Theory of electromagnetic interactions: from few- to many-body systems

Sonia Bacca

October 3rd, 2017

2nd Workshop of the SFB 1245
Schloss Waldhausen, Budenheim

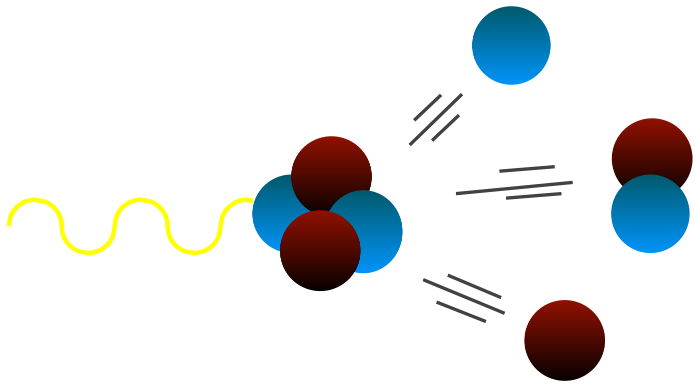
Motivations

- Electromagnetic probes (coupling constant <<1)

"*With the electromagnetic probe, we can immediately relate the cross section to the transition matrix element of the current operator, thus to the structure of the target itself*"

[De Forest-Walecka, Ann. Phys. 1966]

$$\sigma \propto |\langle \Psi_f | J^\mu | \Psi_0 \rangle|^2$$



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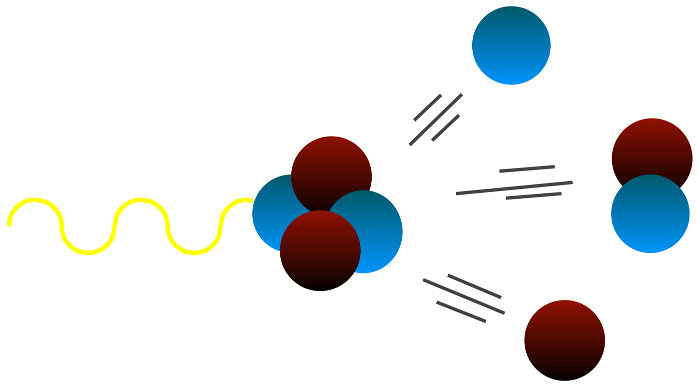

- With ab-initio calculations we can study these observables and provide predictions with error bars, first for light nuclei and then for heavier nuclei

"from few- to many-body systems"

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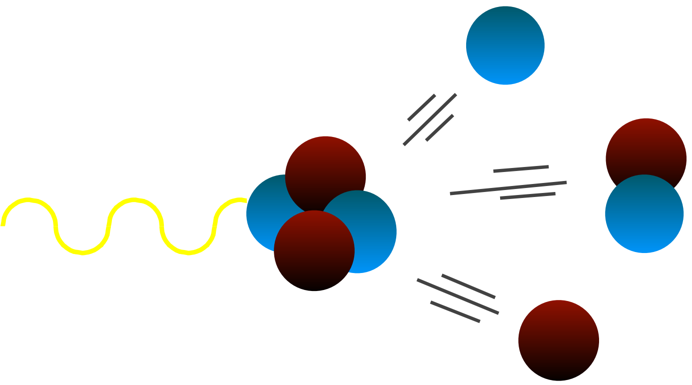

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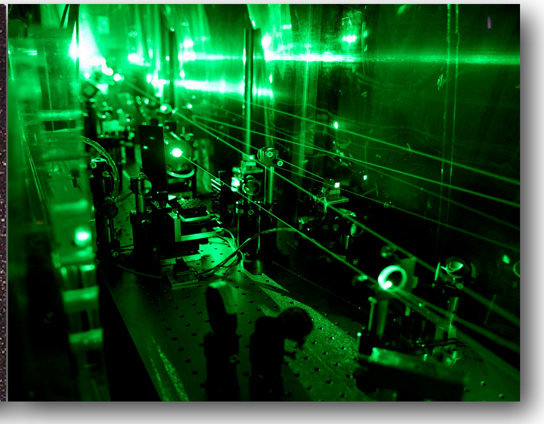
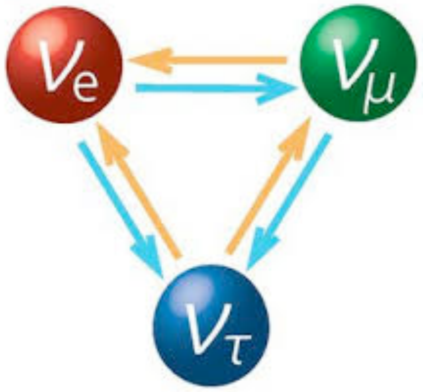
"from few- to many-body systems"

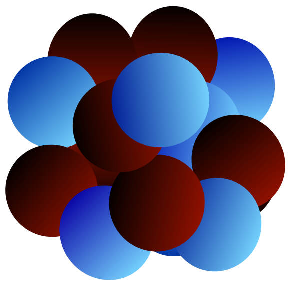
- Provide important informations in other fields of physics, where nuclear physics plays a crucial role:
  - Astrophysics:
  - Atomic physics
  - Particle physics





"Ab initio" theory

- Start from neutrons and protons as building blocks
  (centre of mass coordinates, spins, isospins)



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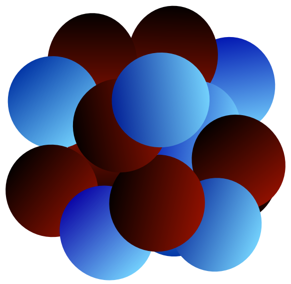

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- Solve the non-relativistic quantum mechanical problem of A-interacting nucleons

$$H|\psi_i\rangle = E_i|\psi_i\rangle$$

$$H = T + V_{NN}(\Lambda) + V_{3N}(\Lambda) + \dots$$

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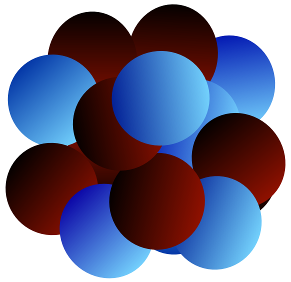

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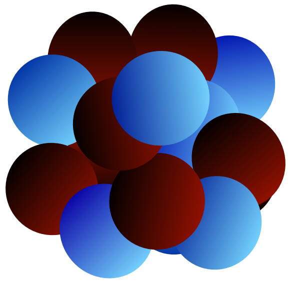

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- Calibrate the force and then calculate A-body nuclei to compare with experiment or to provide predictions for observables which are hard or impossible to measure

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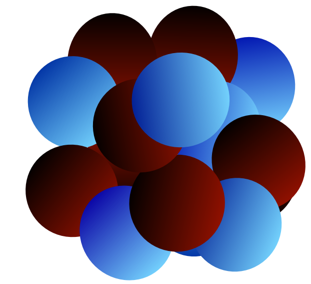

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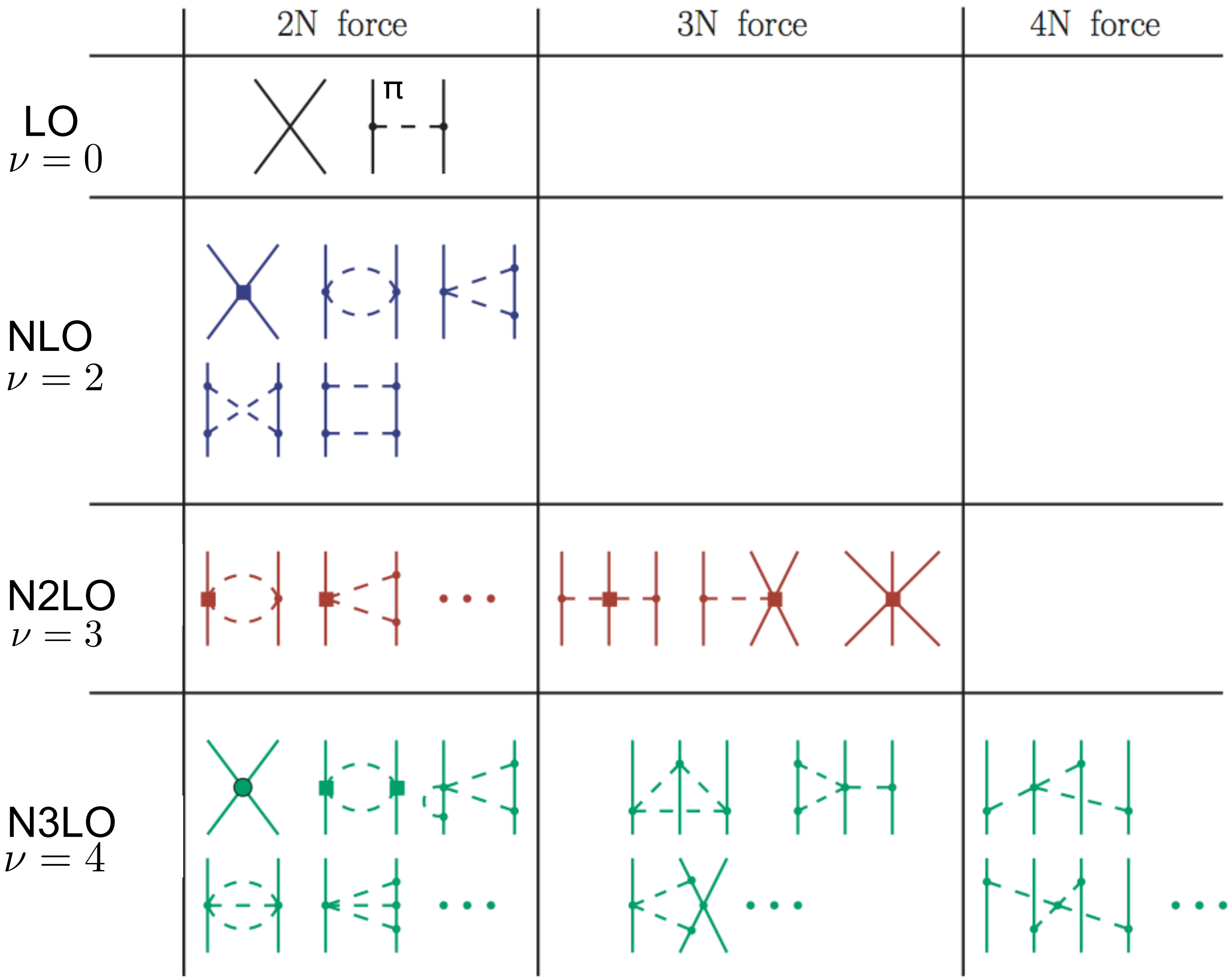
Chiral Effective Field Theory

Weinberg, van Kolck, Epelbaum, Meissner, Machleidt



Systematic expansion

$$\mathcal{L} = \sum_{\nu} c_{\nu} \left(\frac{Q}{\Lambda_b} \right)^{\nu}$$

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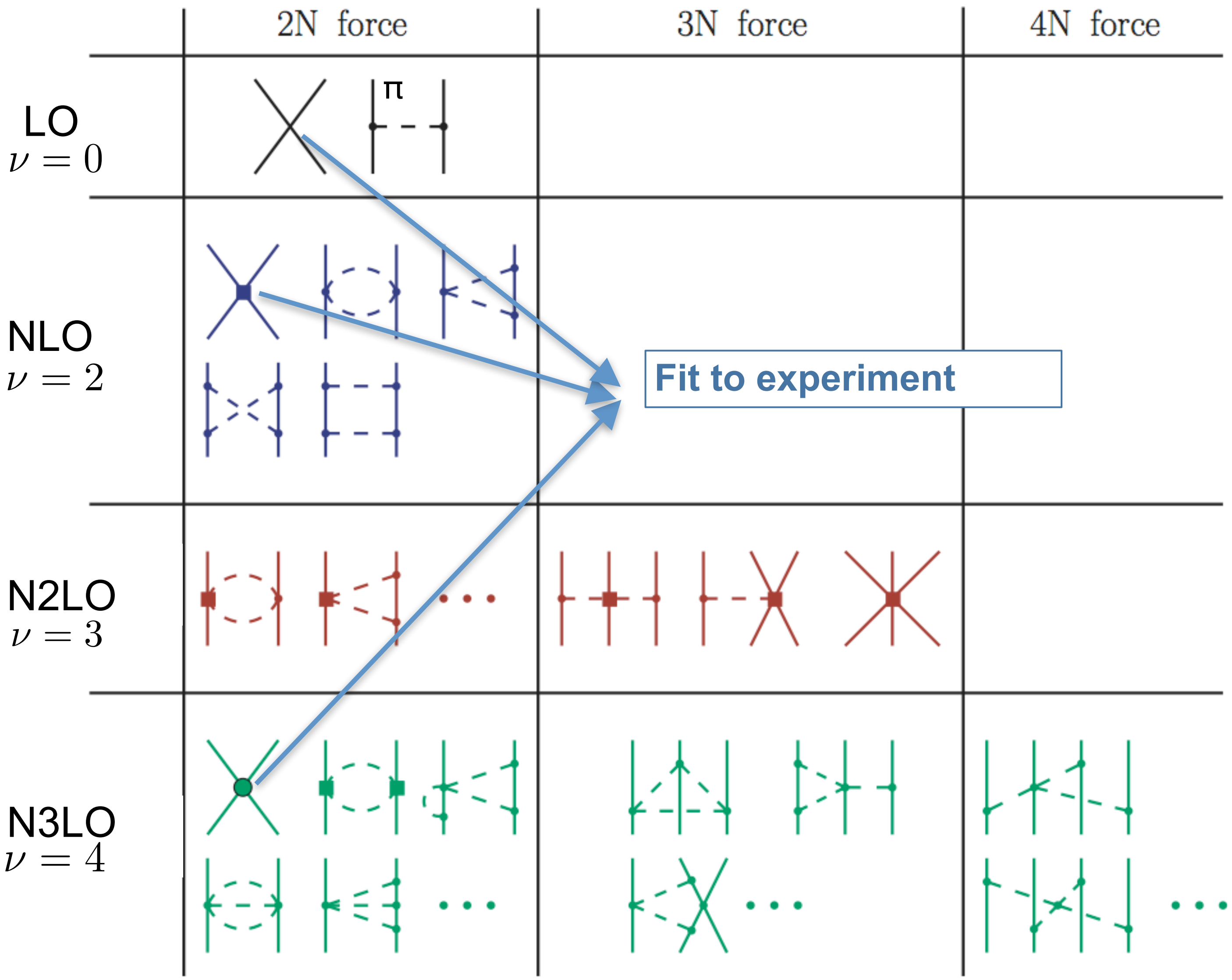
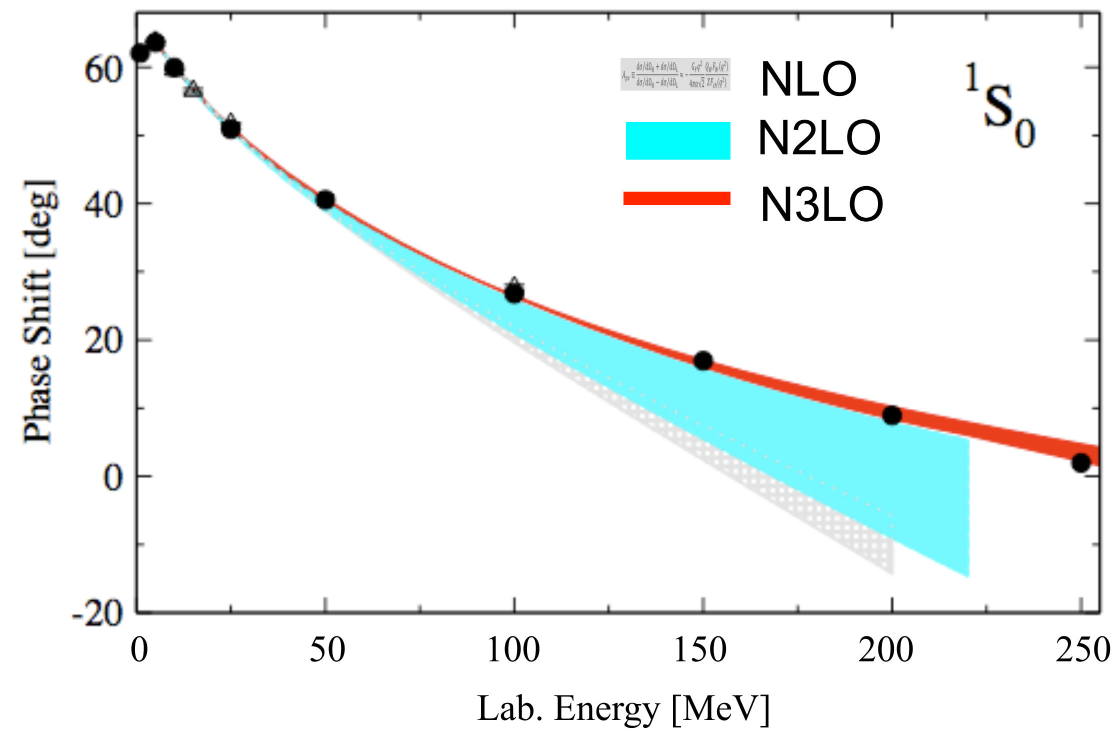


Details of short distance physics not resolved, but captured in low energy constants (LEC)

LEC fit to experiment - NN sector -



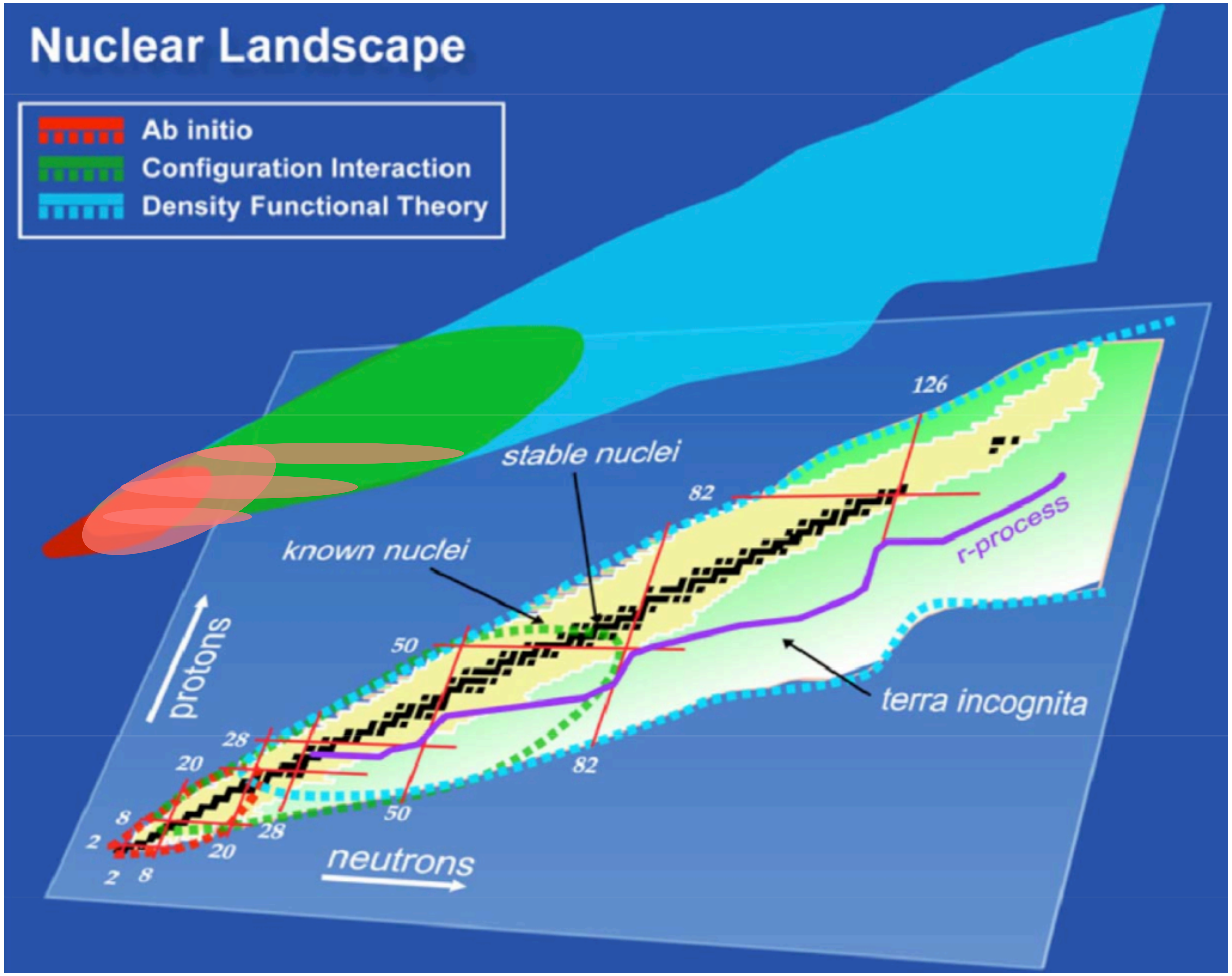
Epelbaum *et al.* (2009)

