Nuclear Matter Equation of State for Astrophysical Applications

Christian Drischler

Report B05, First Workshop of the SFB 1245

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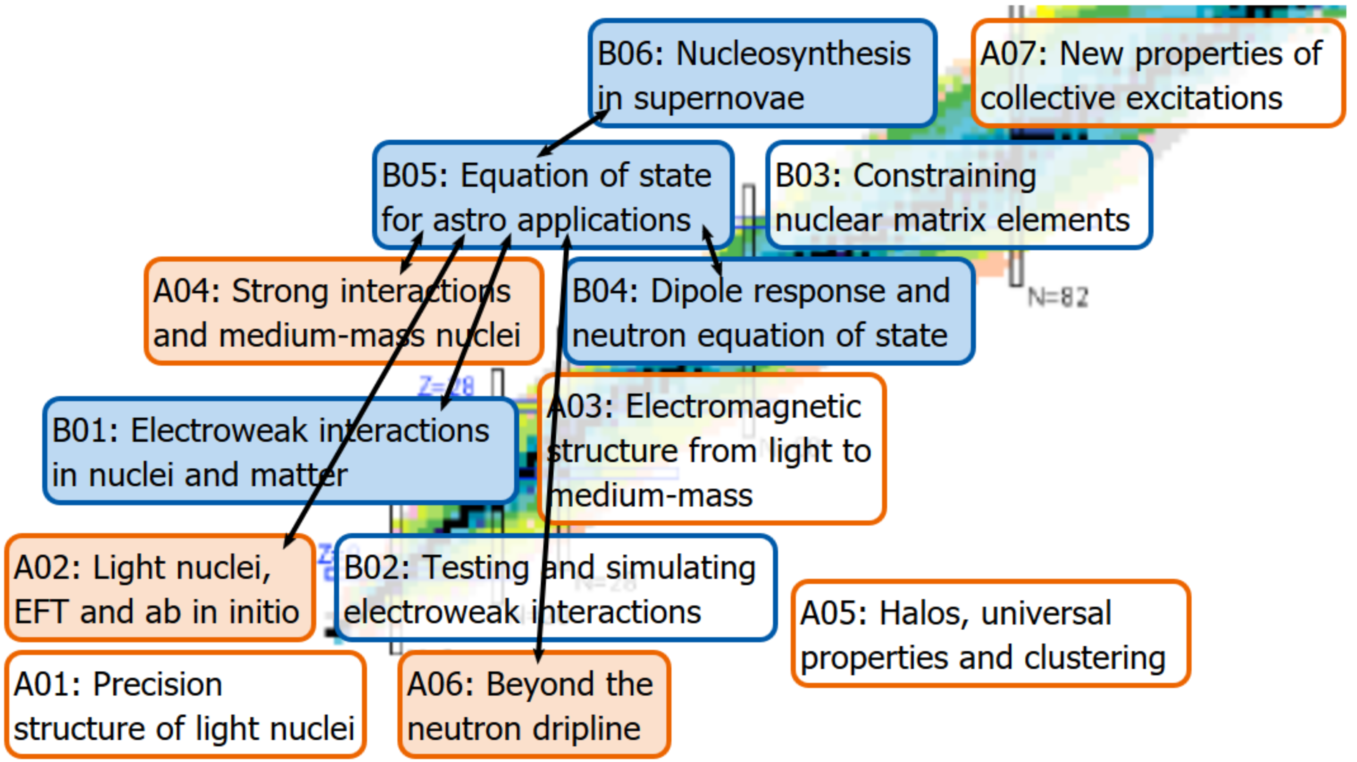

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Nuclear Matter Equation of State for Astrophysical Applications

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Motivation

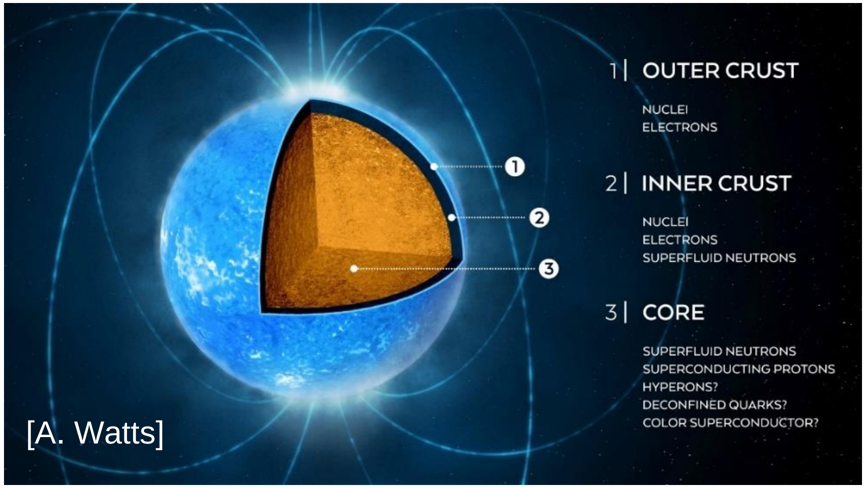
Energy per particle: $\frac{E}{A}(n, x, T)$

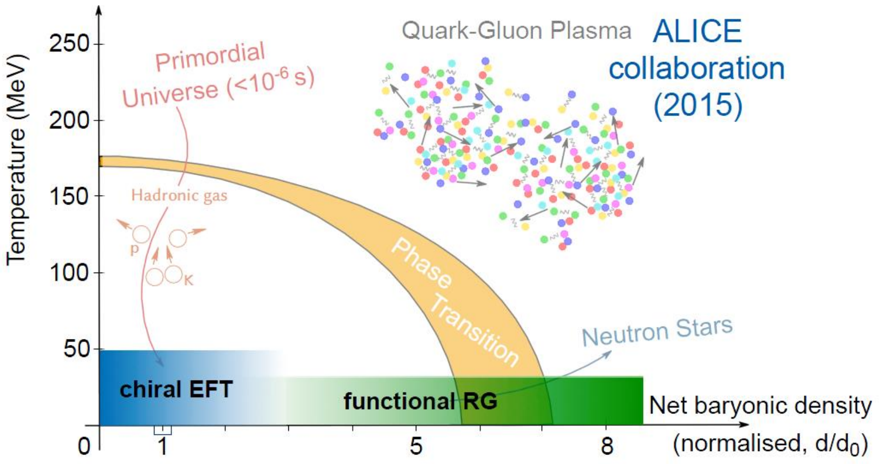
density
proton fraction
temperature

challenging density range: complementary approaches

- low densities: **chiral effective field theory** (EFT)
- high densities: **functional renormalization group** (fRG)

