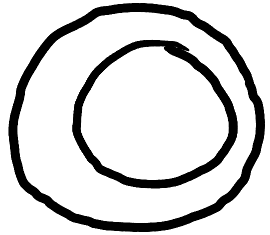



+
−
$\Omega \overset{!}{=} \sigma$

$$F_y = fn(\Omega, \eta, R, \bar{h}, e, B)$$

	F_y	$\overset{\sim p}{\Omega\eta}$	η	R	$\bar{h}$	e	B
F	1	1	1				
L		-2	-2	1	1	1	1
T		-1	1				

$B >> R$



B Lagerbreite.

unendlich lange Lag.

$B << R$

kurze Lag.

$$\frac{F_y}{\Omega\eta BR} = fn\left(\psi, \varepsilon, \frac{B}{R}\right)$$

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$$\frac{F_y}{BR\eta\Omega} = \mathrm{fn}\left(\psi, \varepsilon, \frac{B}{R}\right)$$

4 - Größ

Arnold Sommerfeld 1904

$$\underbrace{\frac{F_y}{BR\eta\Omega}\psi^2} = \mathrm{fn}\left(\varepsilon, \frac{B}{R}\right)$$

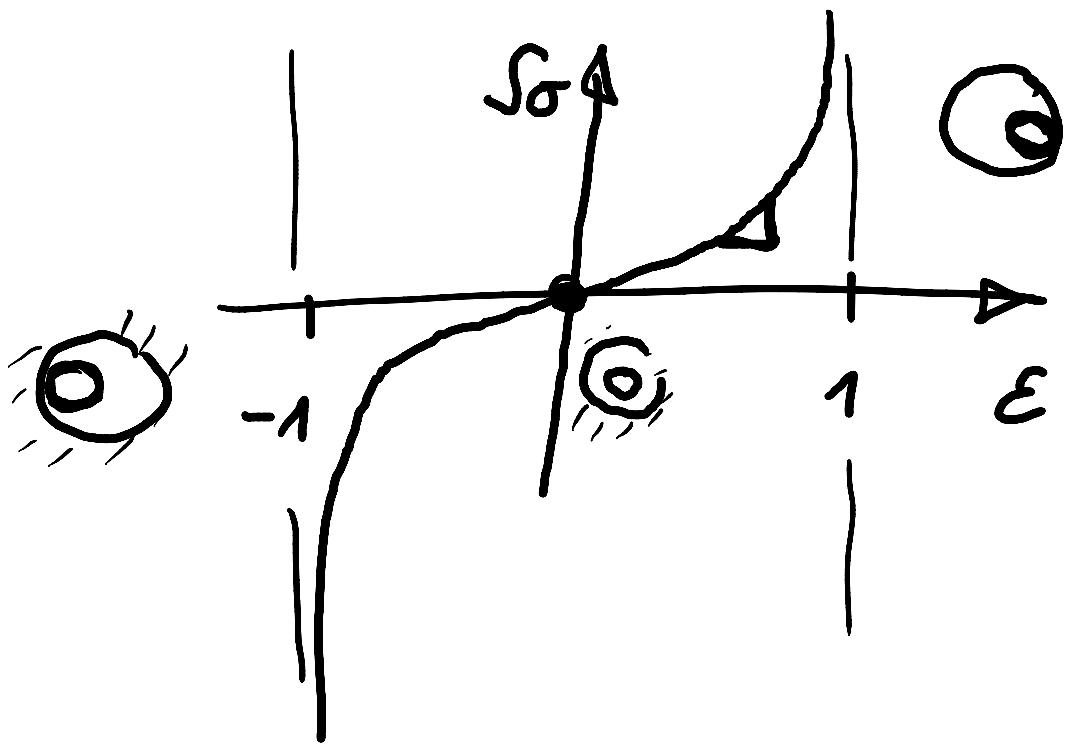
3 Größ?

$$So = So\left(\varepsilon, \frac{B}{R}\right)$$

$$\frac{B}{R} \to \infty : \quad So = So(\varepsilon)$$

unendlich lange Lg.

$$So = \frac{12\pi\varepsilon}{\sqrt{1-\varepsilon^2}\,(2+\varepsilon^2)}$$



$$k_+ = \frac{d\,So}{d\,\varepsilon}$$

Geradsteifigkeit

$$\varepsilon \to 0 : \quad k_+ \to 0$$

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$$\frac{B}{R} \to \sigma: \text{ kurze Lager}$$

$$\cancel{\text{Lg}} \quad So = \frac{\pi \varepsilon^2}{(1-\varepsilon^2)^{3/2}}$$

Lit: Spurk, Strömungslehre Kap. 5

Spurk, Dimensionsanalyse.