Systematic expansion $\mathcal{L} = \sum_\nu c_\nu \left(\frac{Q}{\Lambda_b} \right)^\nu$

LEC fit to experiment - 3N sector -
Using A=3 data or including other A>3 nuclei

Nuclear Structure Theory

Various methods to solve the many-body problem



http://unedf.org

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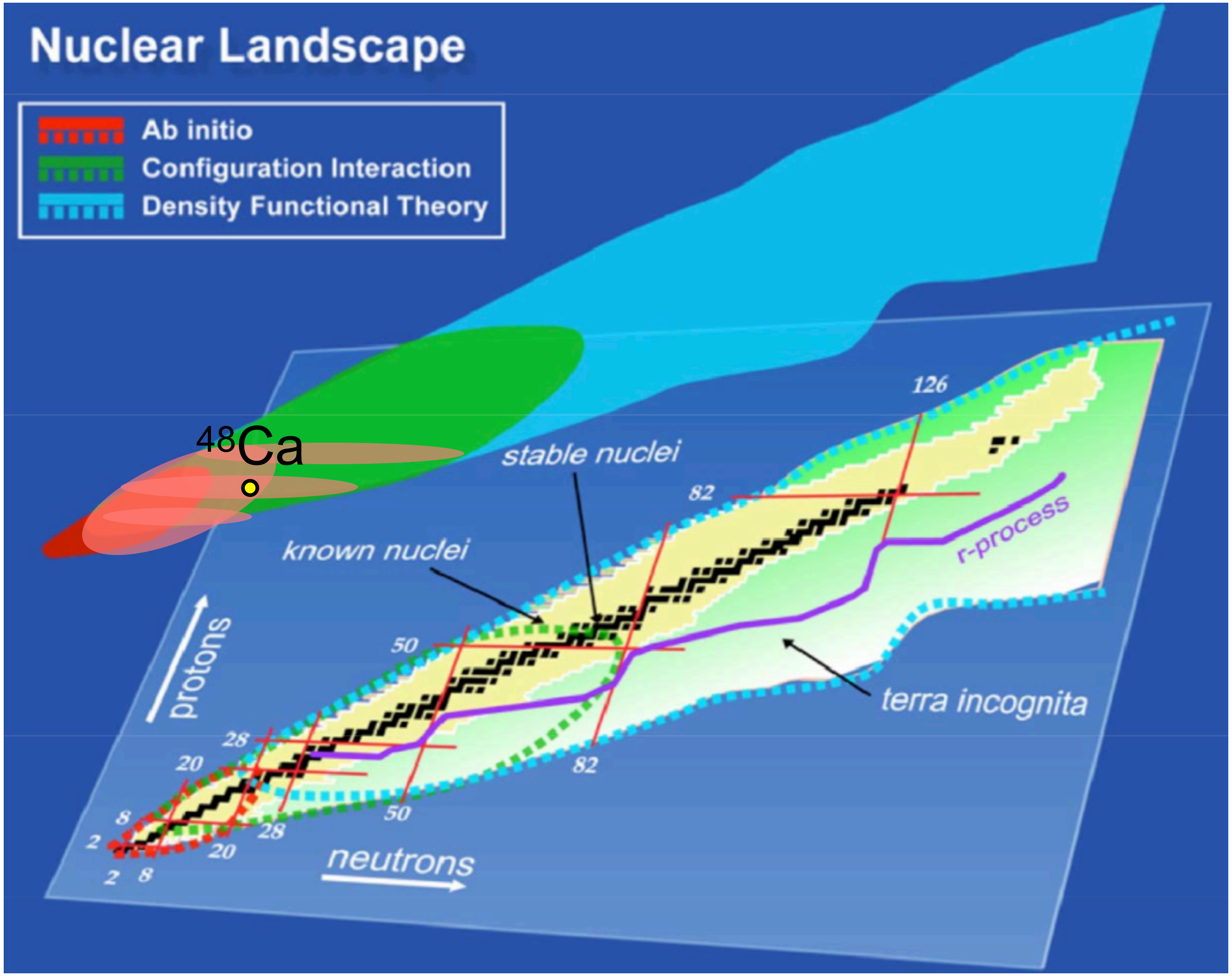

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Nuclear Structure Theory

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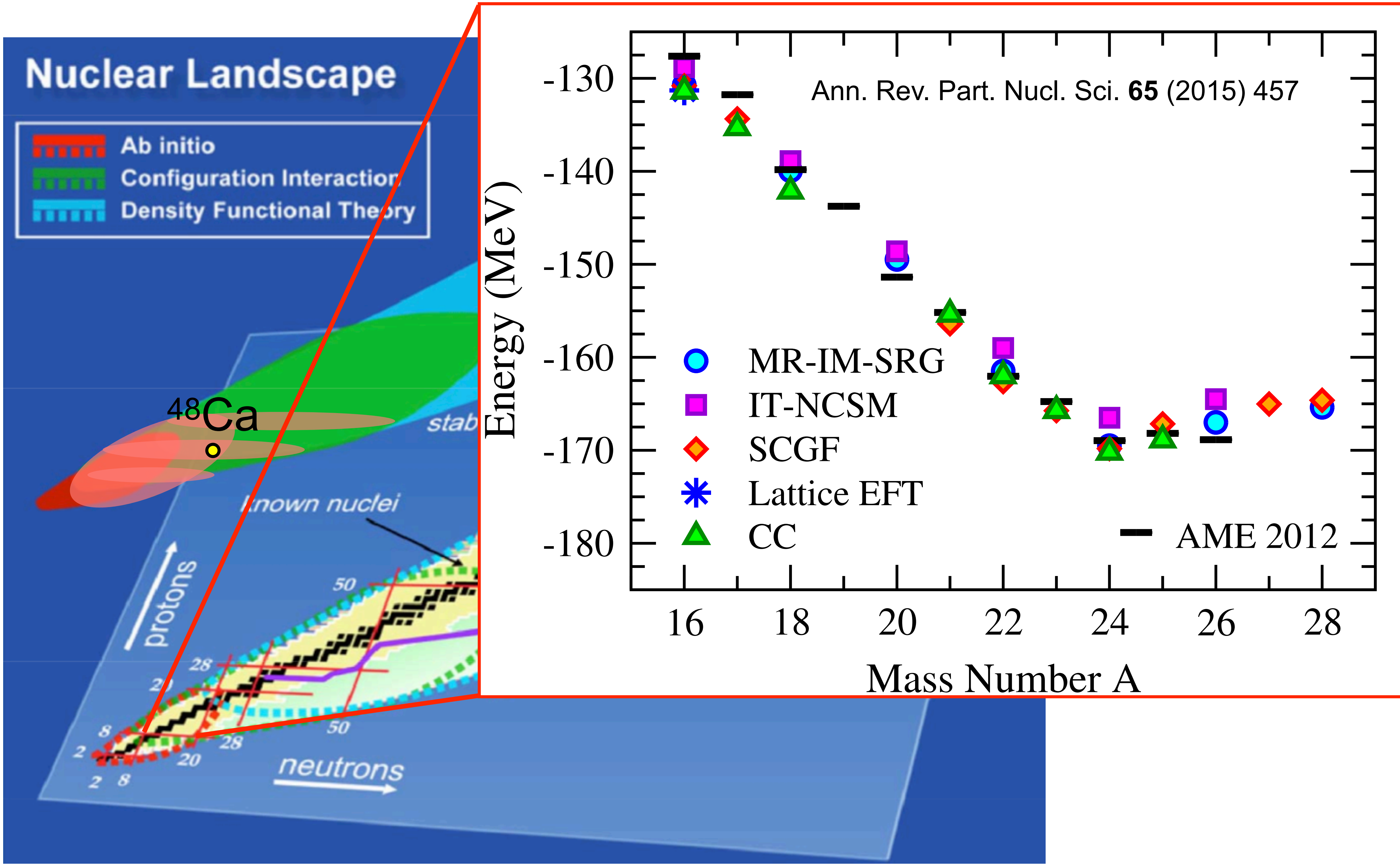
Oxygen chain



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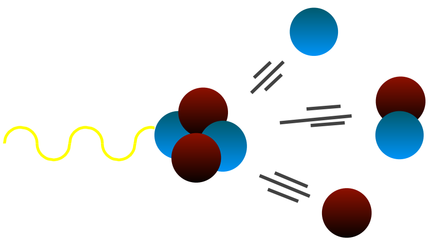
Nuclear Reactions

How does the nucleus respond to external electromagnetic excitations?



$$R(\omega) = \sum\!\!\!\!\!\!\!\int_f |\langle \Psi_f | \mathcal{O} | \Psi_0 \rangle|^2 \delta(\omega - E_f + E_0)$$

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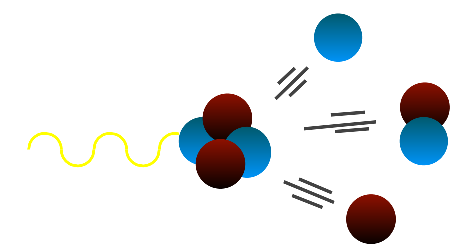

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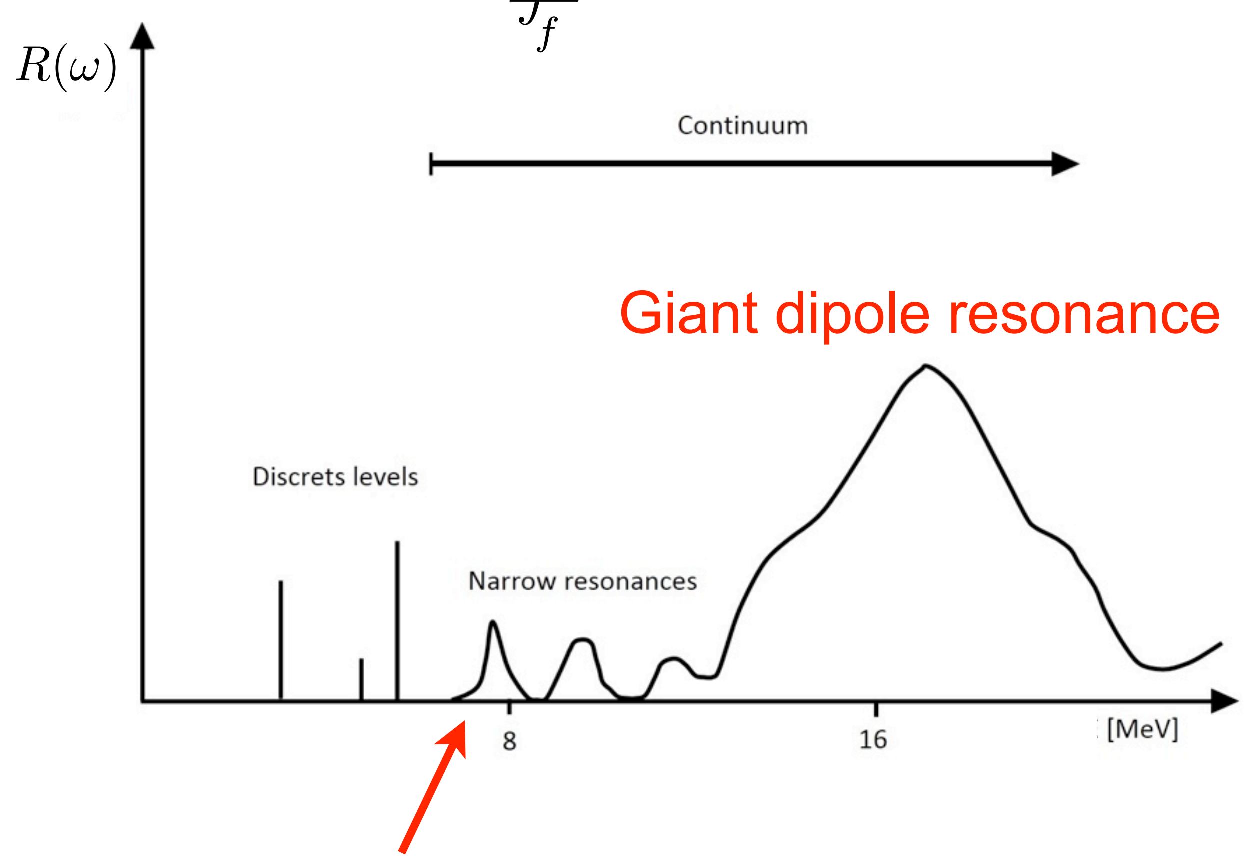
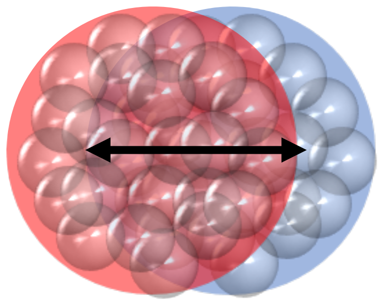
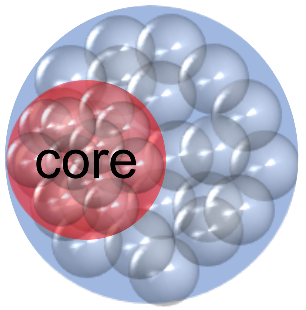




Pigmy dipole resonance in neutron-rich nuclei



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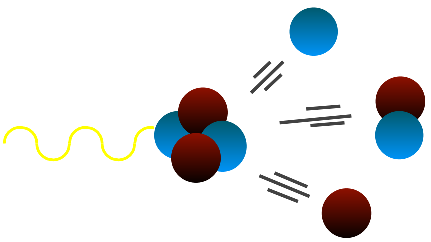

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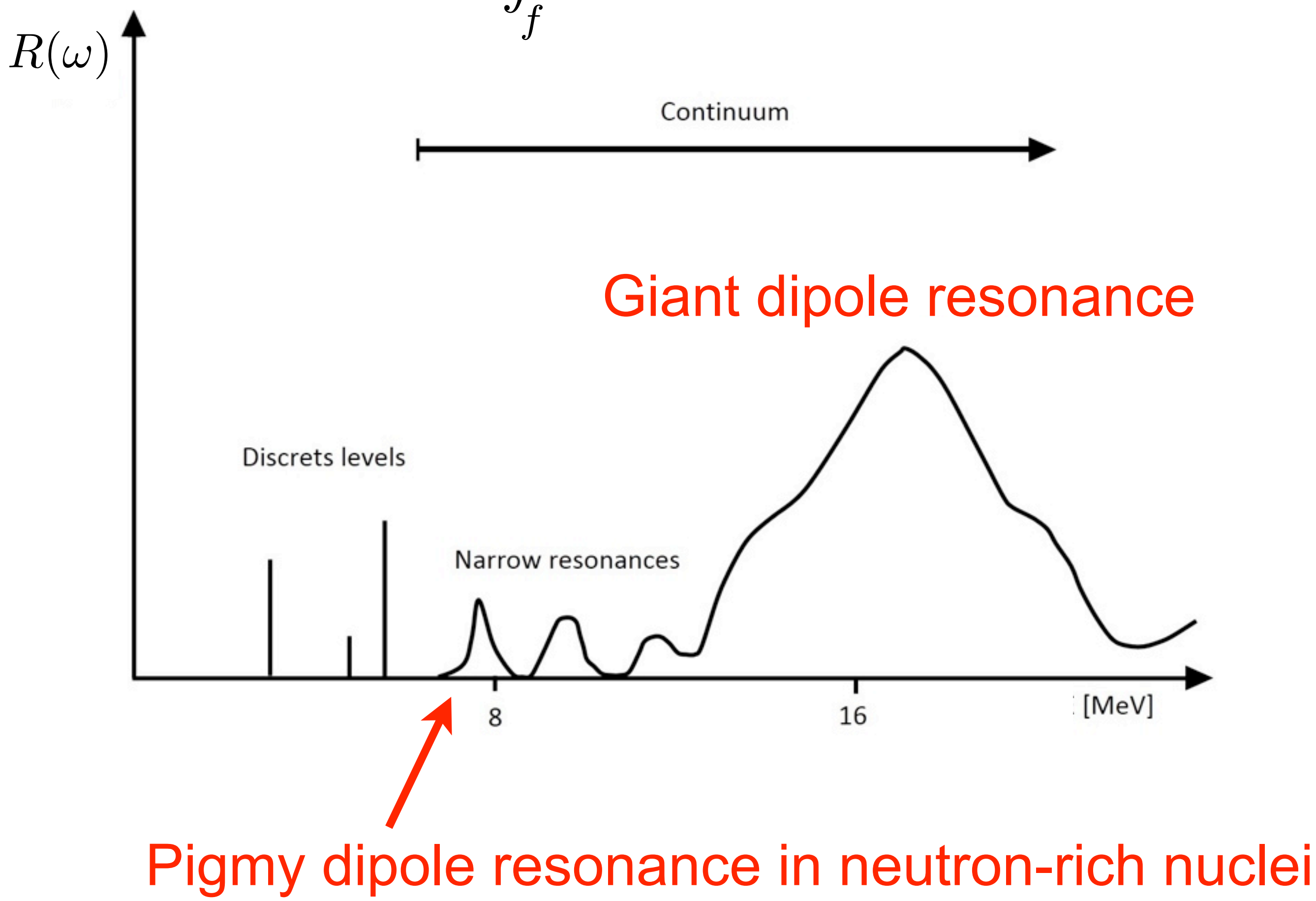
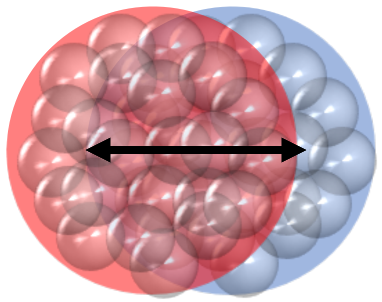
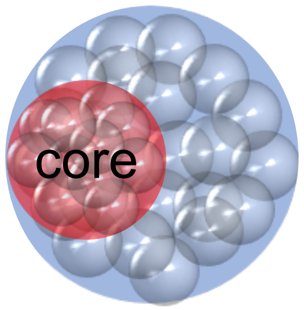


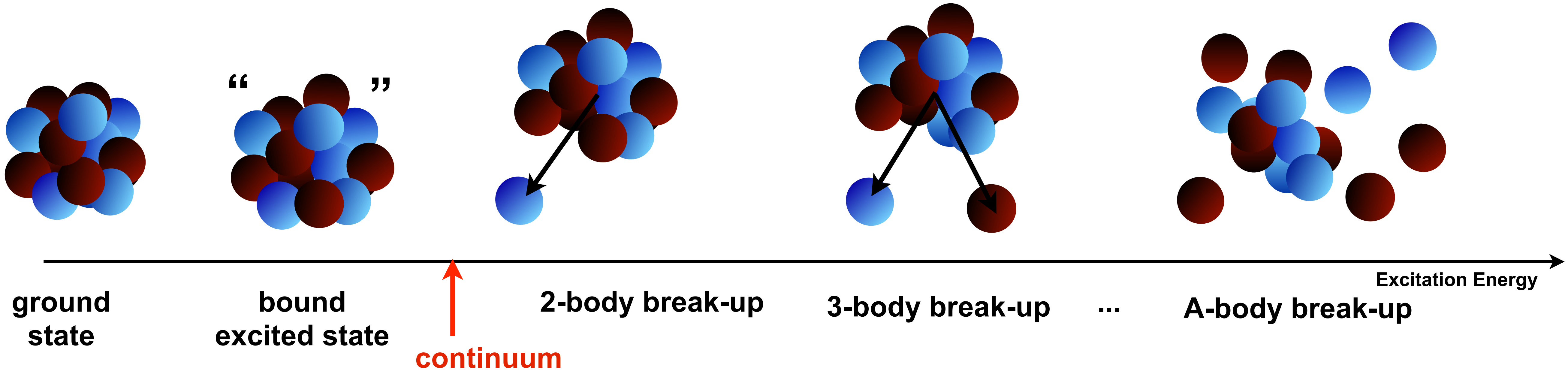




$$\alpha_D = 2\alpha \int_{\omega_{th}}^{\infty} d\omega \frac{R(\omega)}{\omega}$$

→ Low-energy part of strength dominates

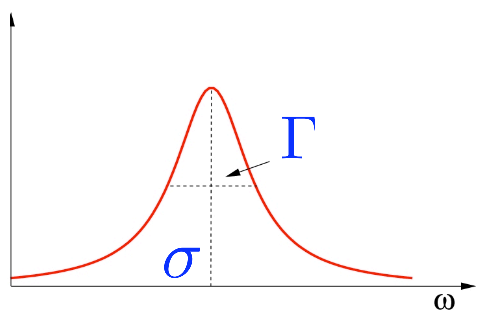
The Continuum Problem



$$R(\omega) \propto \left|\langle \Psi_f | \mathcal{O} | \Psi_0 \rangle\right|^2$$