→ future goal: combine EOS from chiral EFT and fRG





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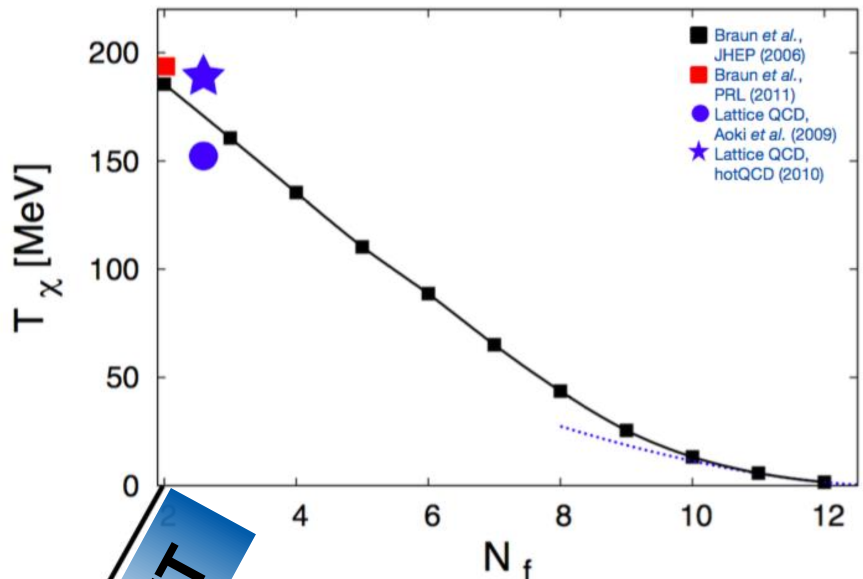
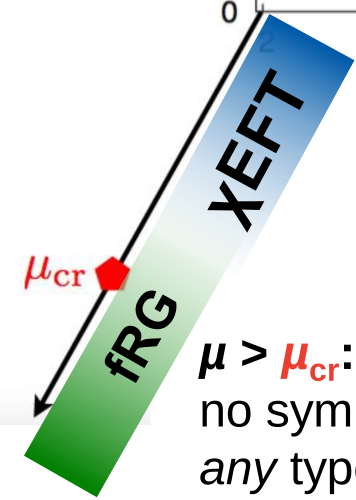
Functional Renormalization Group





$\mu > \mu_{cr}$:
no symmetry breaking of
any type occurs in matter

- **software development:** automatized derivation of fRG equations
- mechanism based on **quark-gluon dynamics** identified which underlies the existence of a critical quark chemical potential μ_{cr}
- exact determination of μ_{cr}: **~100 diagrams** at LO (Fierz-complete analysis)
- **next step:** development of syst. approx. for EOS ($\mu > \mu_{cr}$)

Springer, Braun, Rechenberger, Rennecke, arxiv:1611.06020
Leonhardt, Pospiech, Braun, work in progress

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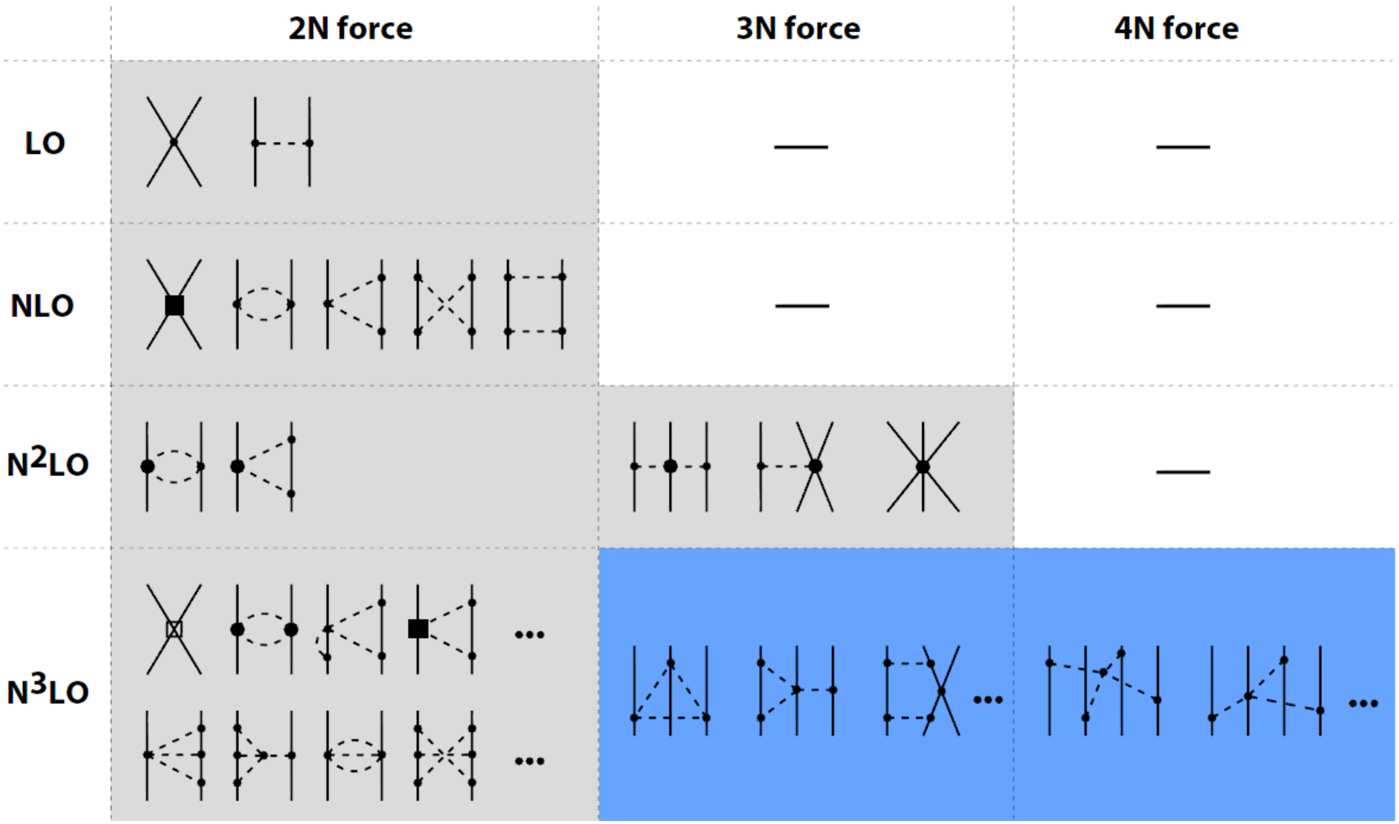
Hierarchy of Nuclear Forces in Chiral EFT



... and ongoing work at N^{4}LO ...