Sommerfeld Mechanik der deformierbaren Medien.

Vogelpohl.

Pinkus & Sternlicht.

$$\frac{F_y}{B R \eta \Omega} = f_4\left(\varphi, \varepsilon, \frac{B}{R}\right)$$

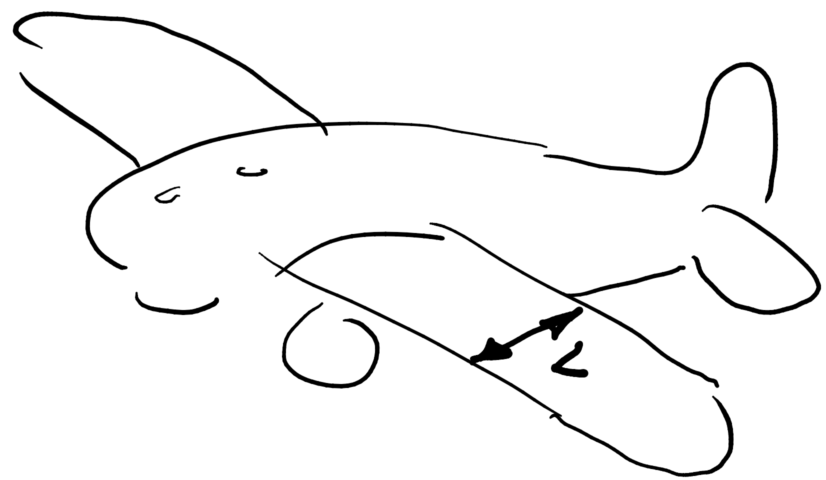
allgemeine Dimensionsgln.

Inspektionelle Dimensionsanalyse od. (Spurk)

Methode der Differentialgleichung (Zierep)

$$\frac{\partial}{\partial x}\left(\frac{h^3}{\eta}\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{h^3}{\eta}\frac{\partial p}{\partial y}\right)=6\,\frac{\partial(Uh)}{\partial x}$$

$$y = y_+ B$$
$$x = x_+ R$$
$$h = h_+ \bar{h}$$
$$p = p_+ \bar{p}$$
$$U = \Omega R$$



$(\ )_+$ dimensionslose Grö...

$$\cdot\ \frac{R}{\Omega R}$$

$$\frac{\bar{h}^3\,\bar{p}}{\eta R^2}\,\frac{\partial}{\partial x_+}\left(h_+^3\,\frac{\partial p_+}{\partial x_+}\right)+\frac{\bar{h}^3\,\bar{p}}{\eta B^2}\,\frac{\partial}{\partial y_+}\left(h_+^3\,\frac{\partial p_+}{\partial y_+}\right) = 6\,\frac{\Omega R}{R}\,\frac{\partial h_+}{\partial x_+}$$

$$\left(\frac{\bar h}{R}\right)^2 \frac{\bar p}{\Omega\eta}\, \underbrace{\frac{\partial}{\partial x_+}\left(h_+^3\, \frac{\partial p_+}{\partial x_+}\right)}_{O(1)} +$$

$$\underbrace{\left(\frac{\bar h}{R}\right)^2 \frac{\bar p}{\Omega\eta}}_{}\, \underbrace{\left(\frac{R}{B}\right)^2}_{} \frac{\partial}{\partial y_+}\left(h_+^3\, \frac{\partial p_+}{\partial y_+}\right) = \underbrace{6\, \frac{\partial h_+}{\partial x_+}}_{O(1)}$$

So $\qquad \underbrace{\text{Quadrat von } \frac{R}{B}} = \beta$

Schlußgleichg.

$$\boxed{\frac{\partial}{\partial x}\left(h^3\, \frac{\partial p}{\partial x}\right) + \beta^2\, \frac{\partial}{\partial y}\left(h^3\, \frac{\partial p}{\partial y}\right) = \frac{6}{So}\, \frac{\partial h}{\partial x}}$$