Exact knowledge limited in energy and mass number

$$L(\sigma,\Gamma) = \int d\omega \frac{R(\omega)}{(\omega-\sigma)^2+\Gamma^2} = \left\langle \tilde{\psi} | \tilde{\psi} \right\rangle < \infty$$



Lorentz Integral Transform ⇨ Reduce the continuum problem to a bound-state-like equation

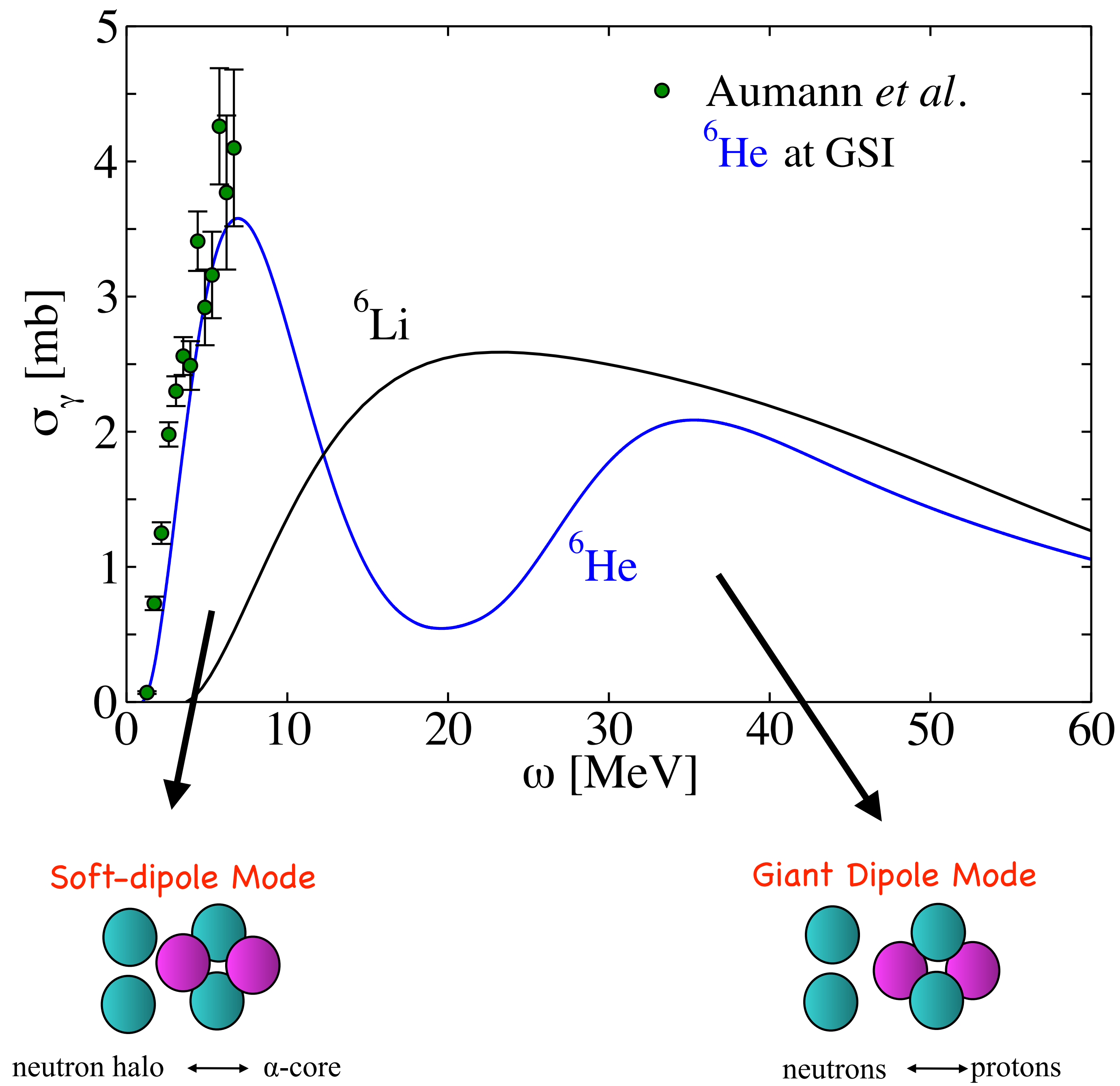
Efros, *et al.*, JPG.: Nucl.Part.Phys. **34** (2007) R459

$$(H - E_0 - \sigma + i\Gamma) \mid \tilde{\psi} \rangle = \hat{O} \mid \psi_0 \rangle$$

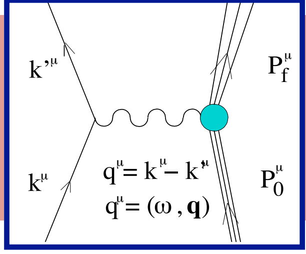
Few-body nuclei with hyper-spherical harmonic expansions

Dipole Response Function

S.Bacca et al, PRL **89** 052502 (2002)



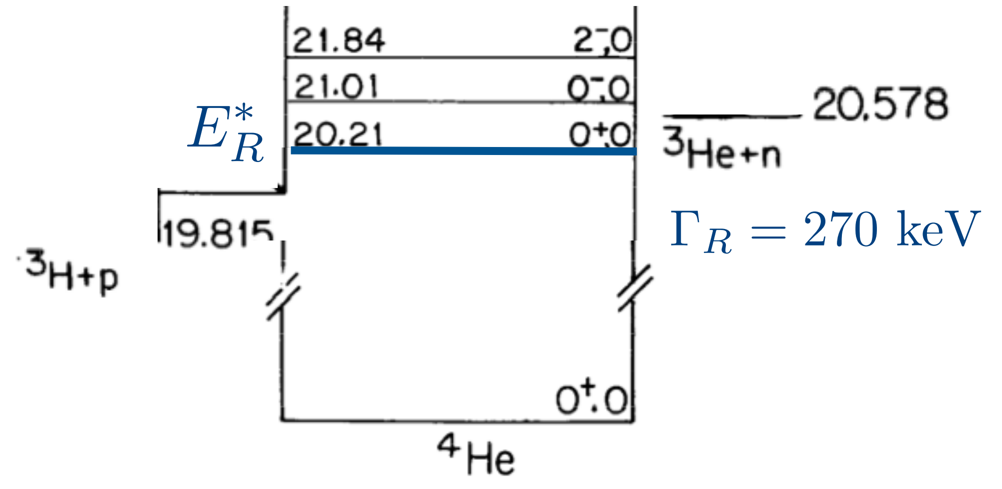
Monopole Resonance $^4He(e,e')0^+$



Resonant Transition Form Factor

$0_1^+ \longrightarrow 0_2^+$

$$|F_{\mathcal{M}}(q)|^2 = \frac{1}{Z^2} \int d\omega R_{\mathcal{M}}^{\rm res}(q,\omega)$$



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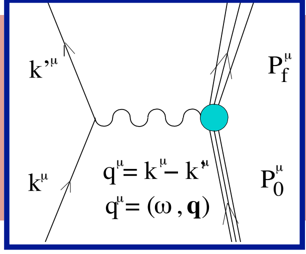

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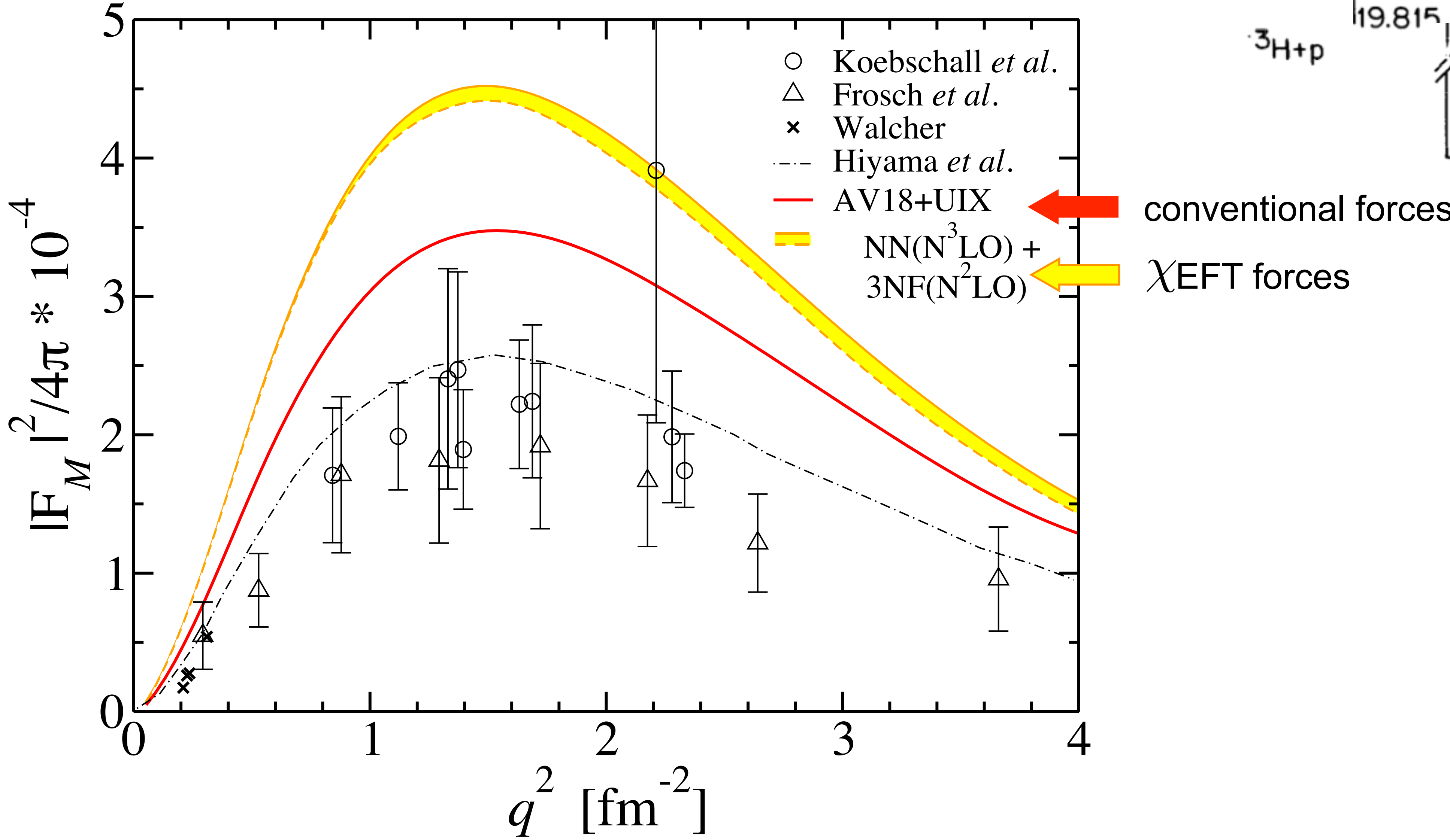
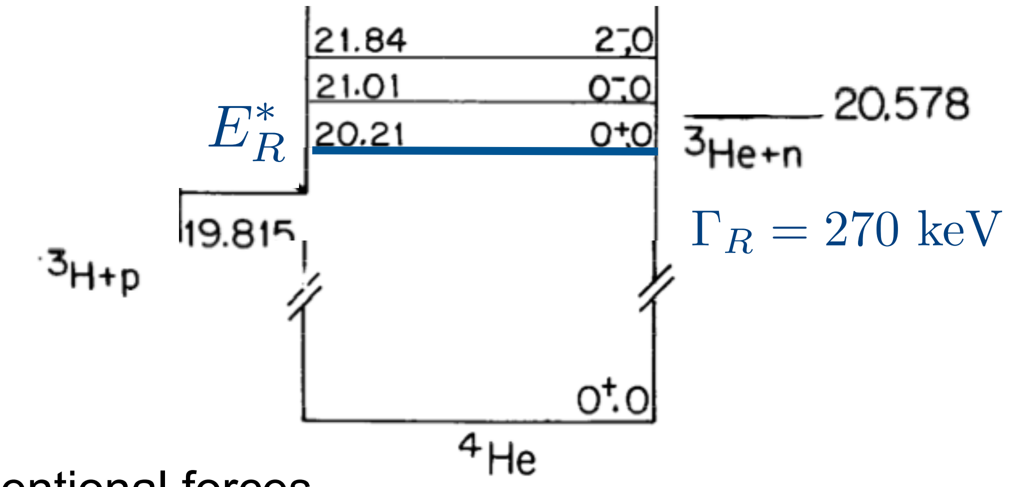
First ab-initio calculation with realistic three-nucleon forces

S.B. *et al.*, PRL **110**, 042503 (2013)





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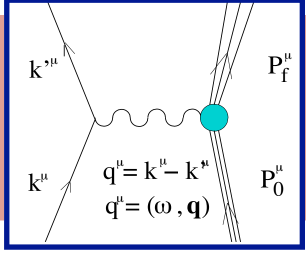

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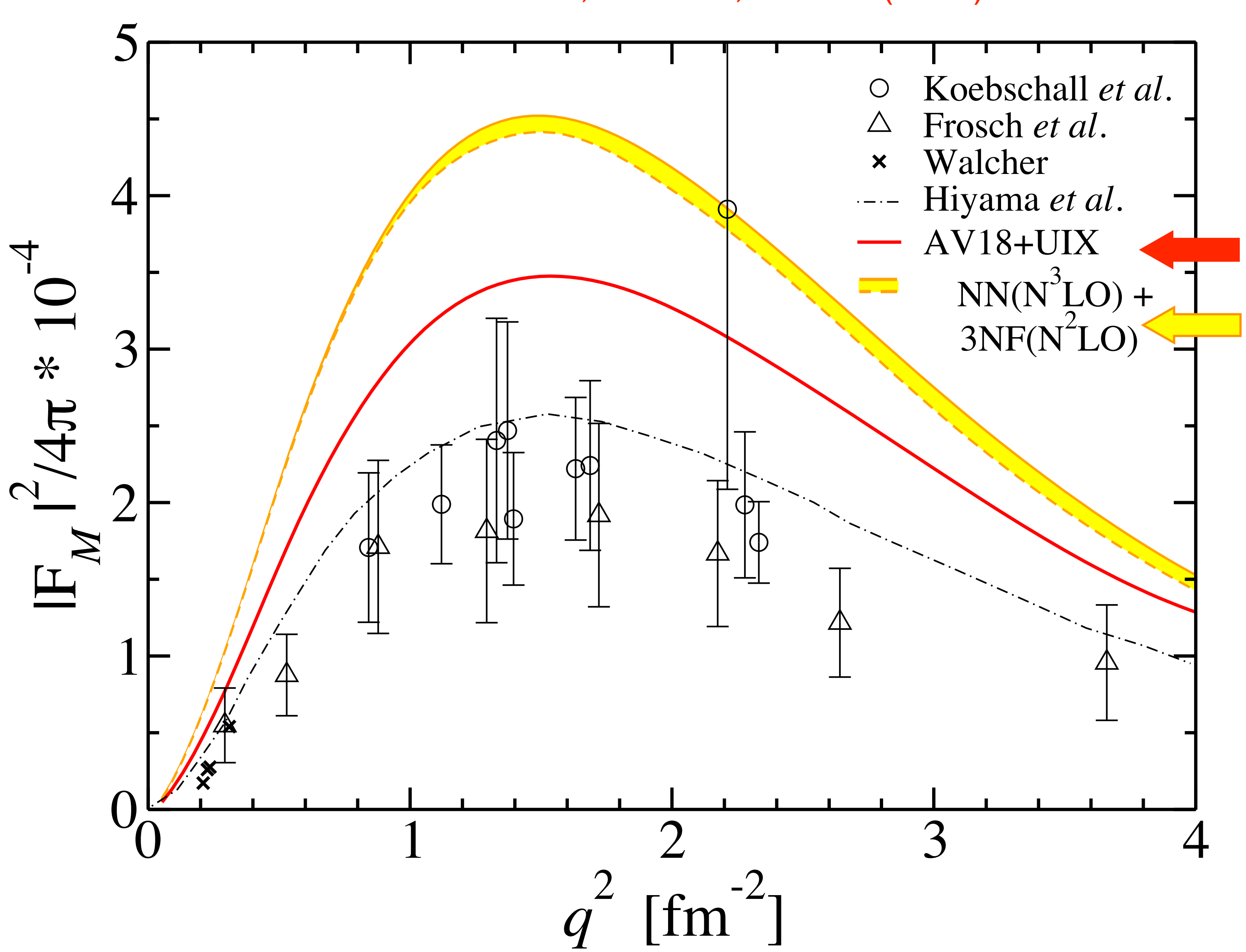
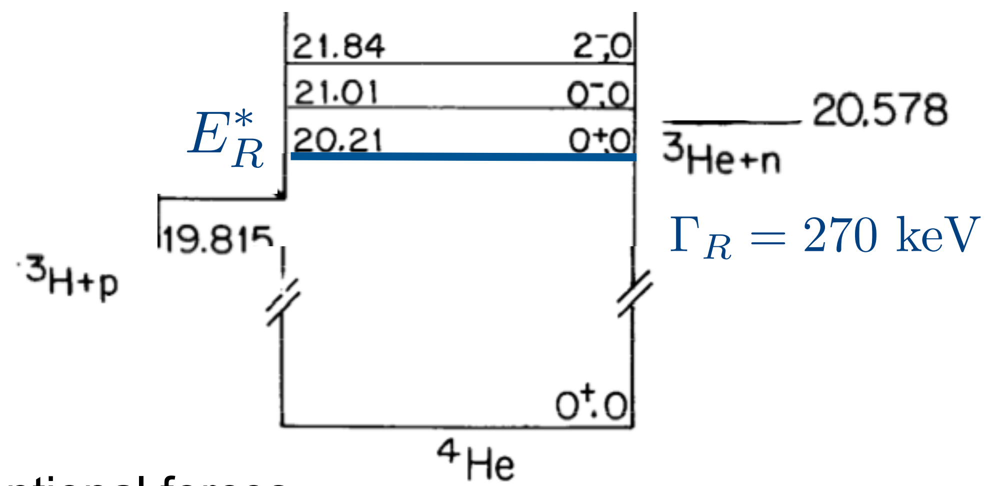


conventional forces

χEFT forces



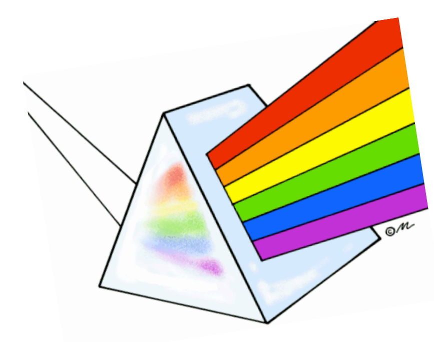
E_R^*

$\Gamma_R = 270$ keV

AV8' + central 3NF	E_0 = -28.44 MeV
AV18+UIX	E_0 = -28.40 MeV
NN(N^{3}LO)+3NF(N^{2}LO)	E_0 = -28.357 MeV