Weinberg, van Kolck, Kaplan, Savage, Wise, Epelbaum, Kaiser, Machleidt, Meißner, ...

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3N forces beyond Hartree-Fock?

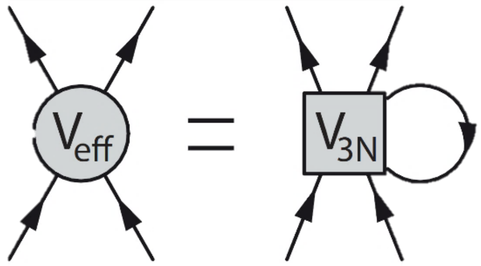
CD, Hebeler, Schwenk, PRC **93**, 054314

Effective NN potentials

by summing *one* particle over the occupied states of the Fermi sea

» dominant 3N contributions

Holt *et al.*, PRC **81**, 024002
Hebeler *et al.*, PRC **82**, 014314



more applications:
Carbone *et al.*, PRC **81**, 024002
Wellenhofer et al., PRC **82**, 014314

so far: only N^2LO 3N and $P = 0$

Improved Method

- applicable to all nuclear forces
- N^3LO 3N forces due to recent PW decomposition

Hebeler *et al.*, PRC **91**, 044001

towards *consistent* N^3LO calculations

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Outline

0 **Improved Normal-Ordering Method**

1 **Isospin-Asymmetric Nuclear Matter**

2 **Many-Body Convergence?**

3 **BCS $^1S_0$ and $^3P_2$-$^3F_2$ Pairing Gaps**

see also:
Vidaña *et al.*, PRC **80**, 045806
CD *et al.*, PRC **89**, 025806
Drews, Weise, PRC **91**, 035802
Wellenhofer *et al.*, PRC **93**, 055802

CD, Hebeler, Schwenk, PRC **93**, 054314.

ISOSPIN-ASYMMETRIC NUCLEAR MATTER

Objectives: equation of state, saturation point, incompressibility, symmetry energy

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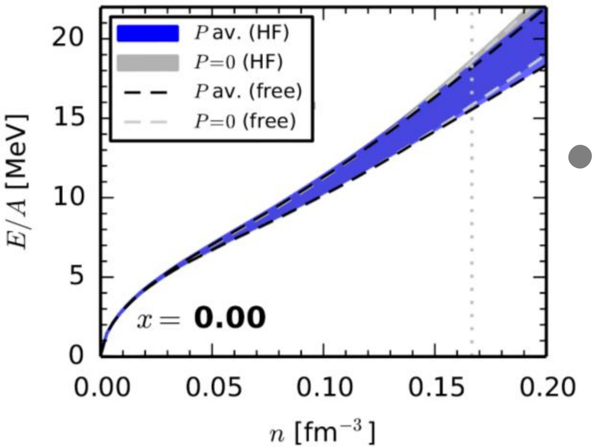
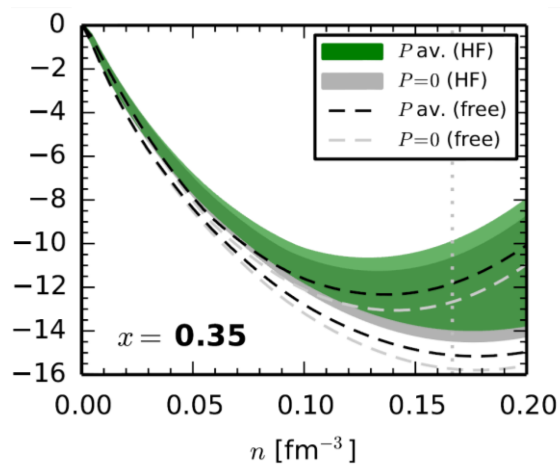
Equation of State

CD, Hebeler, Schwenk, PRC **93**, 054314



Neutron Matter



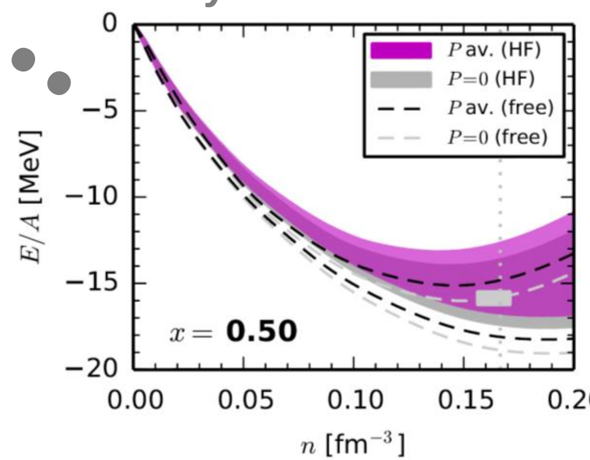
11 proton fractions

x = 0.0, 0.05, ..., 0.5

up to second order

$$x = \frac{n_p}{n_n + n_p}$$

Symmetric Matter



Uncertainty bands: Hebeler *et al.*, PRC **83**, 031301(R)

- 7 Hamiltonians: evolved N^3LO NN + bare N^2LO 3N
- different combinations of λ/Λ_{3N}
- c_D, c_E fit *only* to few-boy data
- free and Hartree-Fock spectrum