E_0^{exp}= -28.30 MeV

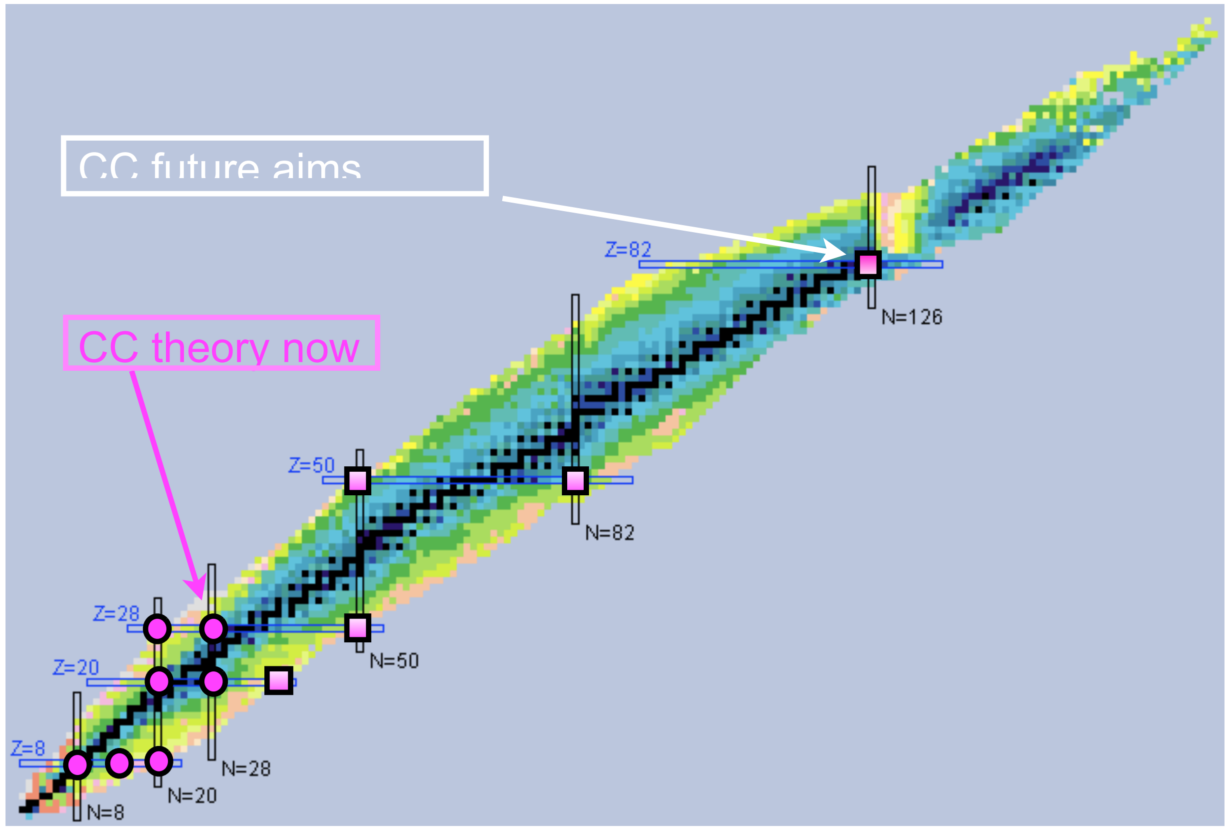
H reproduce $E_0^{\rm exp}$



Pushing the limits in mass number

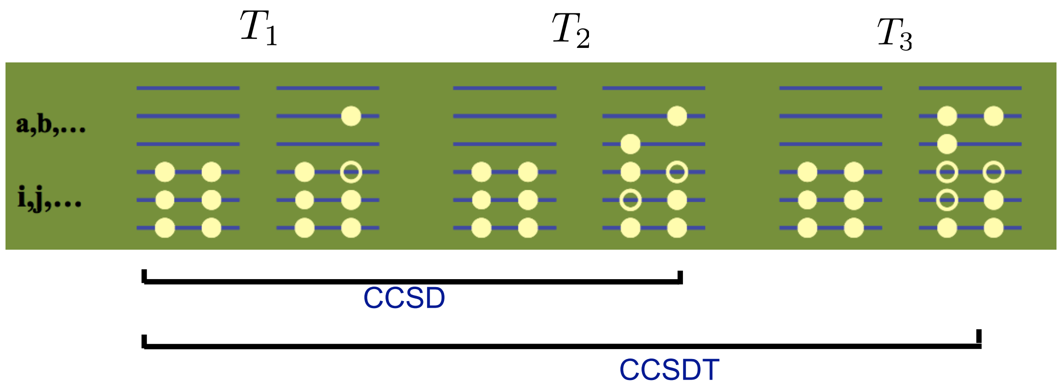
Coupled Cluster Theory

Many-body method that can extend the frontiers of ab-initio calculations to heavier and neutron nuclei



$$|\psi_0(\vec{r}_1,\vec{r}_2,...,\vec{r}_A)\rangle = e^T|\phi_0(\vec{r}_1,\vec{r}_2,...,\vec{r}_A)\rangle$$

$$T = \sum T_{(A)}$$ cluster expansion



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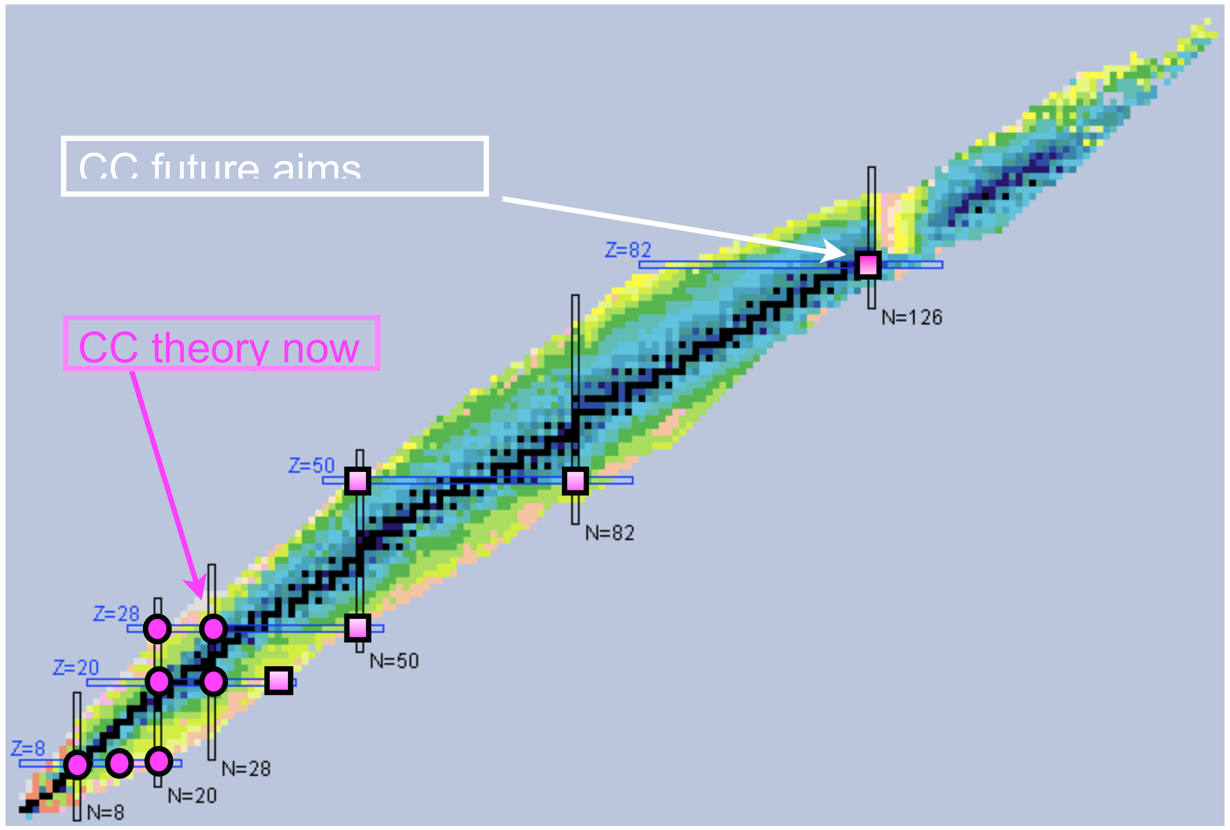

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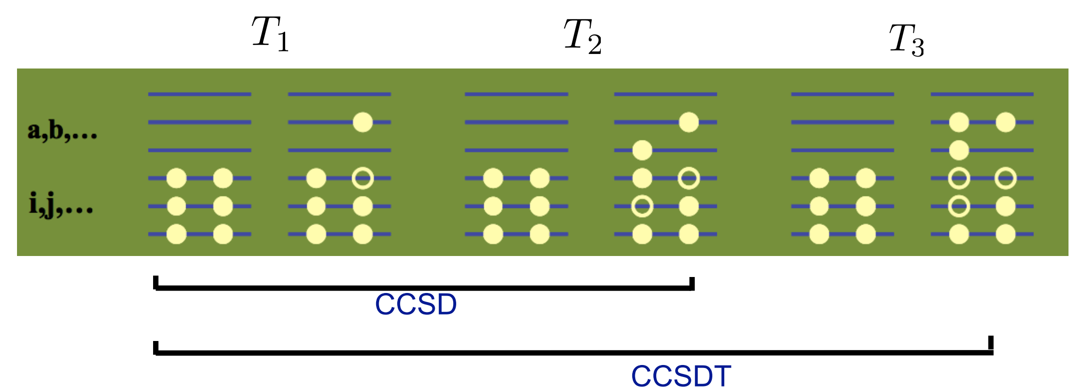

Mature theory for bound states, but what about electromagnetic reactions?

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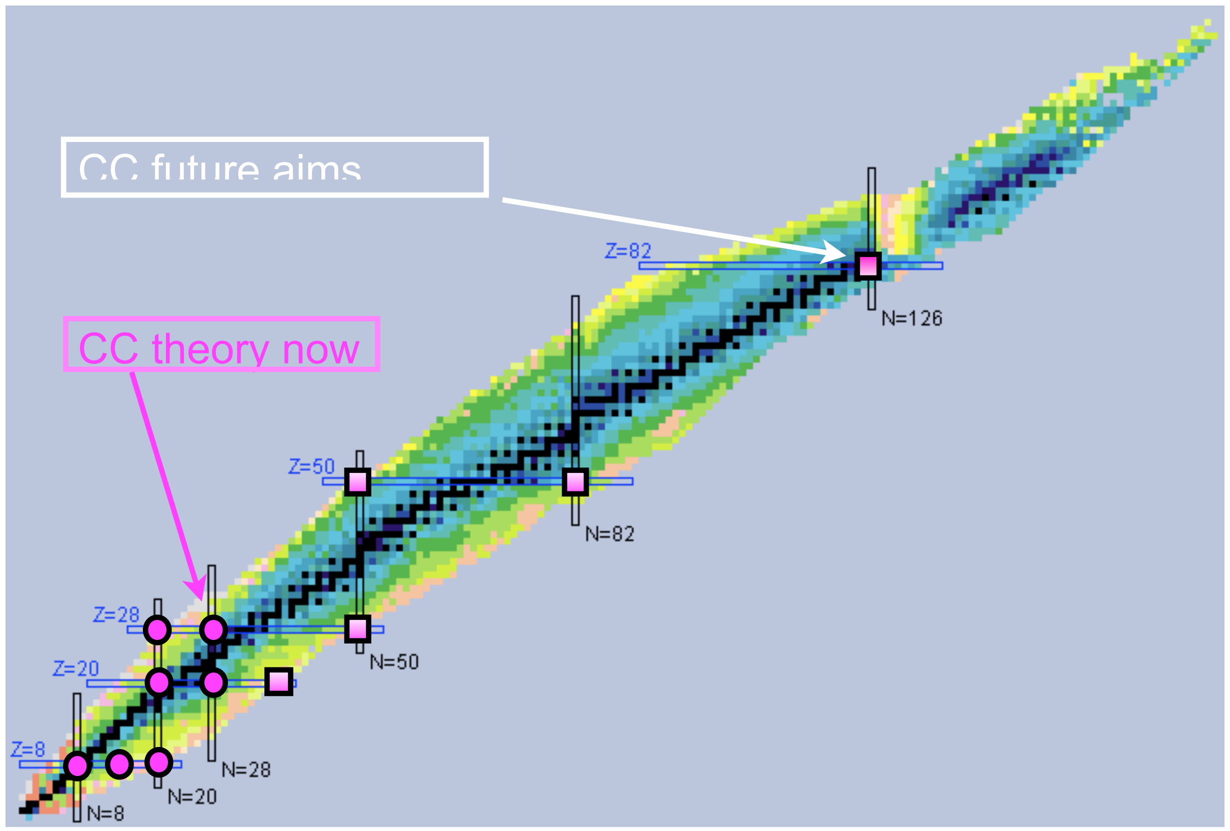

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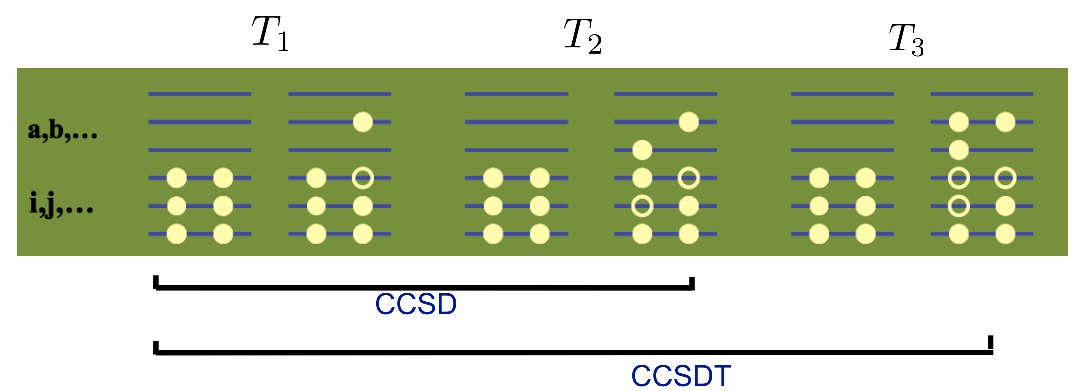


Mature theory for bound states, but what about electromagnetic reactions?

Merge Lorentz integral transform method with coupled-cluster theory

$$(\bar{H} - E_0 - \sigma + i\Gamma)|\tilde{\Psi}_R\rangle = \bar{\Theta}|\Phi_0\rangle$$

$$\bar{H} = e^{-T}He^T$$
$$\bar{\Theta} = e^{-T}\Theta e^T$$
$$|\tilde{\Psi}_R\rangle = \hat{R}|\Phi_0\rangle$$

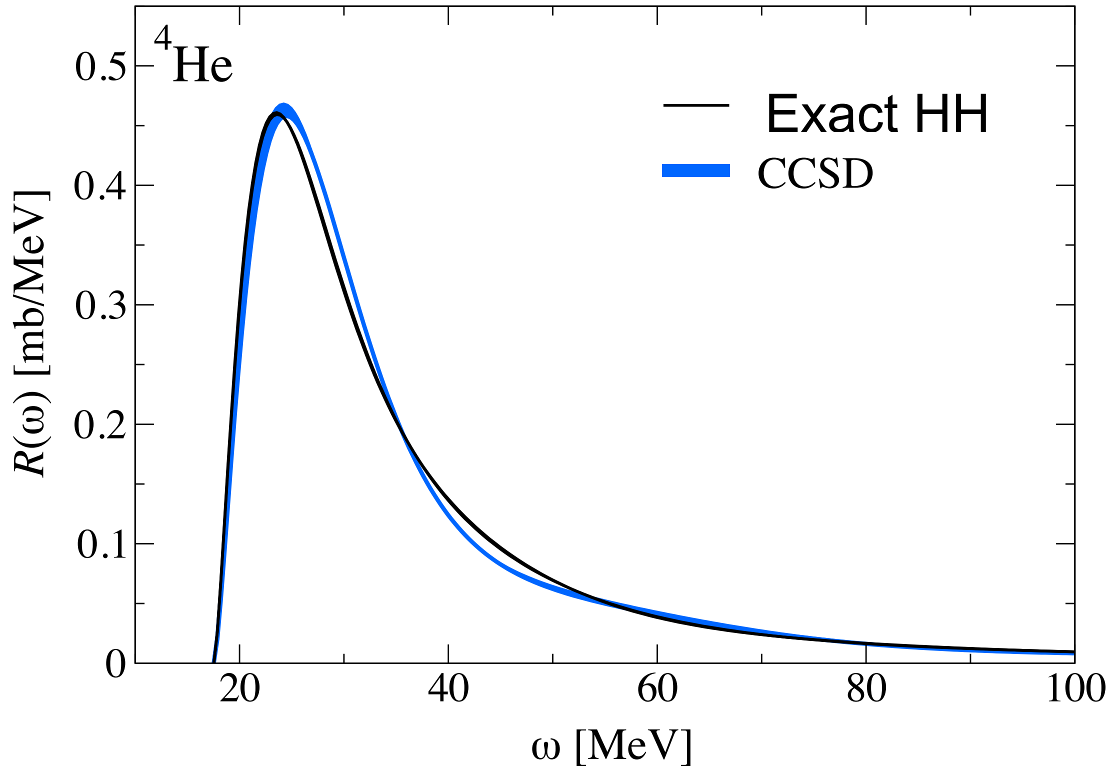
I will show mostly results at CCSD truncation scheme

Photoexcitation of stable nuclei

S.B. *et al.*, Phys. Rev. Lett. **111**, 122502 (2013)

Dipole Response Functions with NN forces from χEFT (N^3LO)

Validation ^4He



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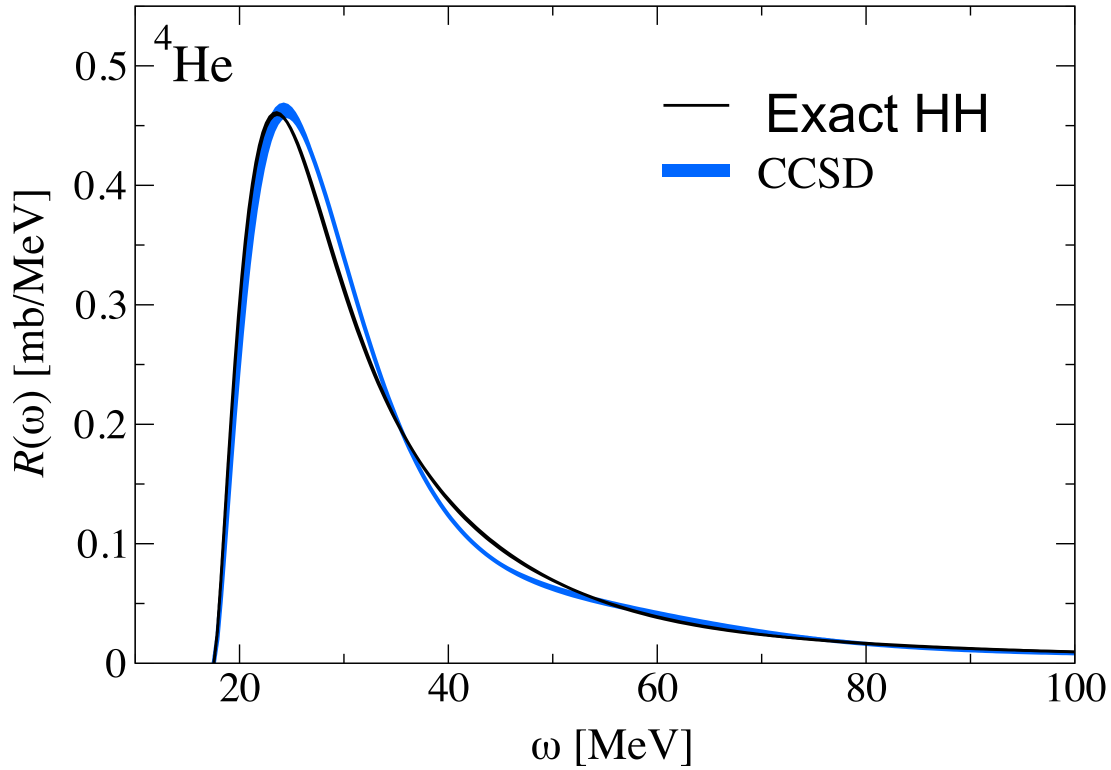
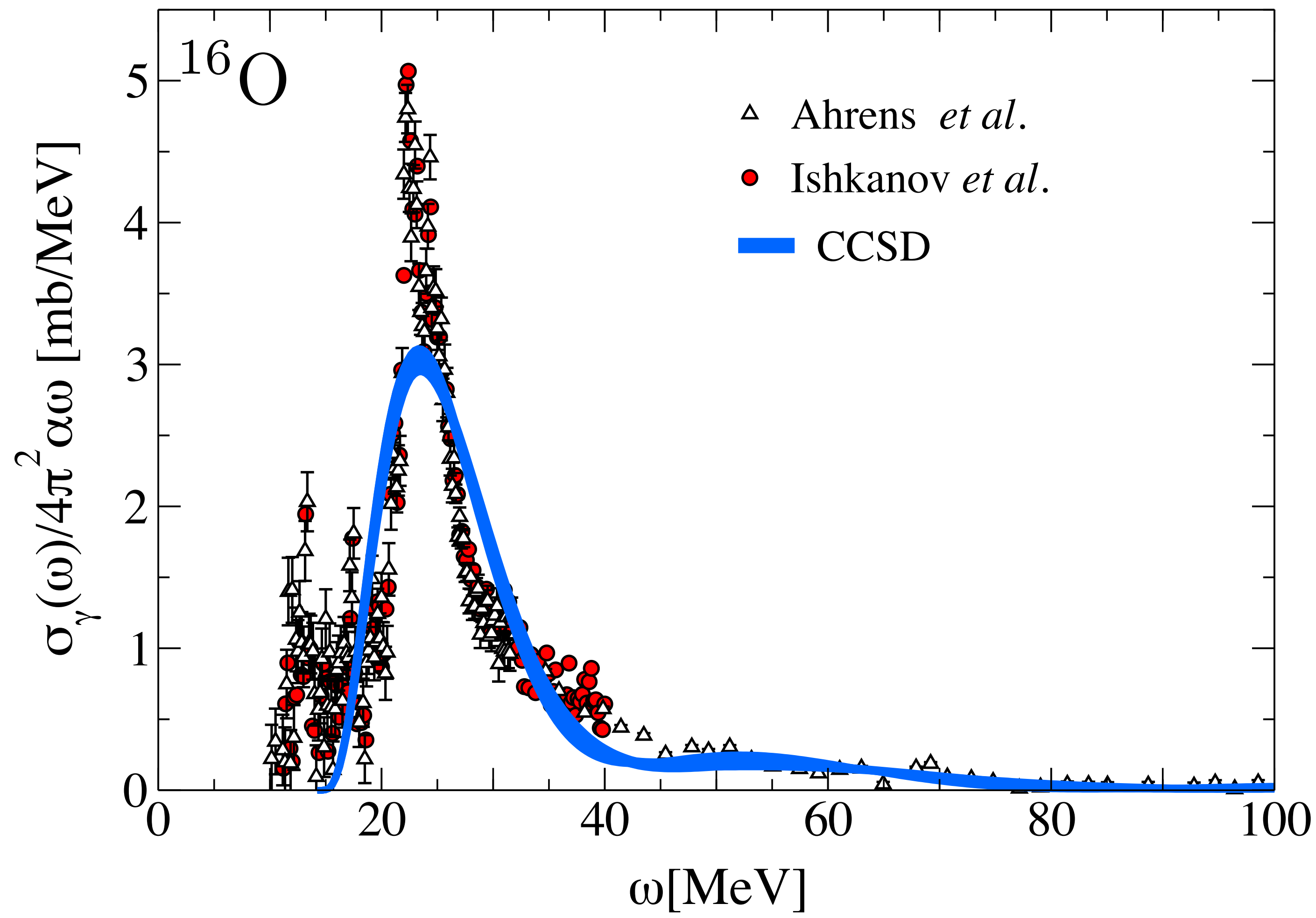
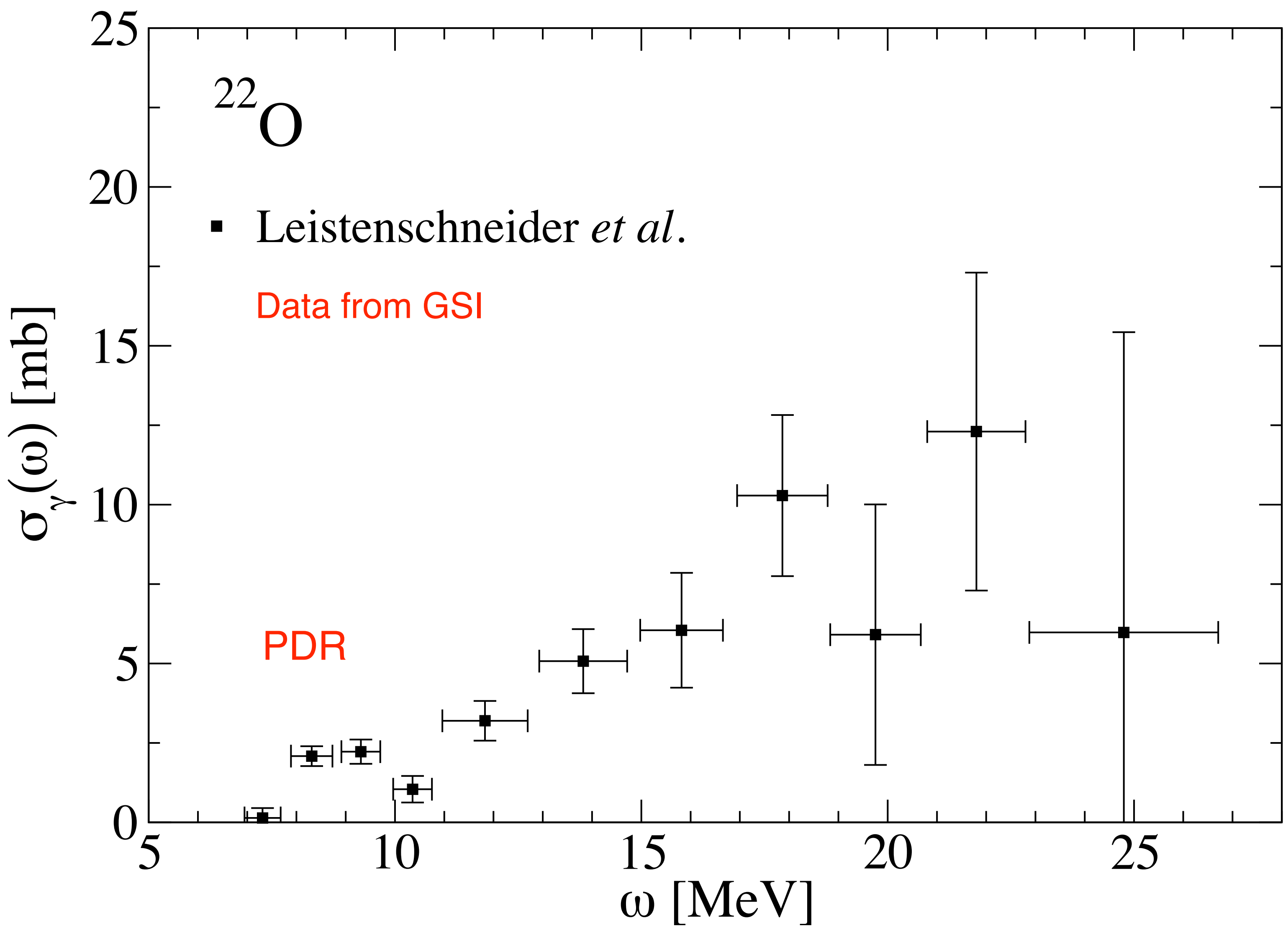
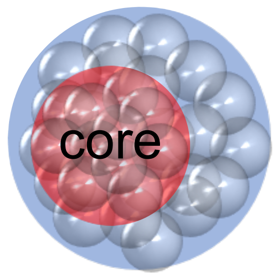


Extension to ^{16}O



Photoexcitation of neutron-rich nuclei

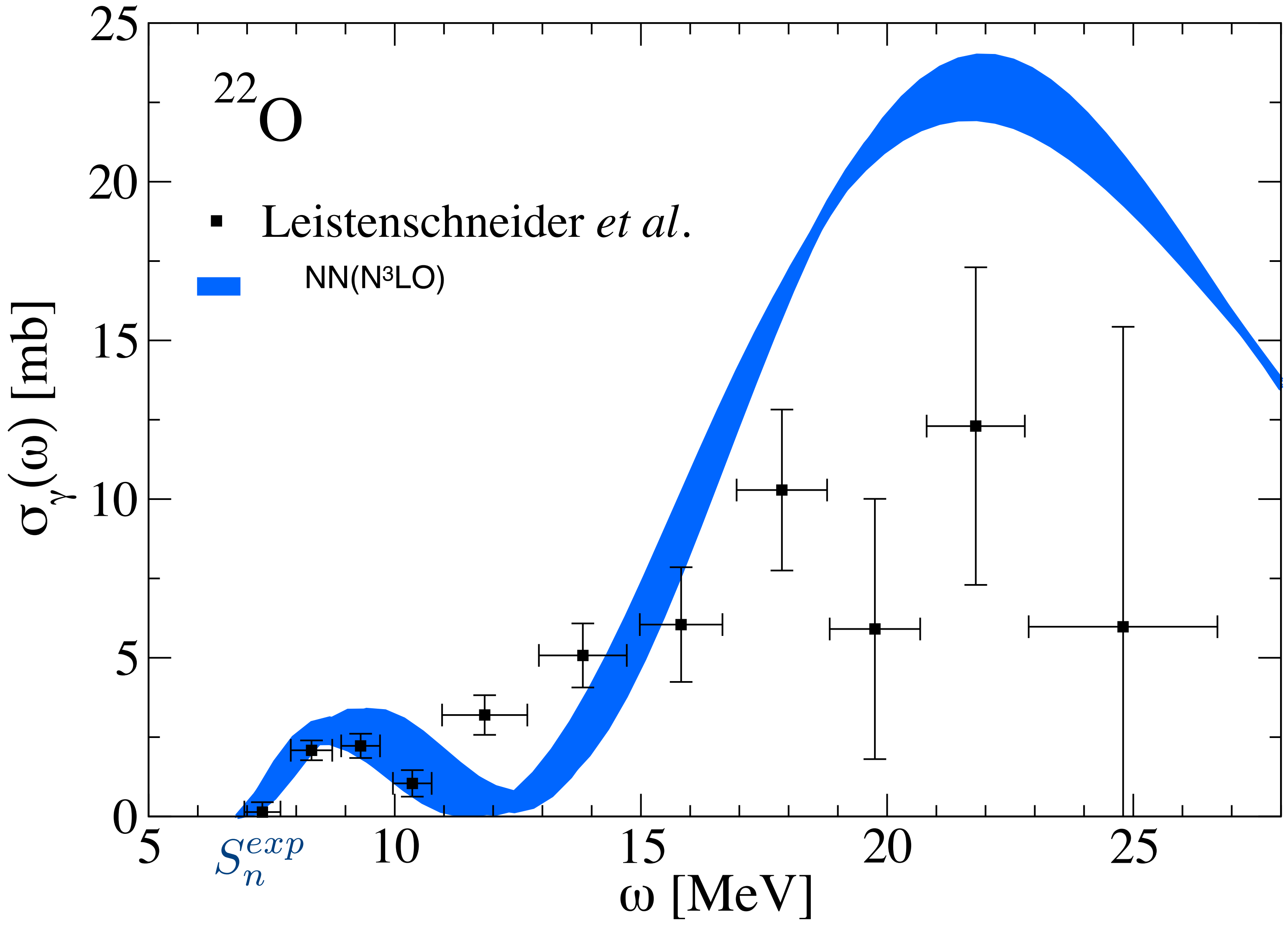


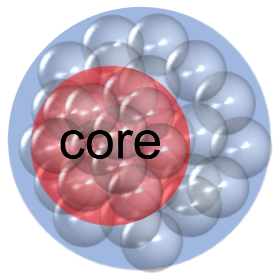


Pigmy Dipole Resonance (PDR)

Photoexcitation of neutron-rich nuclei

S.B. *et al.*, PRC **90**, 064619 (2014)



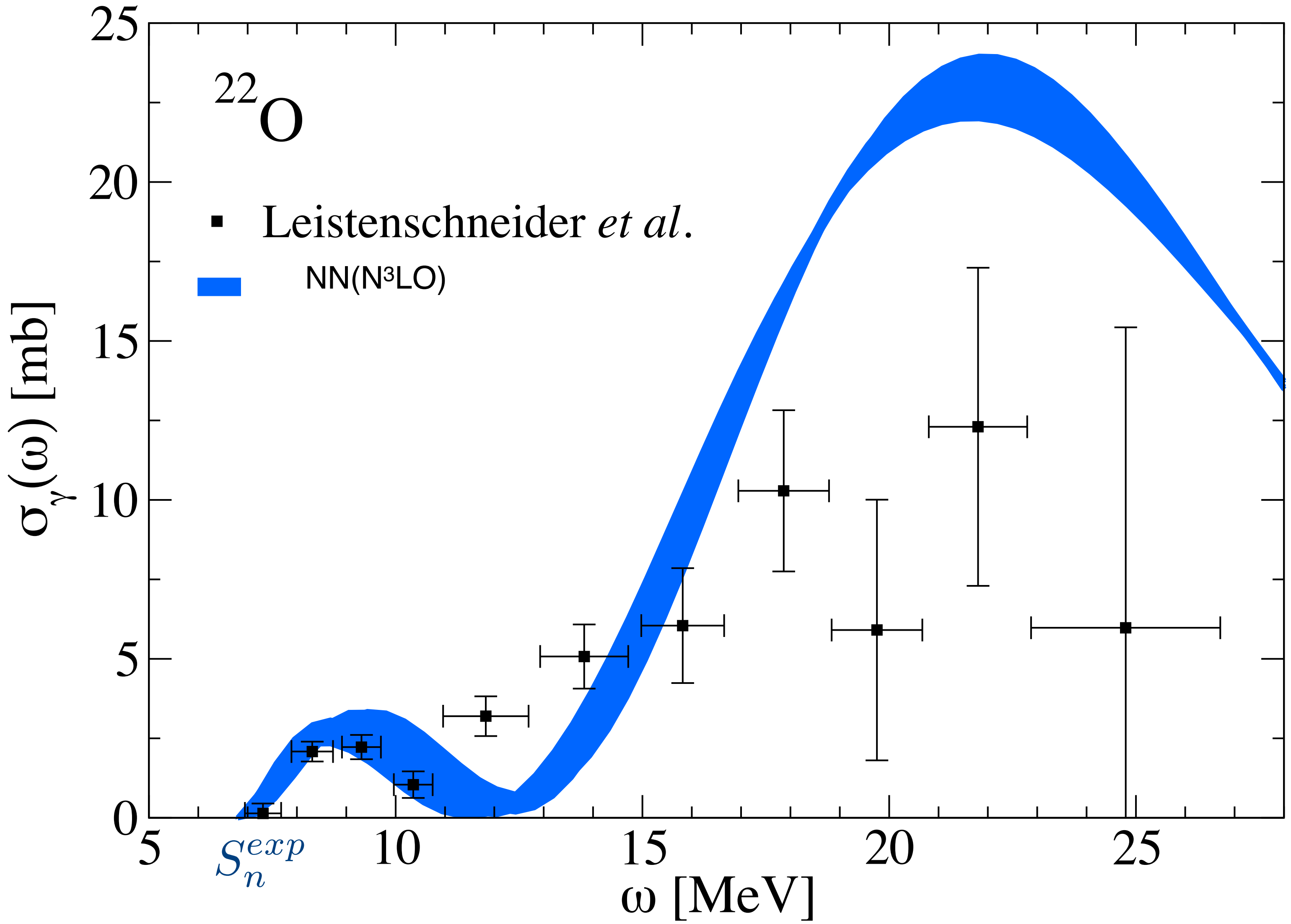


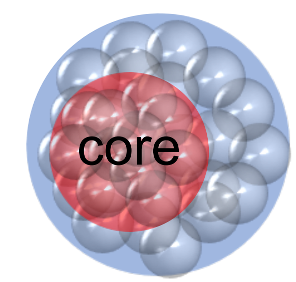
Pigmy Dipole Resonance (PDR)

Nicely described by a first principle calculation

Photoexcitation of neutron-rich nuclei

S.B. *et al.*, PRC **90**, 064619 (2014)





Pigmy Dipole Resonance (PDR)

Nicely described by a first principle calculation

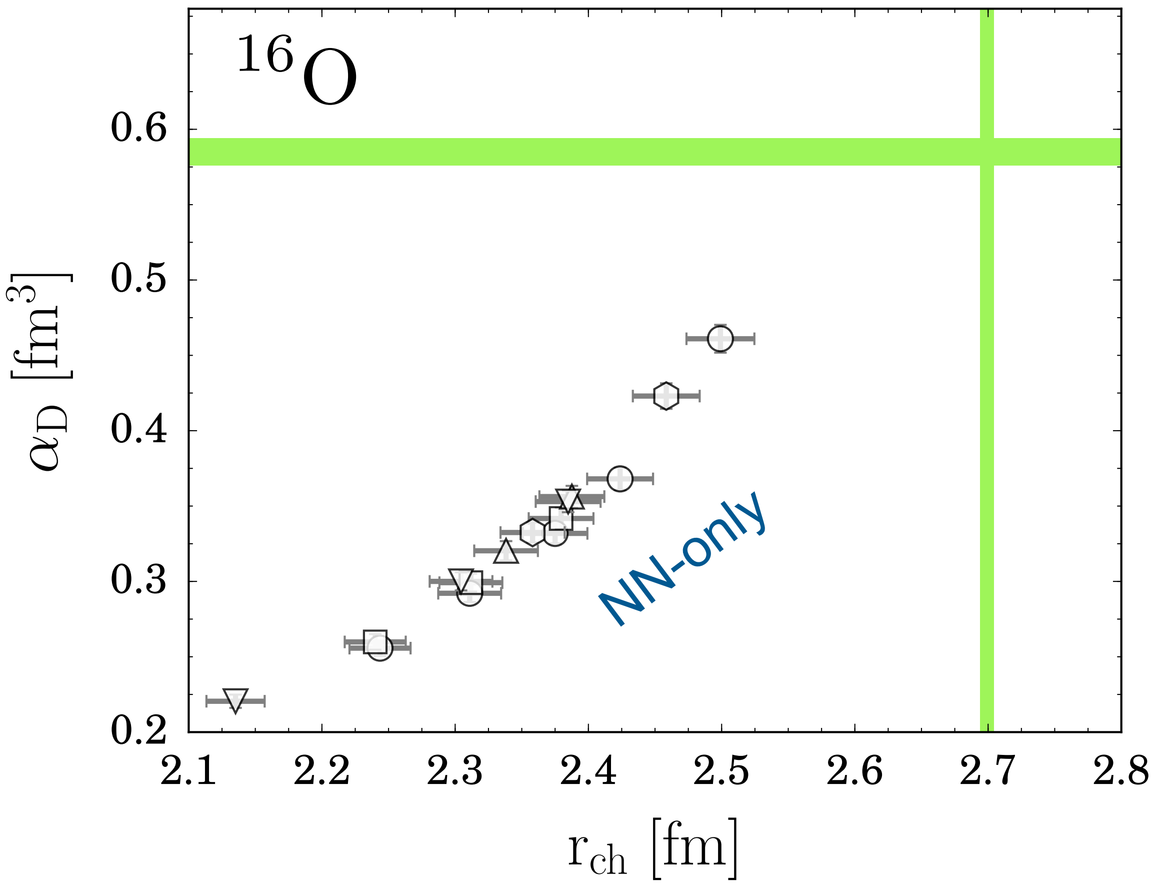
Theory provides a deeper understanding: microscopic interpretation of collective phenomena

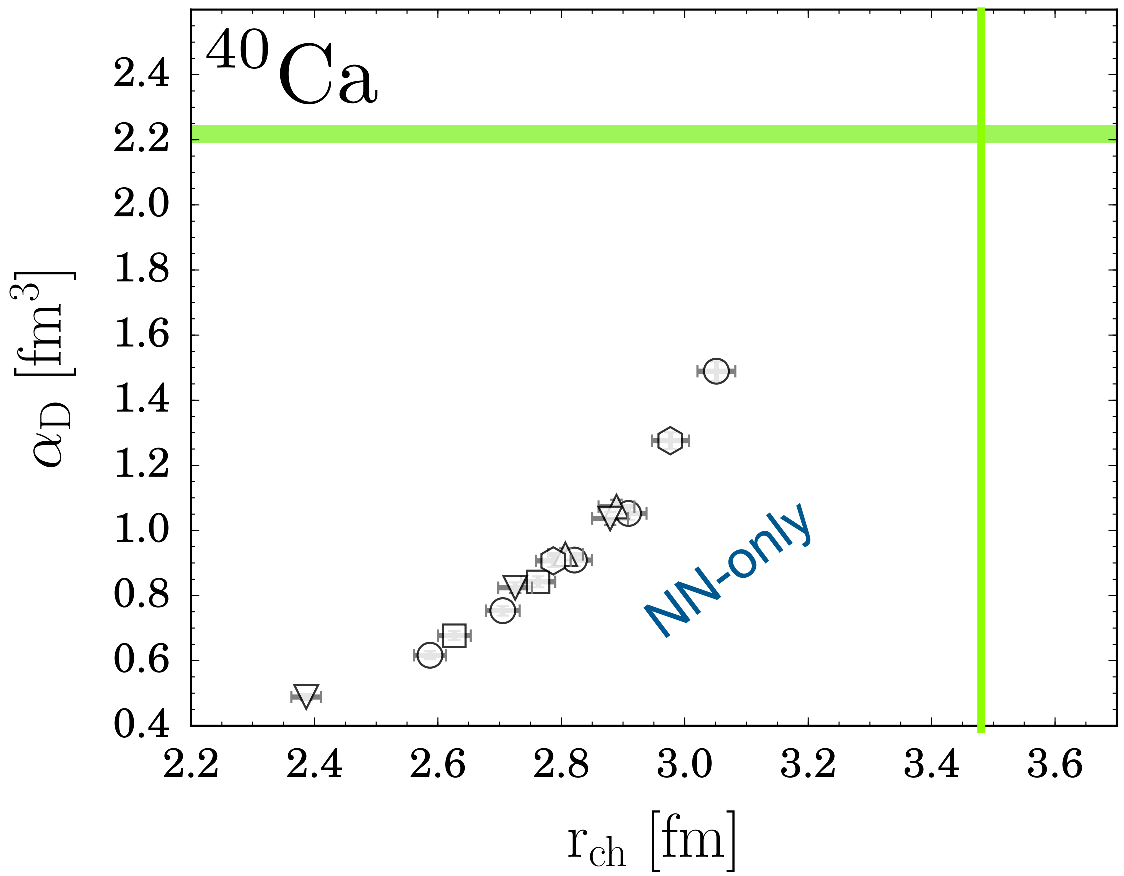
Theory motivates new experiments: e.g. ^{8}He will be measured in RIKEN by T. Aumann