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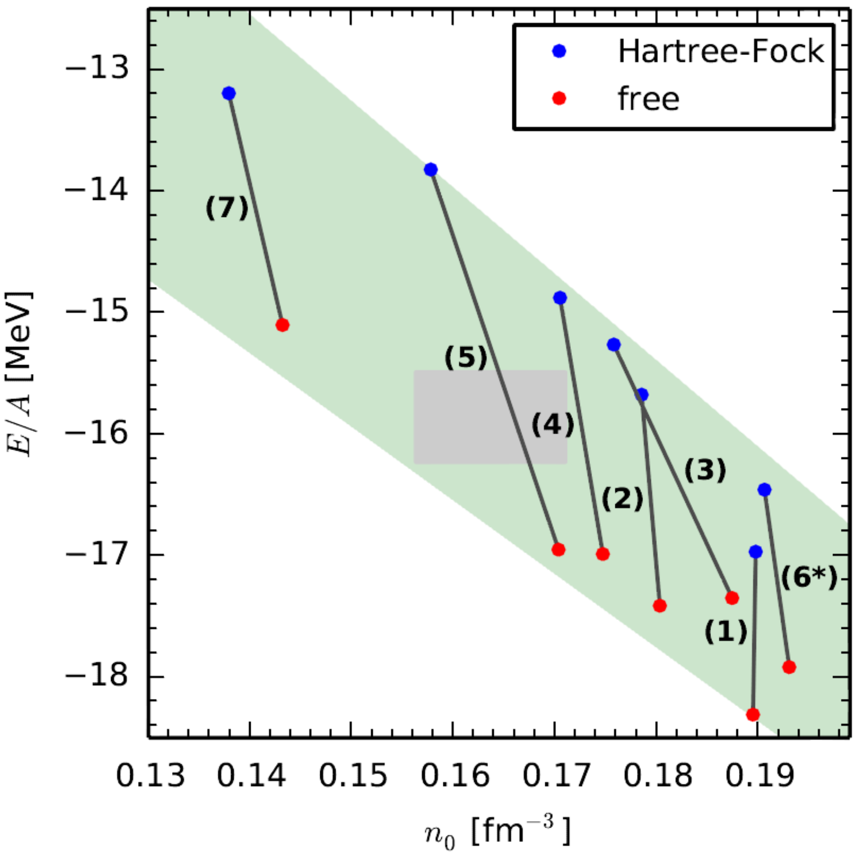
Saturation Properties

CD, Hebeler, Schwenk, PRC **93**, 054314



Coester-like correlation

- covers the empirical range due to 3N contributions

Coester *et al.*, PRC **1**, 769

empirical saturation point:

max. range of 14 EDF's

Dutra *et al.*, PRC **85**, 035201
Kortelainen *et al.*, PRC **89**, 054314

$$n_0 = (0.138 - 0.193)\ \mathrm{fm}^{-3}$$

$$K = (182 - 254)\ \mathrm{MeV}$$

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Symmetry Energy and Slope Parameter

see also: Hagen *et al.*, Nat. Phys. **12**, 186

standard expansion: $\beta = 1 - 2x$

$$\frac{E}{A}(n,\beta) = \frac{E_{\text{SNM}}(n)}{A} + S_2(n)\beta^2 + \ldots$$

$$S_2(n) = S_v + \frac{L}{3}\left(\frac{n-n_0}{n_0}\right) + \cdots$$

tight constraints

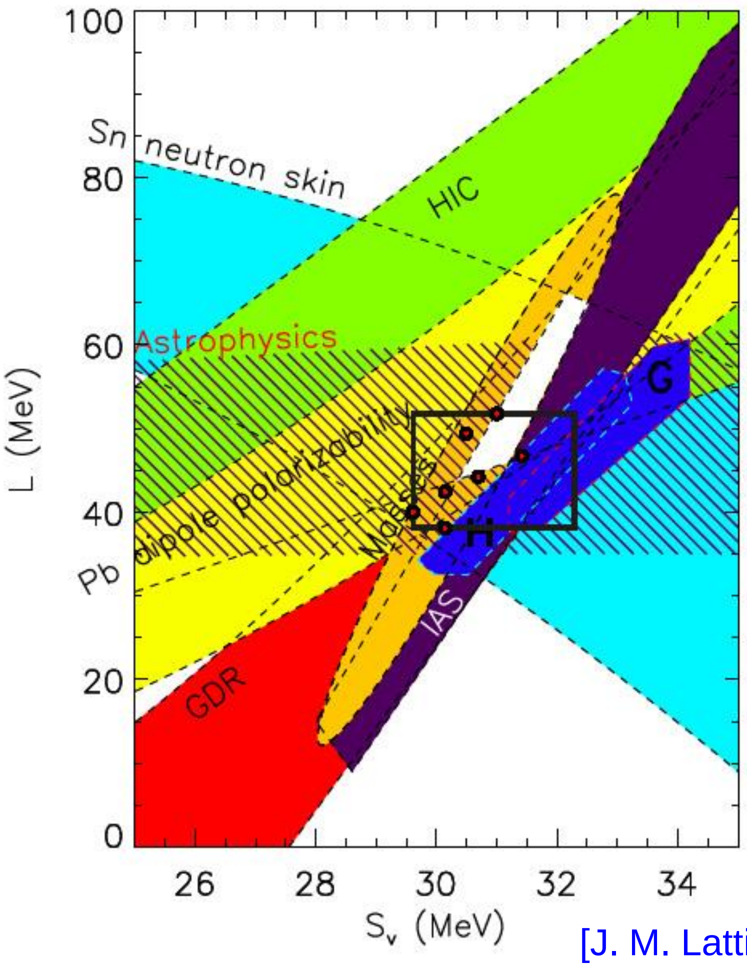
$$S_v = (30.9 \pm 1.4)\ \text{MeV}$$

$$L = (45.0 \pm 7.1)\ \text{MeV}$$

in **agreement** with emp. extractions

quadratic expansion is **reliable**;
nonanalytical quartic term: $\beta^4 \ln|\beta|$

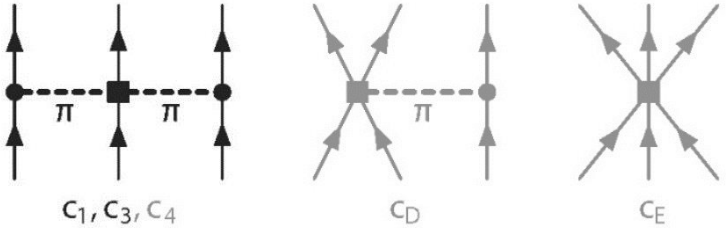
Kaiser, PRC **91**, 065201
Wellenhofer *et al.*, PRC **93**, 055802



[J. M. Lattimer]

see also:
Dickhoff, Barbieri, Prog. Part. Nucl. Phys. **52**, 377
Rios *et al.*, PRC **79**, 025802
Krüger *et al.*, PRC **88**, 025802
Tews *et al.*, PRC **93**, 024305

Neutron Matter:



CD, Carbone, Hebeler, Schwenk, PRC **94**, 054307.