Electric Dipole Polarizability

Medium-mass nuclei with NN + 3NF interactions

M. Miorelli *et al.*, PRC **94** 034317 (2016)





Electric Dipole Polarizability

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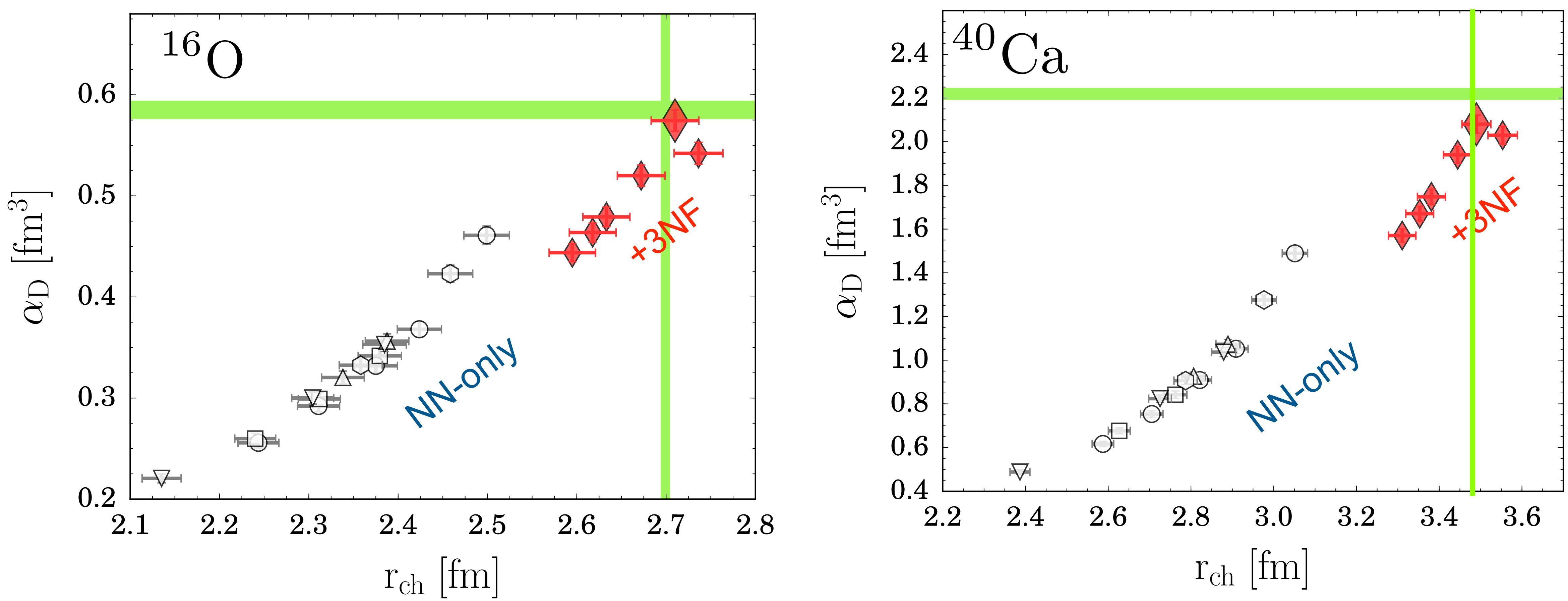

3NF A. Ekström *et al.*, Phys. Rev. C91, 051301 (2015)
K. Hebeler *et al.*, Phys. Rev. C83, 031301 (2011)

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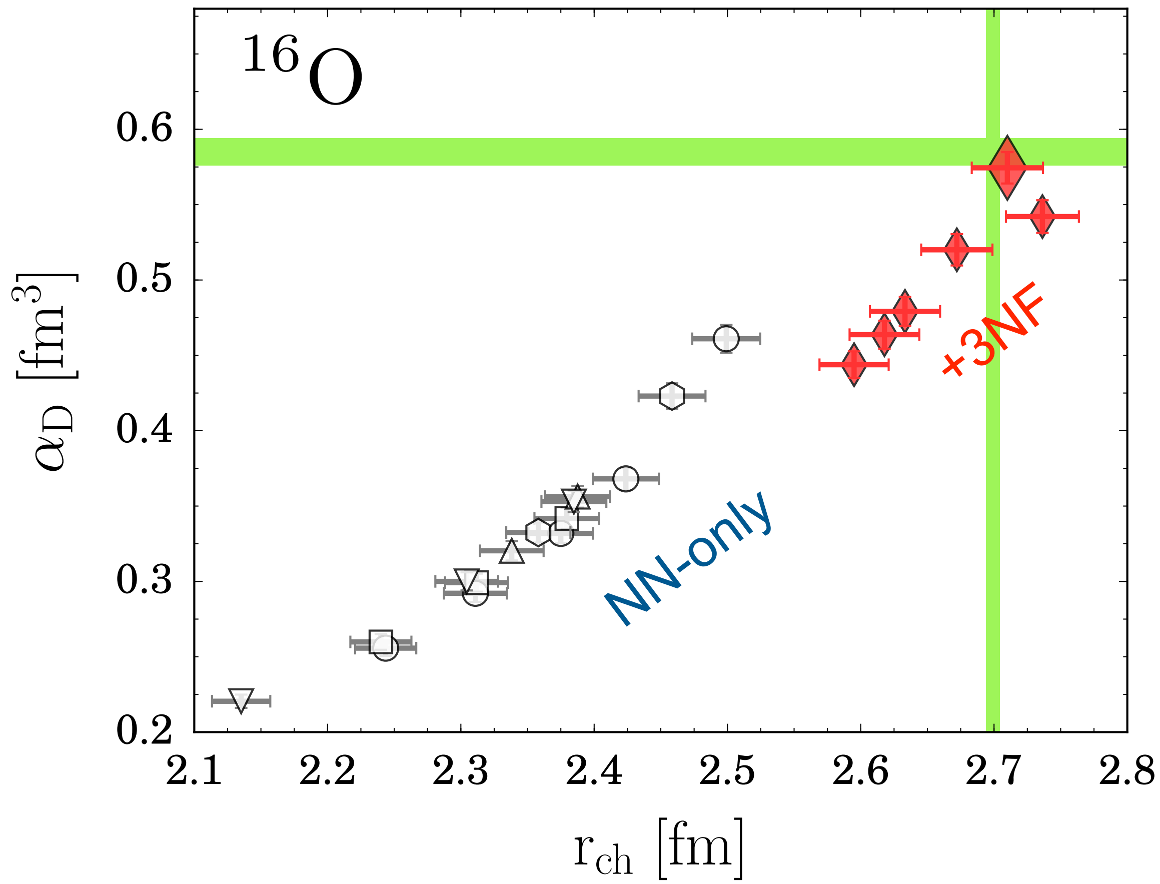
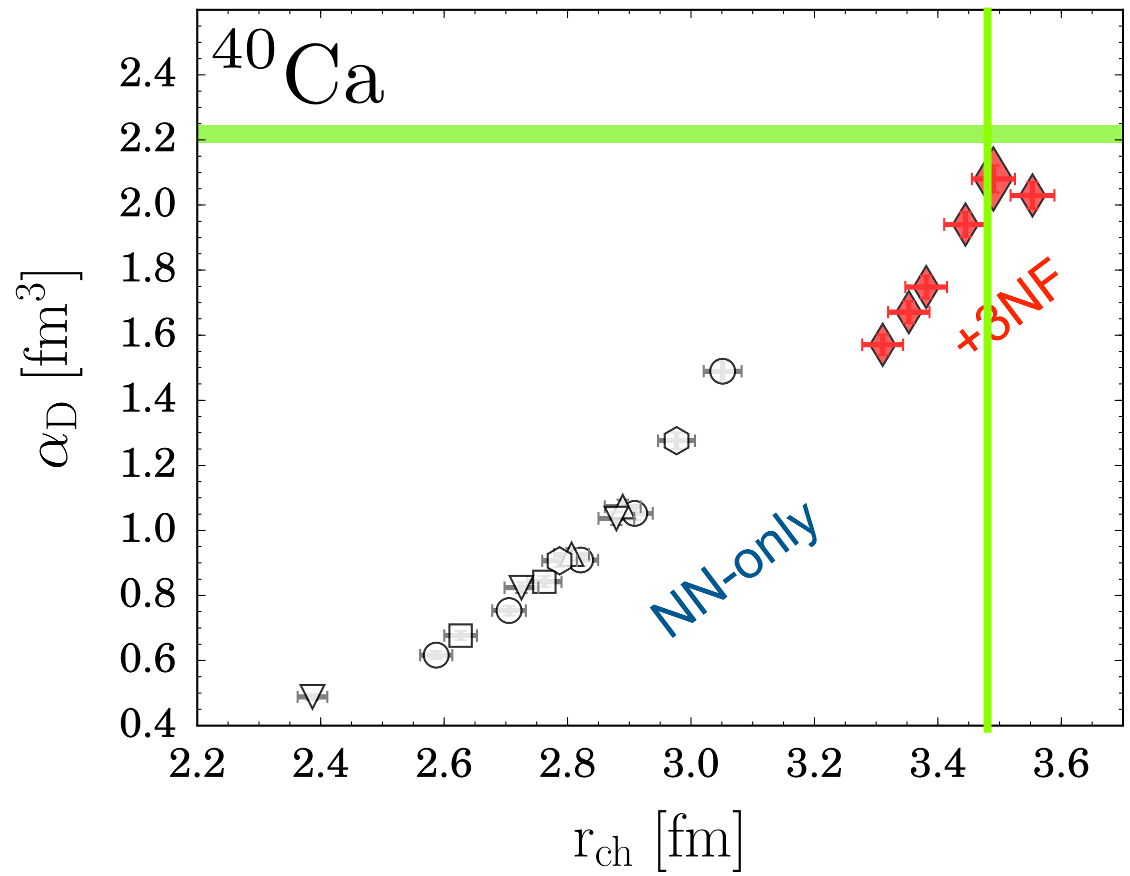


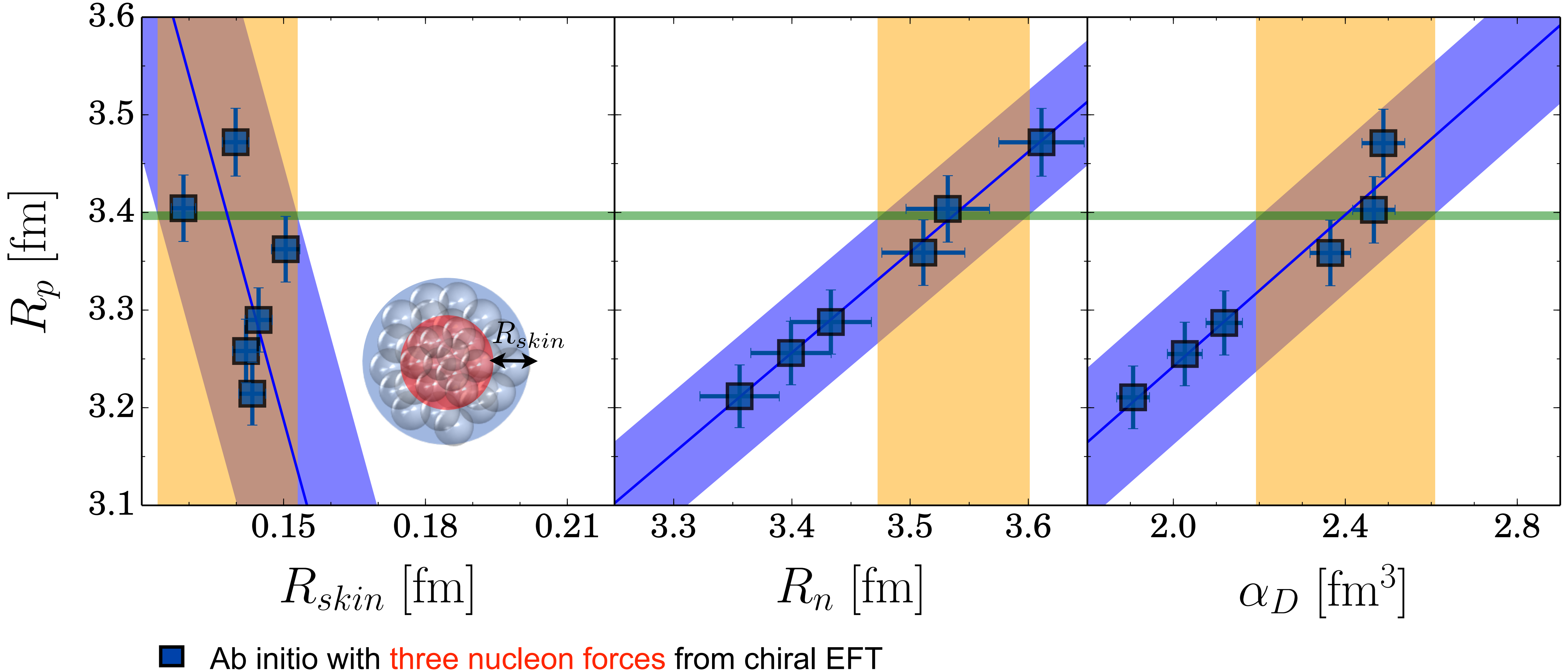


3NF
A. Ekström *et al.*, Phys. Rev. C91, 051301 (2015)
K. Hebeler *et al.*, Phys. Rev. C83, 031301 (2011)

Much better agreement with experimental data
Variation of Hamiltonian can be used to assess the theoretical error bar

^{48}Ca from first principles

International collaboration (USA/Canada/Europe/Israel) using coupled-cluster theory
Hagen *et al.*, Nature Physics **12**, 186 (2016)



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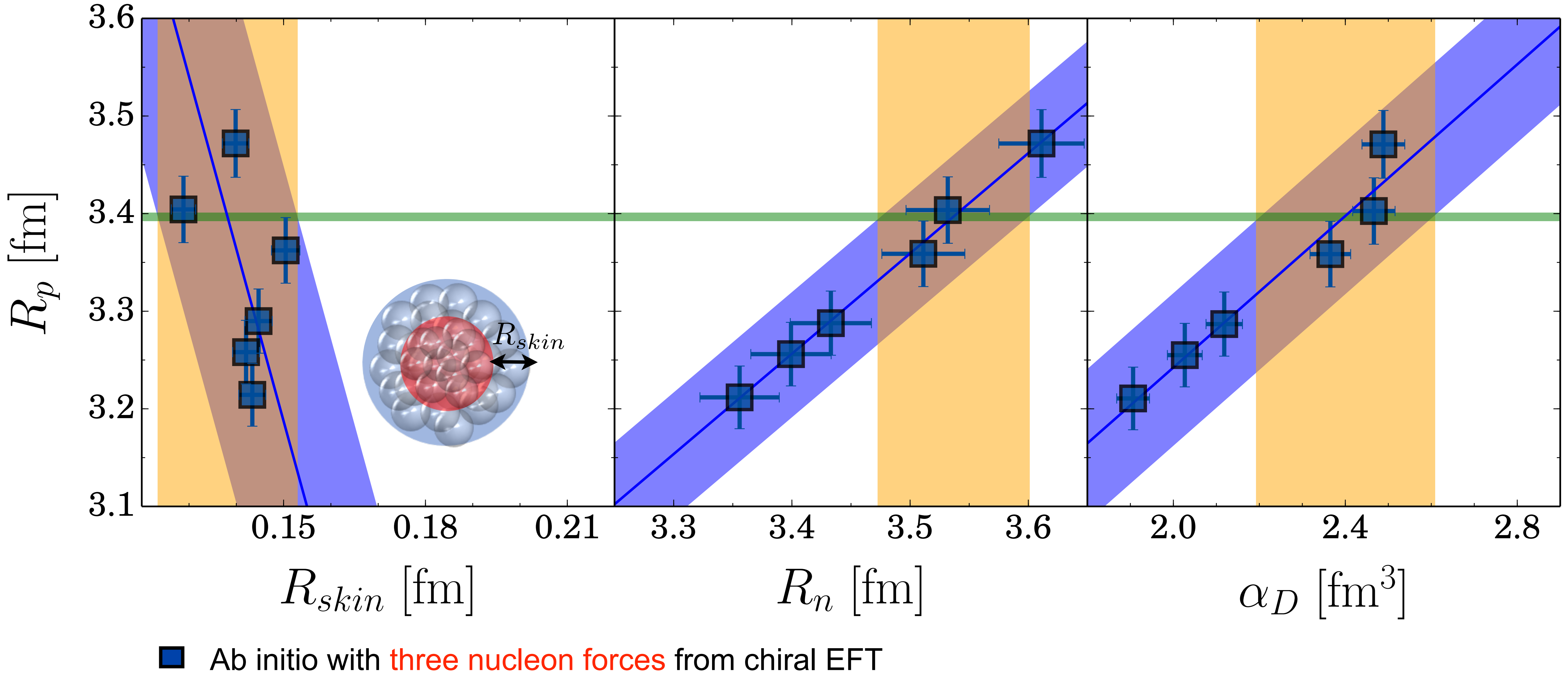

Strong correlations with R_p allow to put narrow constraints to R_{skin} and α_D

Ab-initio predictions:

$$0.12 \leq R_{\text{skin}} \leq 0.15 \text{ fm}$$

$$2.19 \leq \alpha_D \leq 2.60 \text{ fm}^3$$

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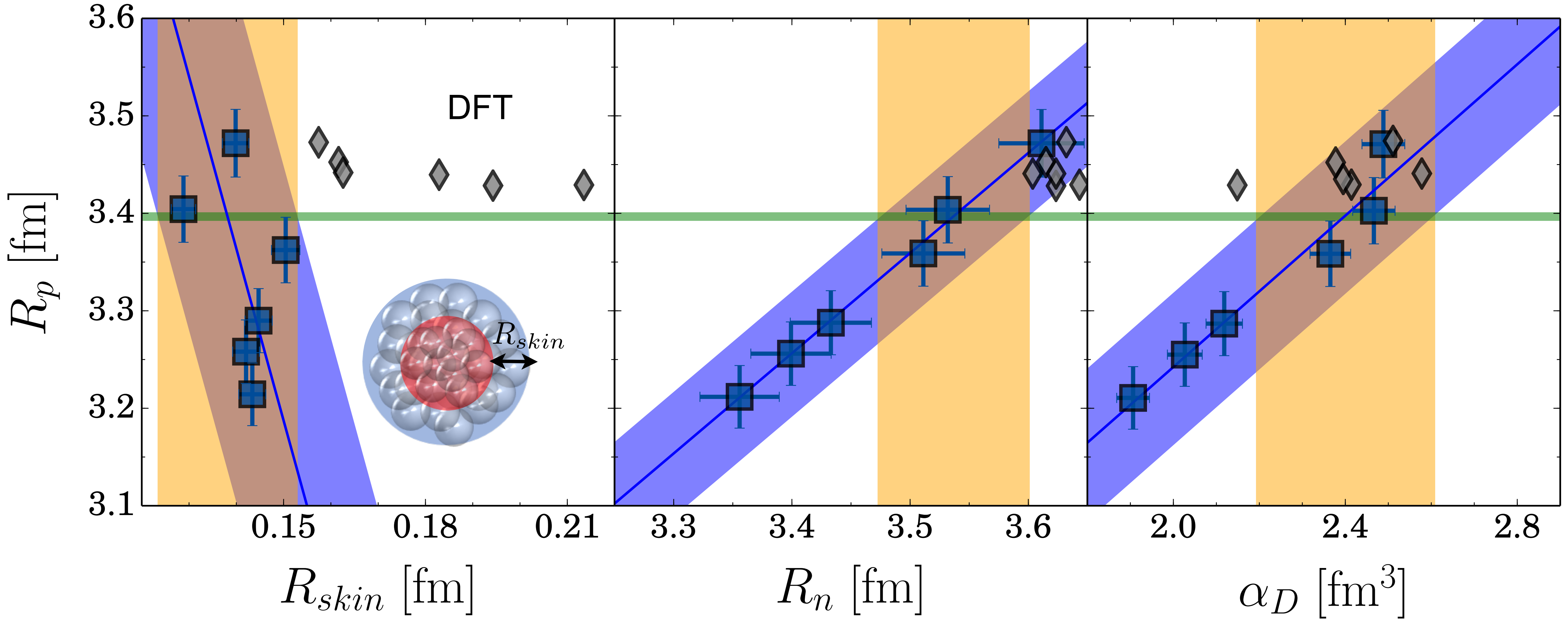