MANY-BODY CONVERGENCE?

Objectives: test many-body convergence
study impact of N^3LO 3N forces

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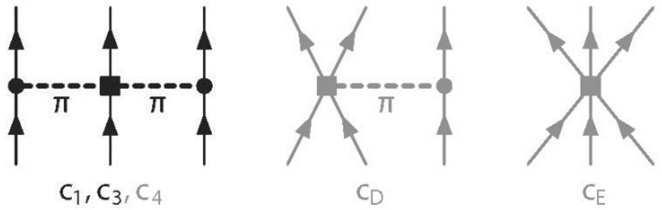
Testing Many-Body Convergence

CD, Carbone, Hebeler, Schwenk, PRC **94**, 054307

- consistent N^3LO NN/3N forces
- finite proton fractions need reliable fits of c_D, c_E at N^3LO

Golak *et al.*, Eur. Phys. J. A **50** 177

Neutron Matter



Uncertainty bands

- use always c_i's recommended for N^3LO calculations
- plus many-body uncertainty

Krebs *et al.*, PRC **85**, 054006

MBPT	SCGF Method
Improved Normal-Ordering Method	
up to third order	**nonperturbative**
free vs. HF spectrum	full spectral function
T=0 MeV	Extrapolated to T=0 MeV

see also: Carbone *et al.*, PRC **90** 054322

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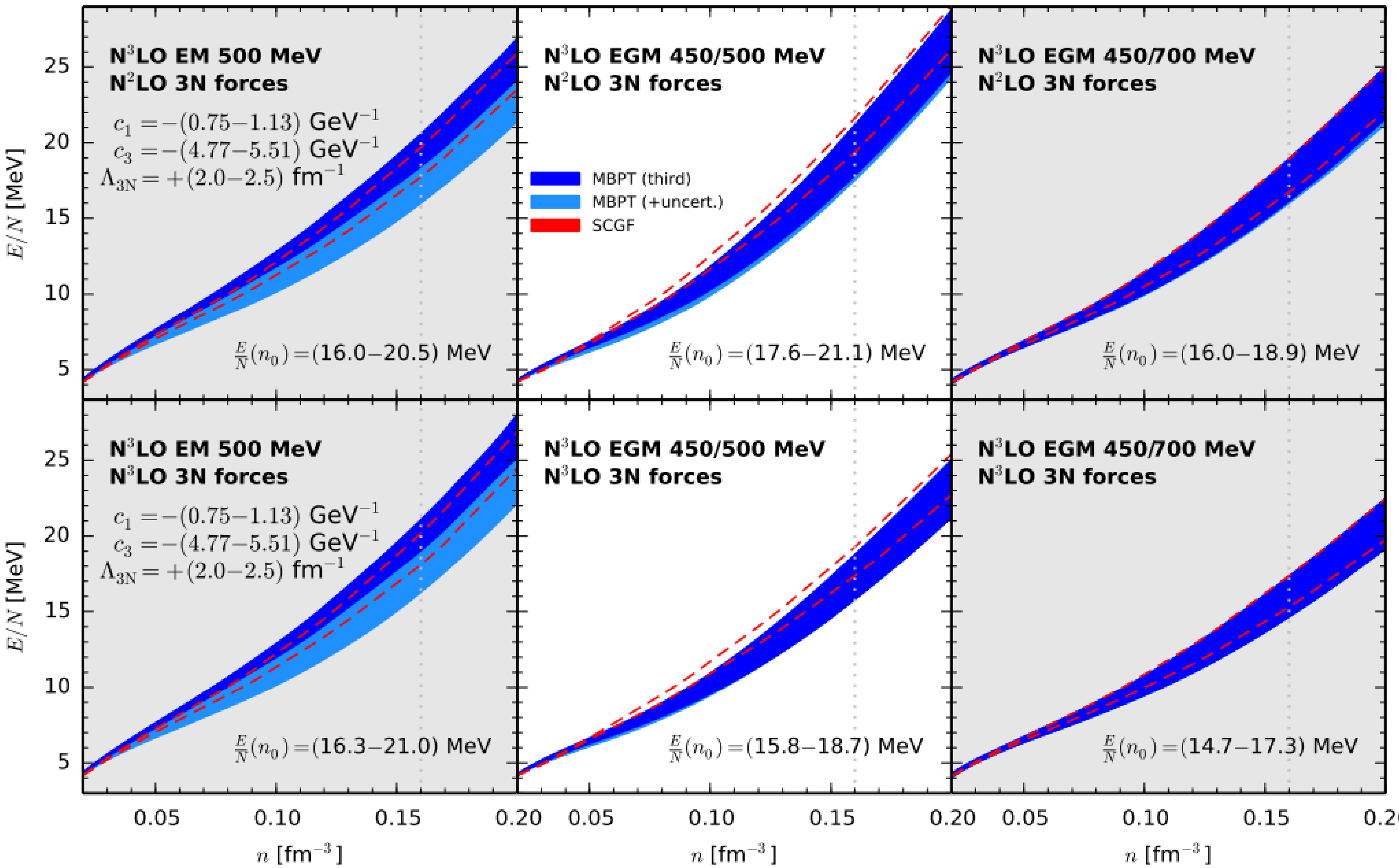
MBPT vs. SCGF Method

CD, Carbone, Hebeler, Schwenk, PRC **94**, 054307



N^2LO 3N

N^3LO 3N

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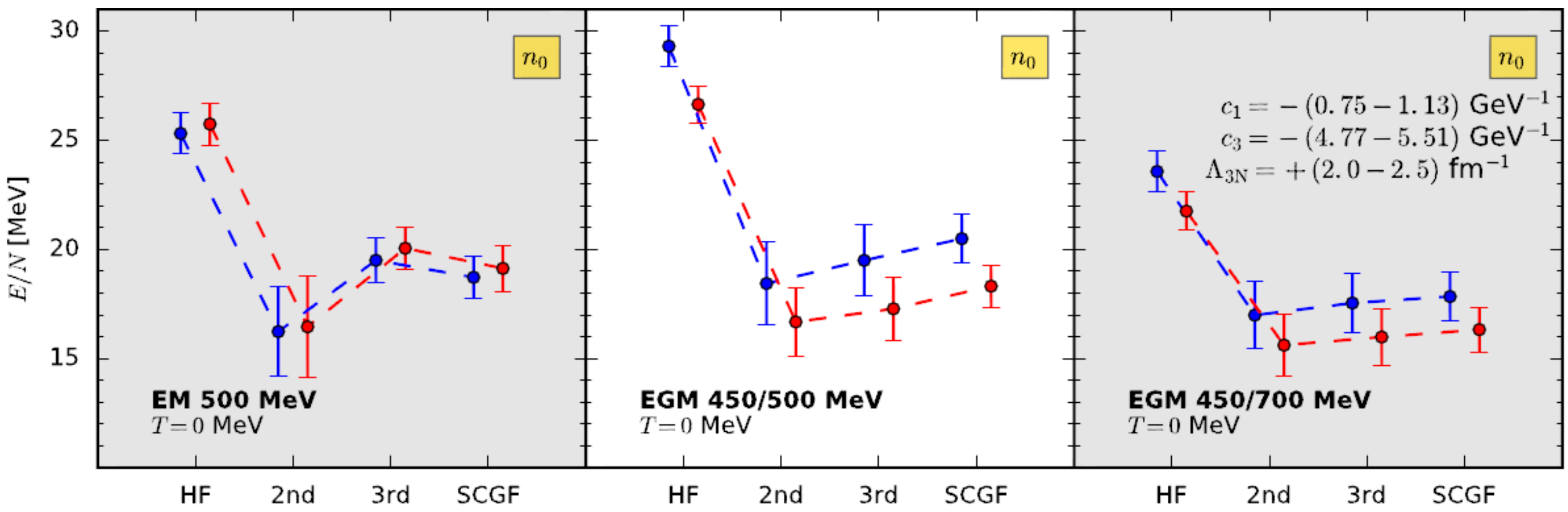
Testing Many-Body Convergence

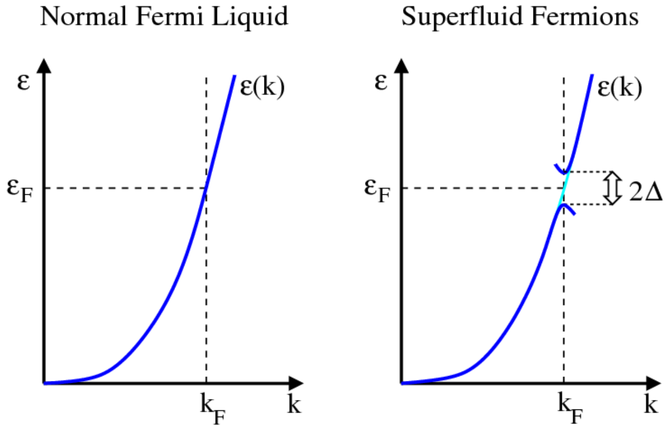
CD, Carbone, Hebeler, Schwenk, PRC **94**, 054307



Order-by-order analysis: (at saturation density)

- attractive second vs. repulsive third order
- MBPT **well converged** for EGM potentials (small third order)
- EM 500 MeV is less perturbative (larger third order)
- small energy shift due to N^3LO 3N w.r.t. N^2LO 3N contributions

see also:
Srinivas, Ramanan, arXiv:1606.09053 (PRC in press)
Ding *et al.*, PRC **94**, 025802
Maurizio *et al.*, PRC **90**, 044003
Page *et al.*, "Novel Superfluids", Oxford University Press



CD, Krüger, Hebeler, Schwenk, arXiv:1610.05213

BCS $^1S_0$ AND $^3P_2$-$^3F_2$ PAIRING GAPS

Objectives: study subleading 3N contributions
recent (semi-)local NN potentials, new uncertainties

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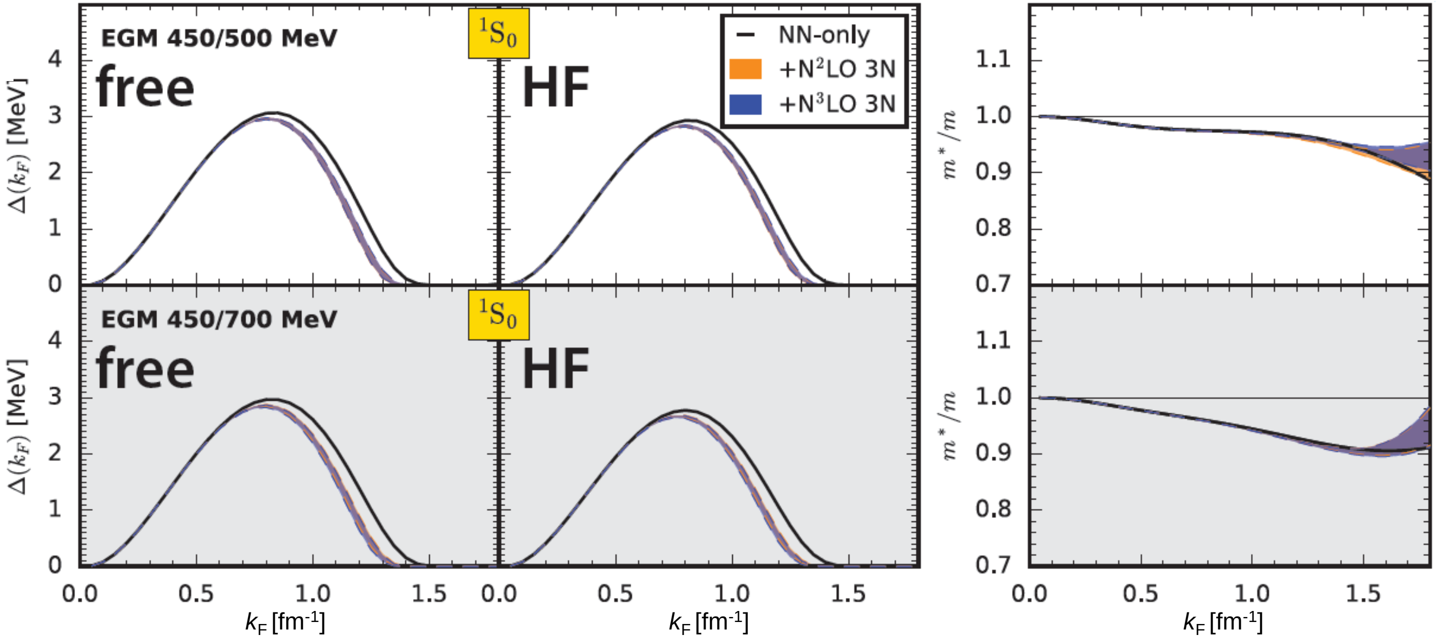
Pairing Gaps: 3N forces in $^1S_0$