■ Ab initio with three nucleon forces from chiral EFT

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JG|U

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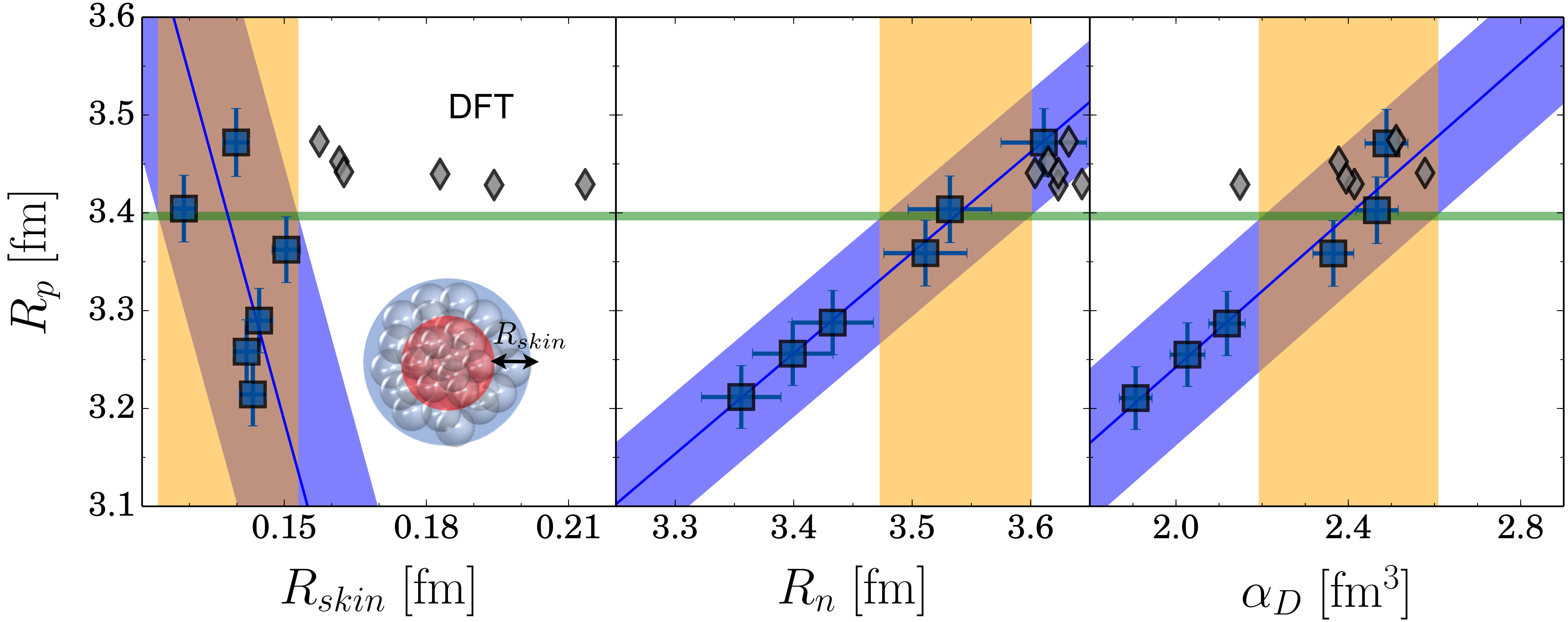

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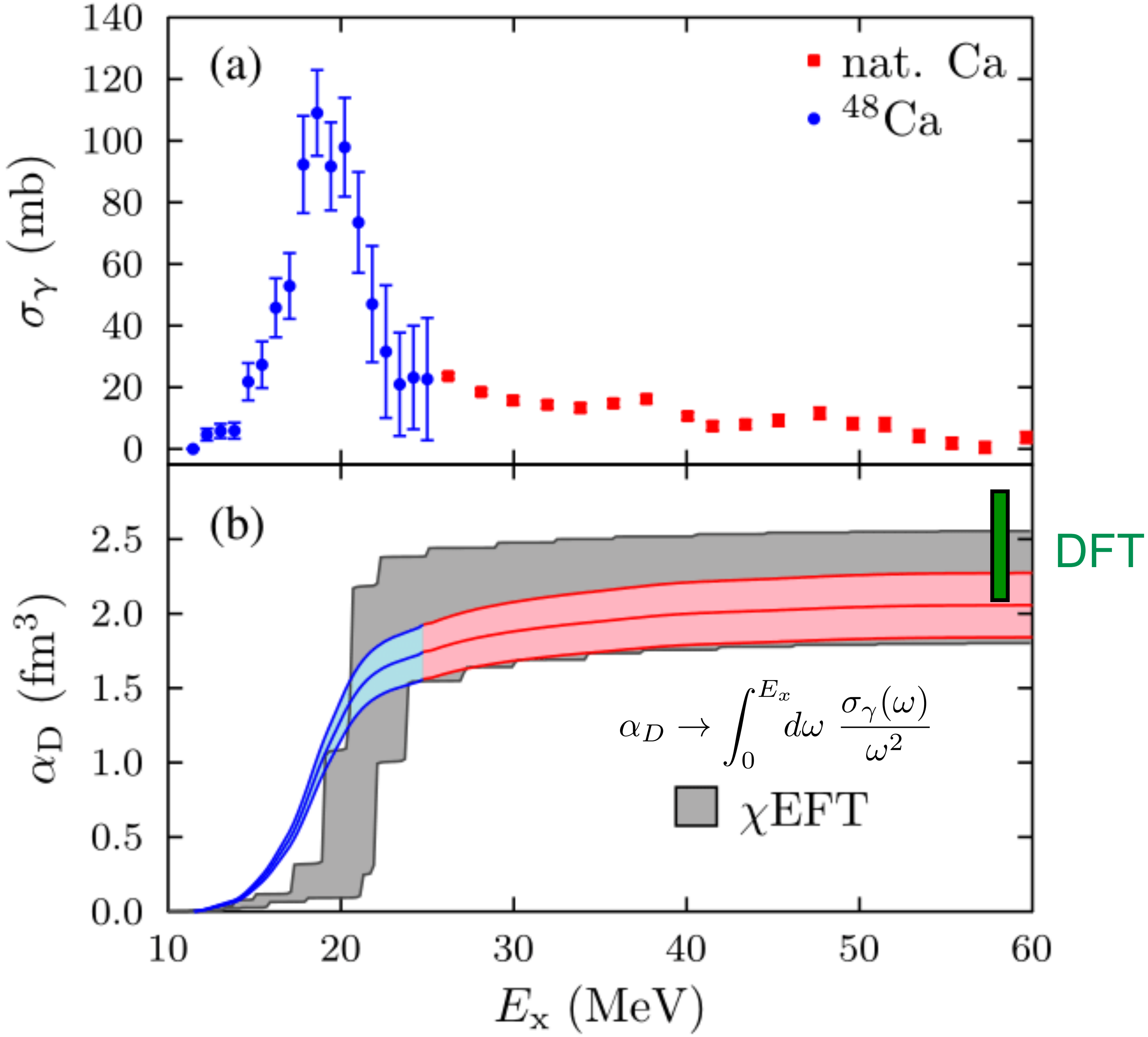
R_{skin} will be measured with Parity violating electron scattering CREX MREX

^{48}Ca electric dipole polarizability

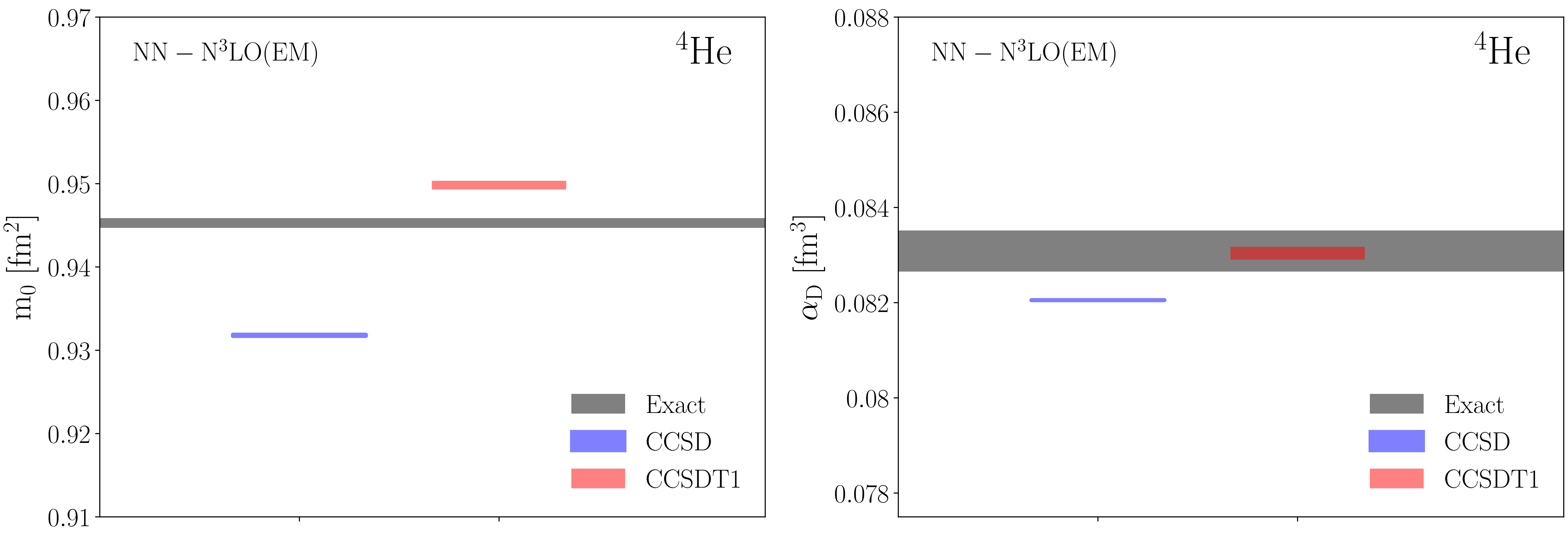
New measurements from the Osaka-Darmstadt collaboration using inelastic proton scattering

J.Birkhan, *et al.*, Phys. Rev. Lett. **118,** 252501 (2017)



How to improve our calculations

M. Miorelli *et al.*, in preparation (2017)



CCSD scheme

$$e^T = e^{T_1+T_2}$$

$$R = R_0 + R_1 + R_2$$

CCSDT1 scheme (linearized triples)

$$e^T = e^{T_1+T_2} + T_3$$

$$R = R_0 + R_1 + R_2 + R_3$$

Exact ⇒ hyperspherical harmonics, all correlations included (up to quadruples)

How to improve our calculations

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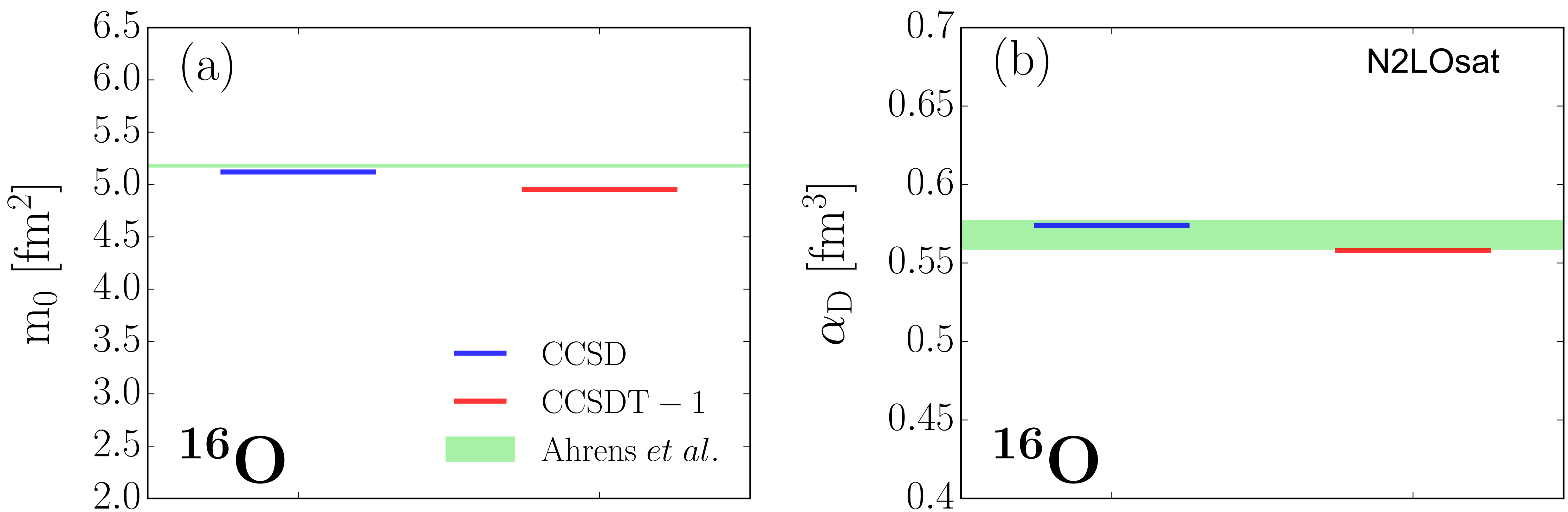

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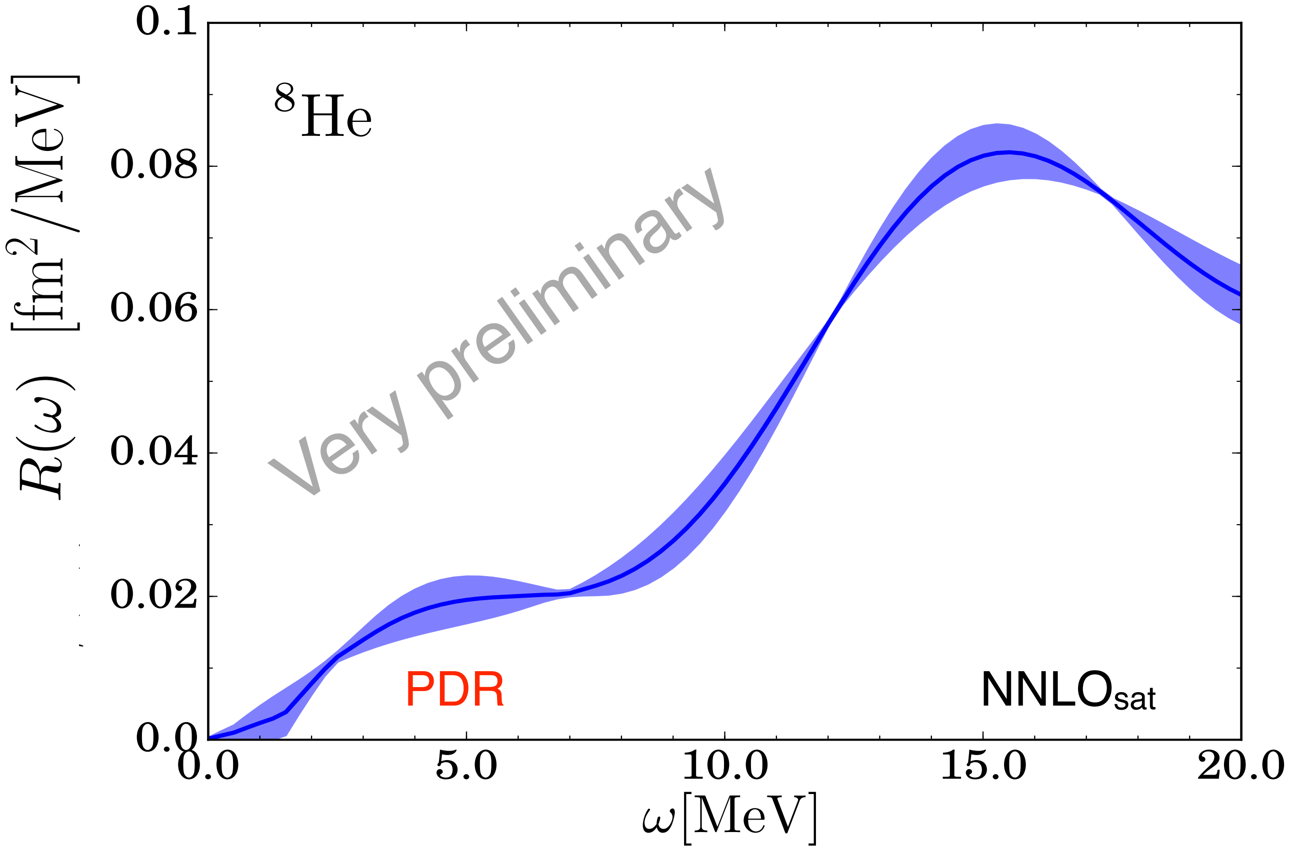
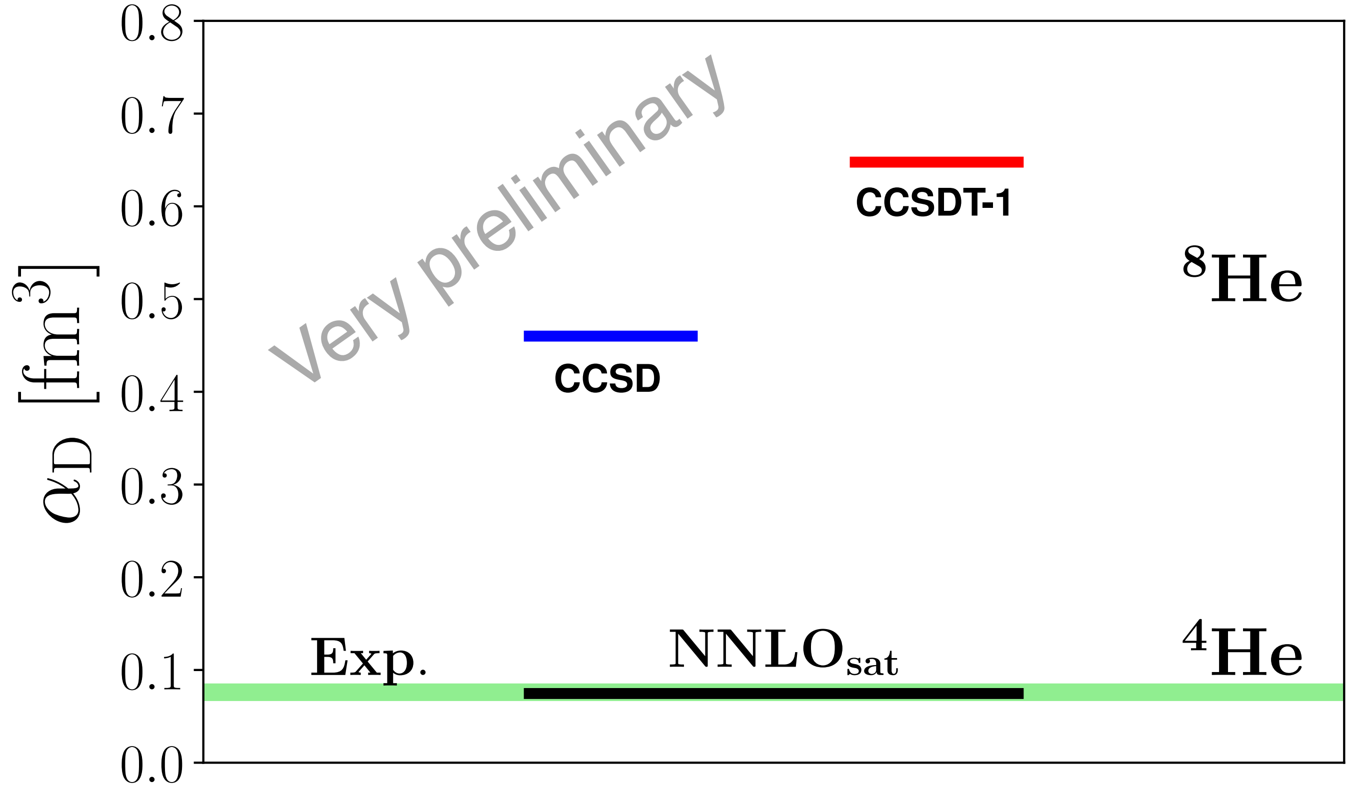
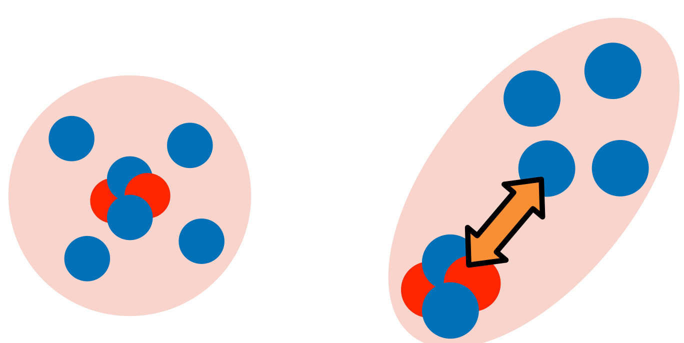
Applications to ^{8}He







$$\alpha_D = 2\alpha \int_{\omega_{th}}^{\infty} d\omega \frac{R(\omega)}{\omega}$$