CD, Krüger, Hebeler, Schwenk, arXiv:1610.05213



- uncertainties: 3N parameter variation (recommended values)
- **pairing gap at low densities**
  - universal gaps: strongly constrained by phase shifts
  - **small 3N contributions**: only small suppression for $k_F > 0.8$ fm^{-1}
  - almost independent of the energy spectrum

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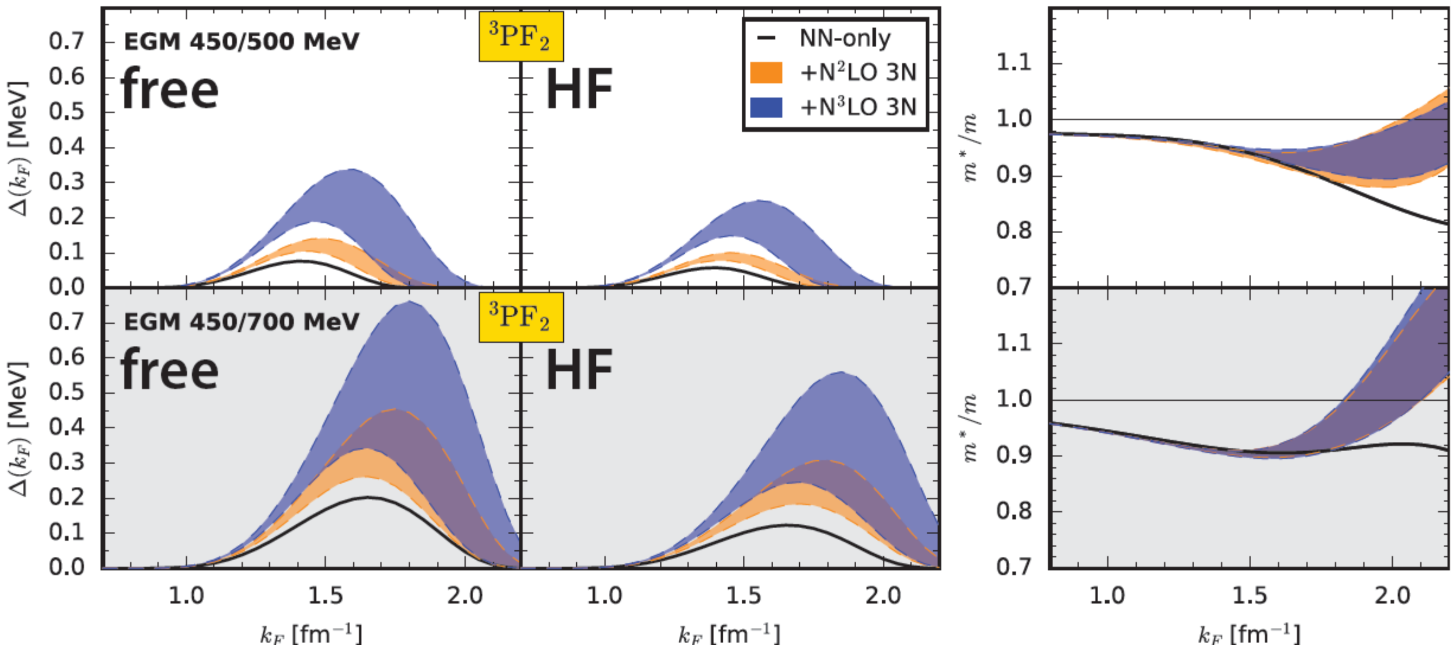
Pairing Gaps: 3N forces in $^3P_2$-$^3F_2$

CD, Krüger, Hebeler, Schwenk, arXiv:1610.05213



- uncertainties: 3N parameter variation (recommended values)
- **pairing gap at high densities**
  - **3N forces add attraction**: larger max. gap and closure at higher densities
  - effective masses are enhanced due to 3N forces
  - chiral EFT still efficient at $k_F > 2$ fm^{-1}**?**

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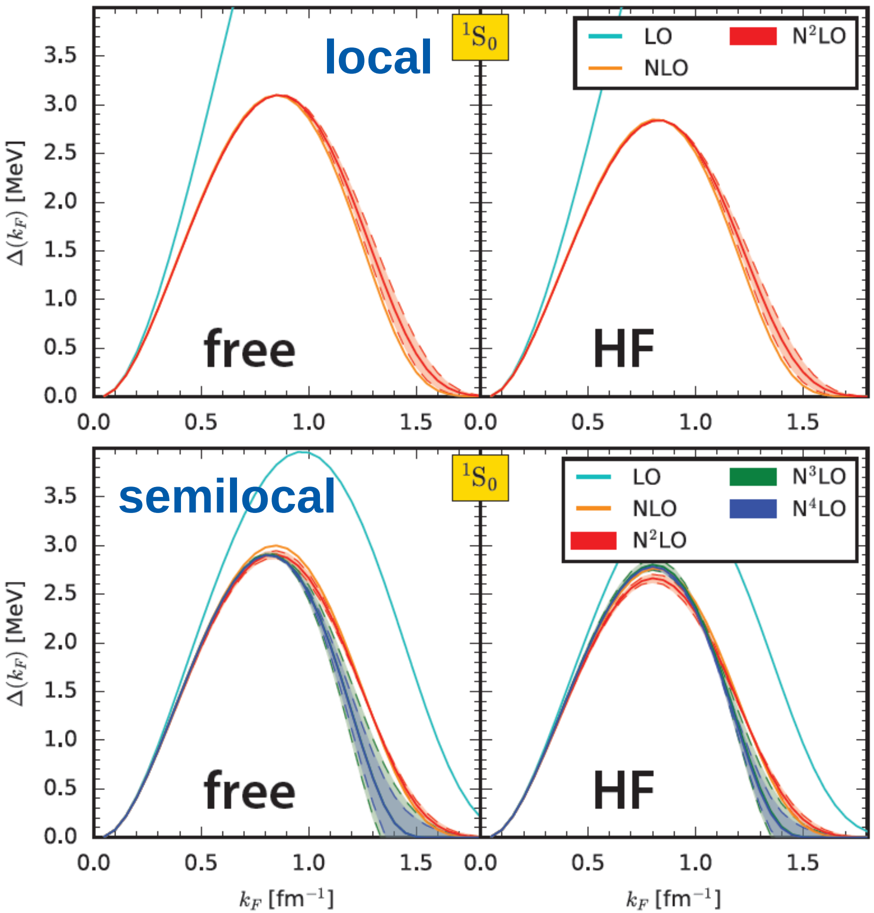
(Semi-)Local NN: $^1S_0$ channel

CD, Krüger, Hebeler, Schwenk, arXiv:1610.05213



local and semilocal NN forces:

- up to N^{2}LO and N^{4}LO
- R_0 = 0.9, 1.0, 1.1 and, 1.2 fm

new uncertainties (Epelbaum *et al.*)

order-by-order analysis in the chiral expansion (LO neglected)

findings:

- at NLO and beyond **gaps agree** up to k_F~(0.6−0.8) fm^{-1}
- sensitivity to spectrum is again small

Gezerlis *et al.*, PRC **90**, 054323
Epelbaum *et al.*, Eur. Phys. J. A **51**, 53

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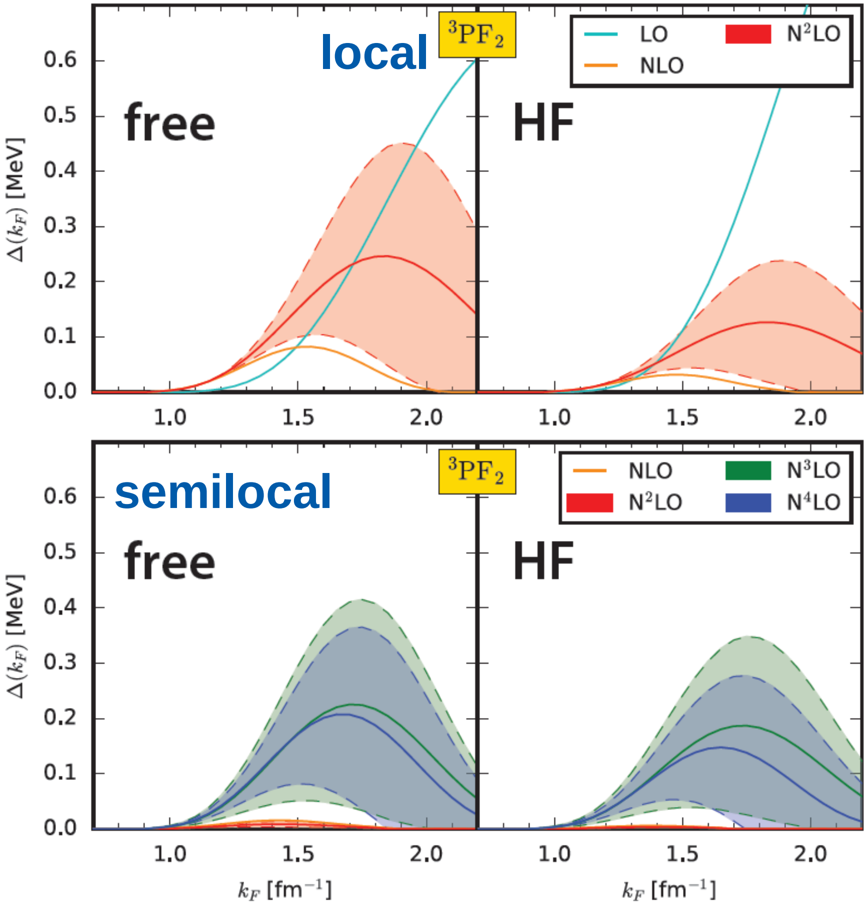
(Semi-)Local NN: $^3P_2$-$^3F_2$ channel

CD, Krüger, Hebeler, Schwenk, arXiv:1610.05213



local and semilocal NN forces:

- up to N^2LO and N^4LO
- R_0 = 0.9, 1.0, 1.1 and, 1.2 fm

new uncertainties (Epelbaum *et al.*)

order-by-order analysis in the chiral expansion (LO neglected)

findings:

- **LO gaps** (local NN) depend strongly on the cutoff: **artifacts**
- large uncertainties: breakdown of the chiral expansion **?**

Gezerlis *et al.*, PRC **90**, 054323
Epelbaum *et al.*, Eur. Phys. J. A **51**, 53

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Summary | Outlook

Improved Normal-Ordering Method

- applicable to all 3N forces (incl. N^3LO)
- **asymmetric matter**: results for EOS, symmetry energy, …

More Applications

- studied **many-body convergence** in neutron matter: N^3LO 3N forces beyond Hartree-Fock and in SCGF method
- **BCS pairing gaps** in $^1S_0$ and $^3P_2$-$^3F_2$:
  - N^3LO 3N contributions to traditional NN potentials
  - recent (semi-)local NN potentials, new uncertainties

Extensions – a selection

- finite temperatures, consistently-evolved forces, ...
- **constrain fits of nuclear forces: saturation**, … Carlsson *et al.*, PRX **6**, 011019

Special Thanks to

Collaborators:

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