$$\alpha_D(^8\text{He}) >> \alpha_D(^4\text{He})$$

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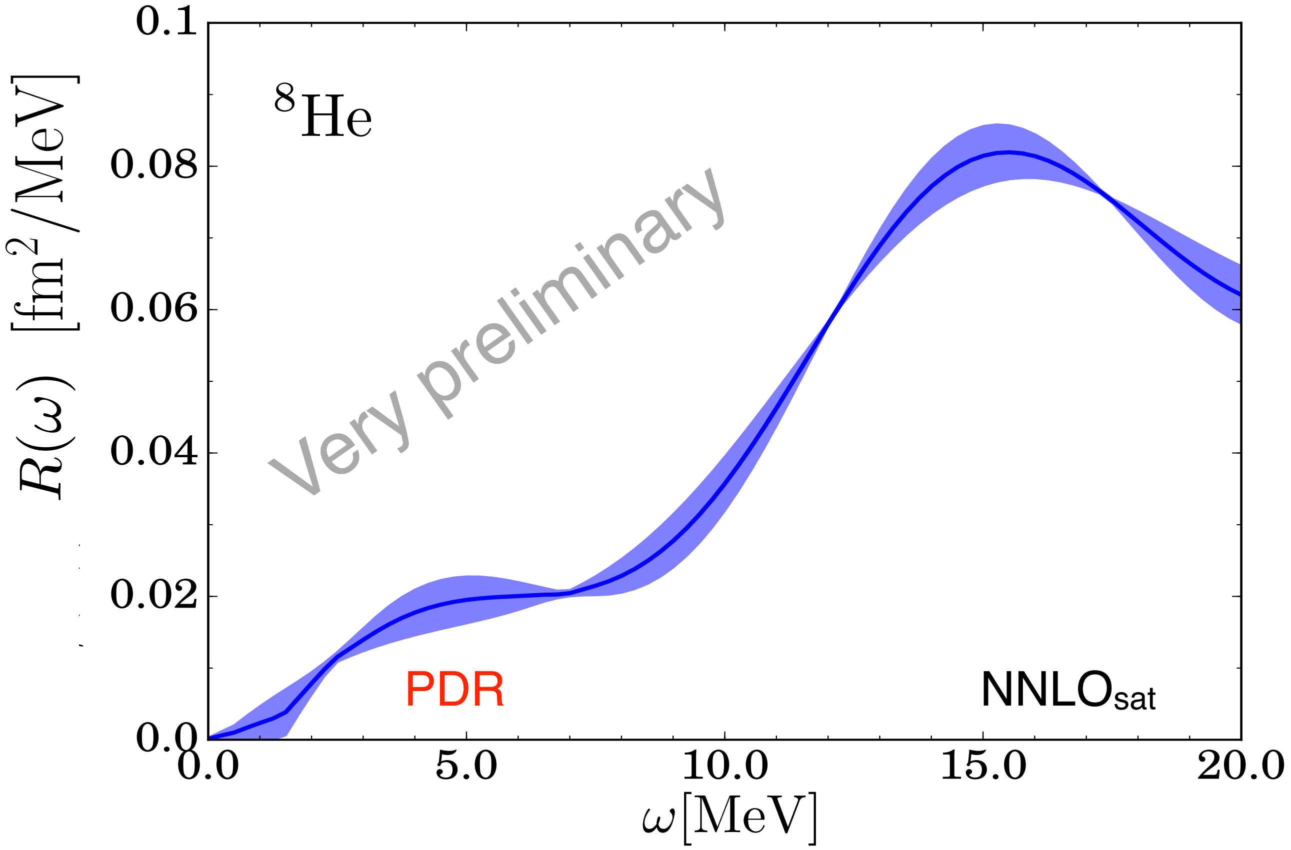

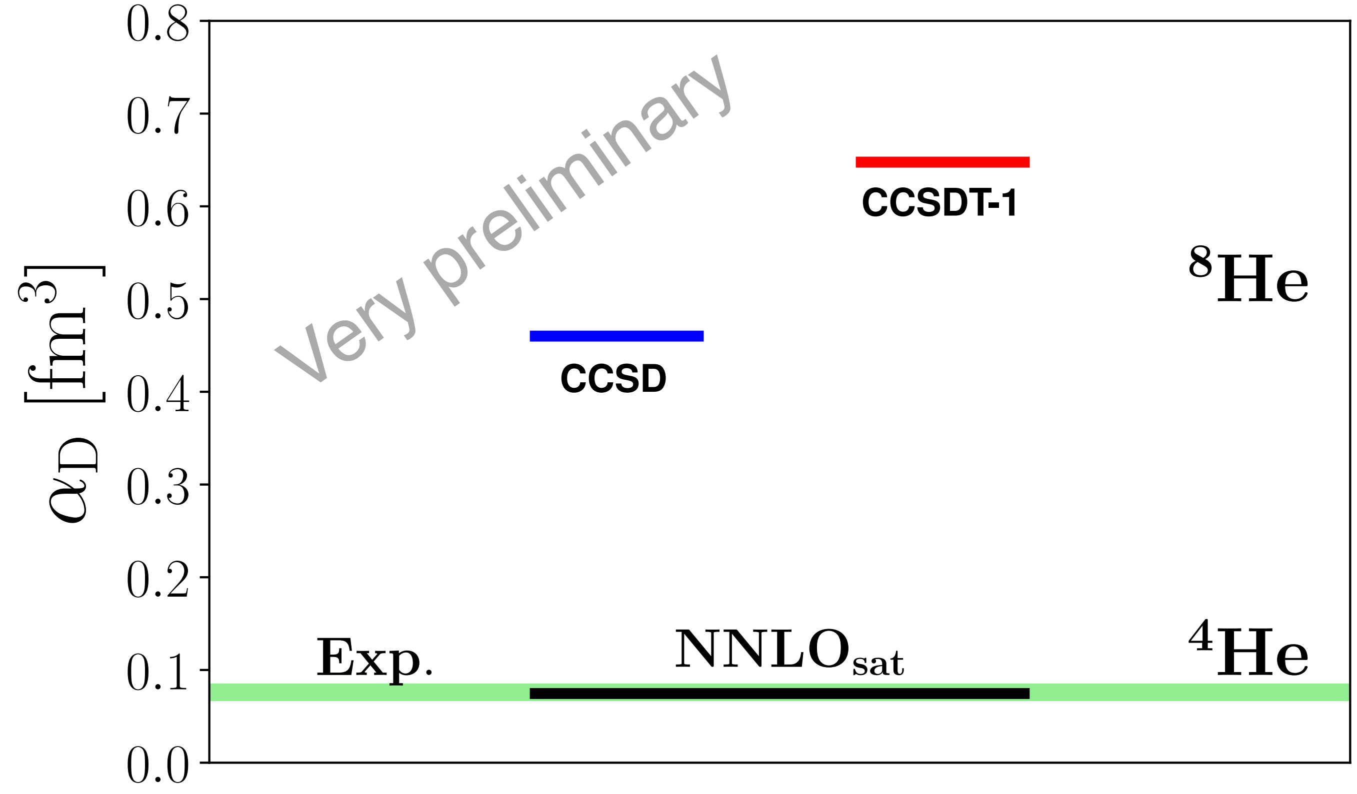

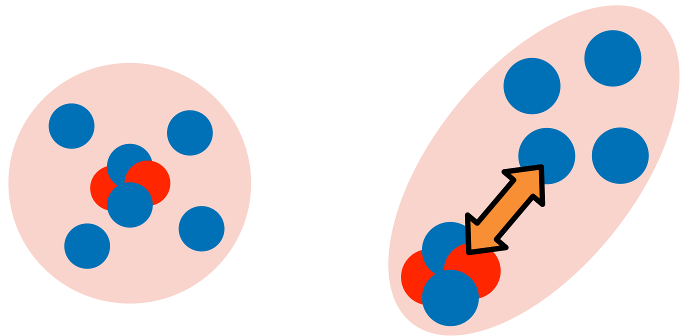

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Theory motivates new experiments

Will be measured by T. Aumann

Outlook

- The ab-initio pathway is arguably the best way develop a strong predictive theory and connect to experiment
- Such connection can be exploited in several areas of physics
- In the future we will address electron-nucleus and neutrino-nucleus scattering

Thanks to all my collaborators

Thanks for your attention!

Connection to Neutron Stars

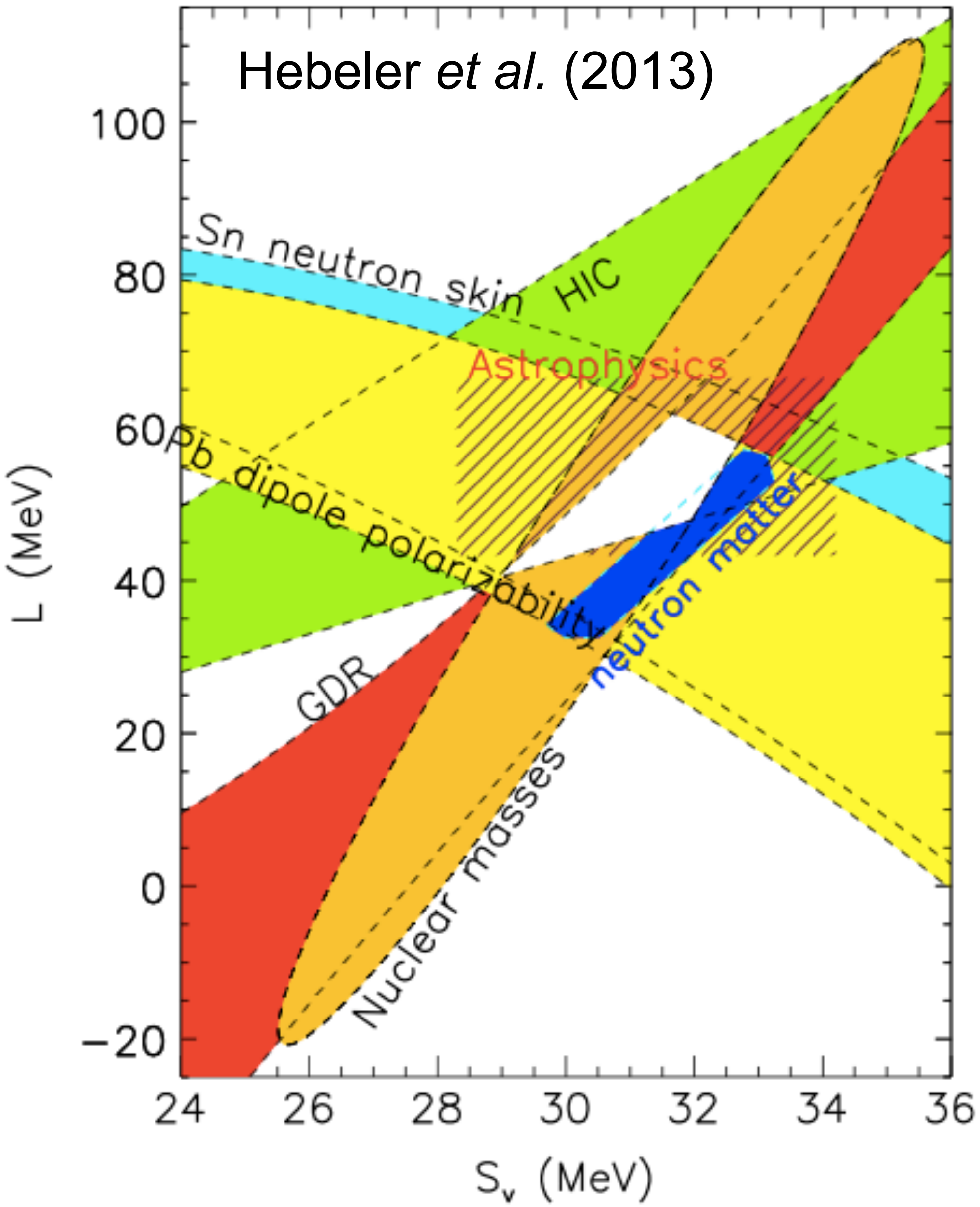
Equation of state

$$E(\rho,\delta) = E(\rho,0) + S(\rho)\delta^2 + \mathcal{O}(\delta^4)$$

$$\rho = \rho_n + \rho_p, \quad \delta = \frac{\rho_n - \rho_p}{\rho_n + \rho_p}$$

$$S(\rho) = S_v + \frac{L}{3\rho_0}(\rho - \rho_0) + \frac{K_{sym}}{18\rho_0^2}(\rho - \rho_0)^2 + \ldots$$

S_v and L can be inferred from heavy ion collisions and are correlated with finite nuclei observables



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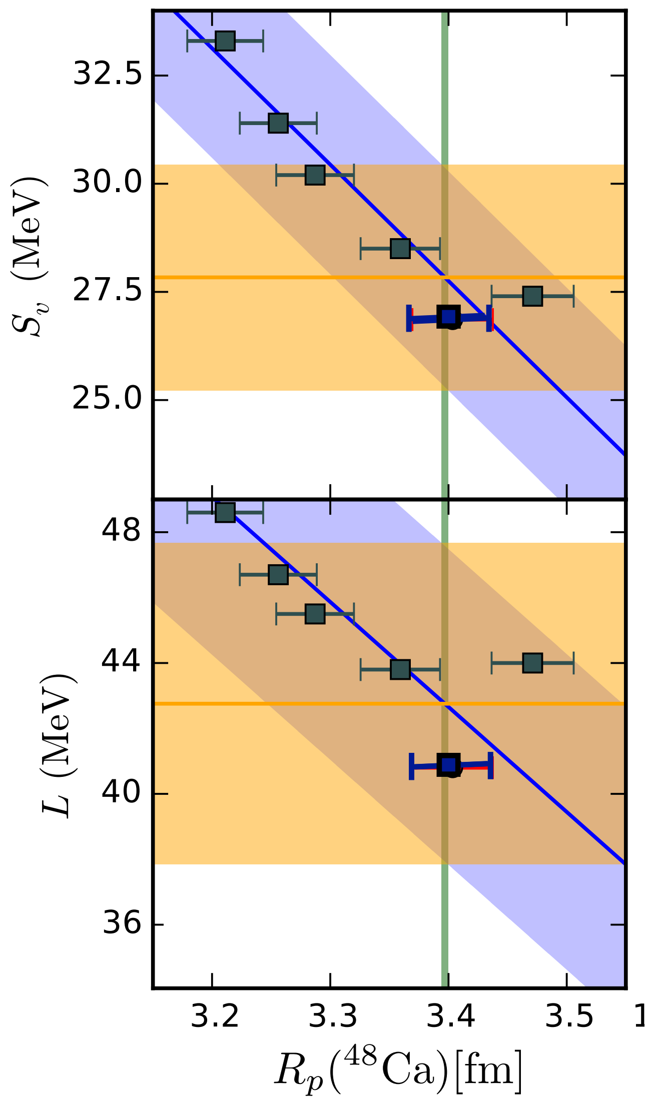
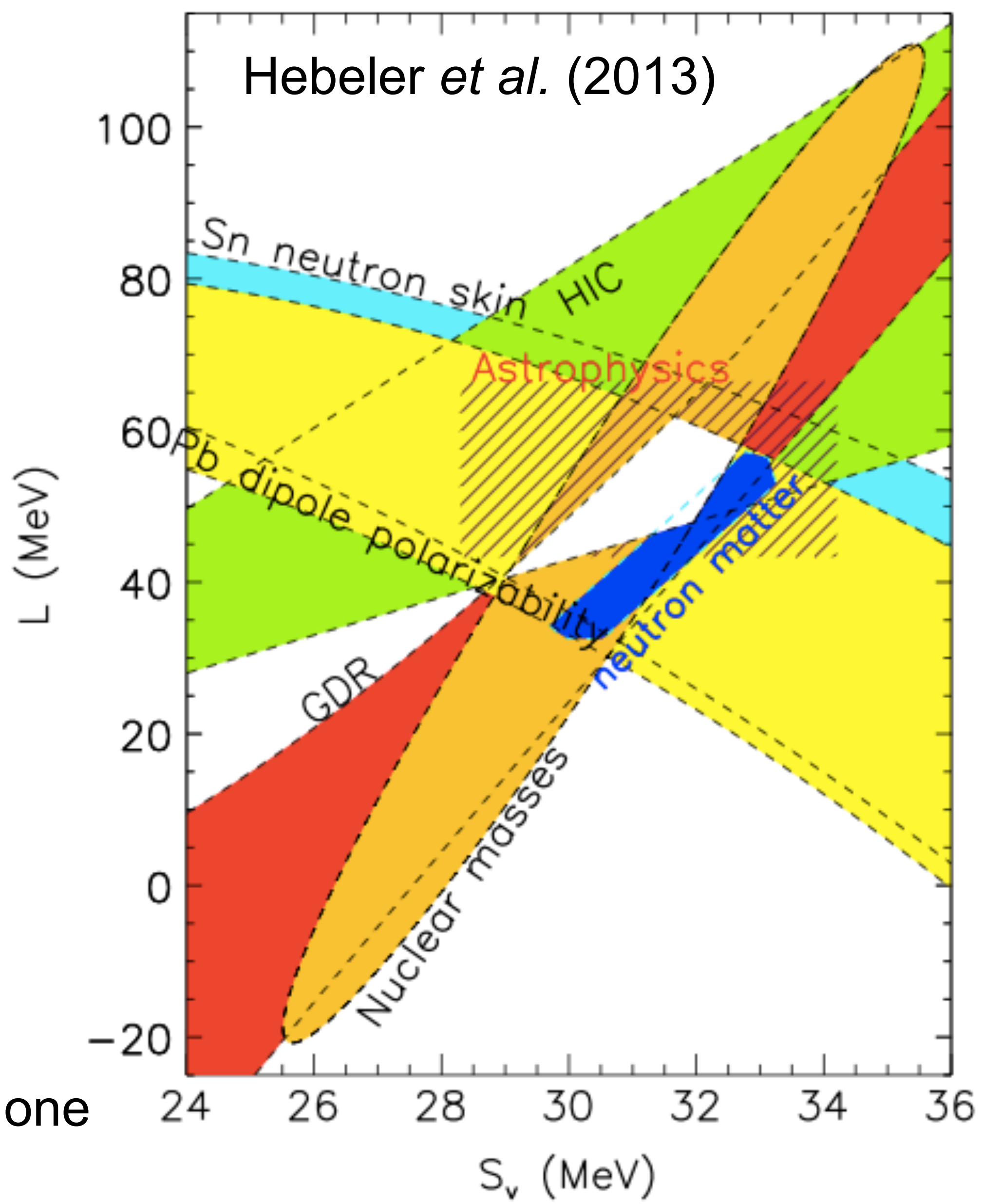


Using correlations with R_p in ^{48}Ca one can constrain nuclear matter



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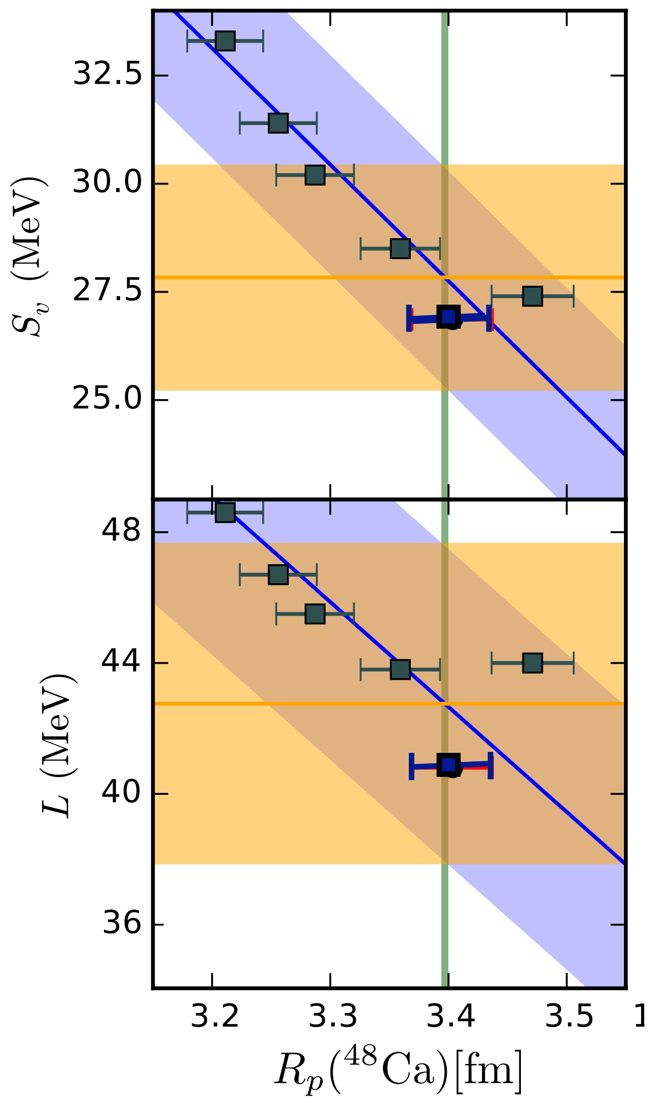

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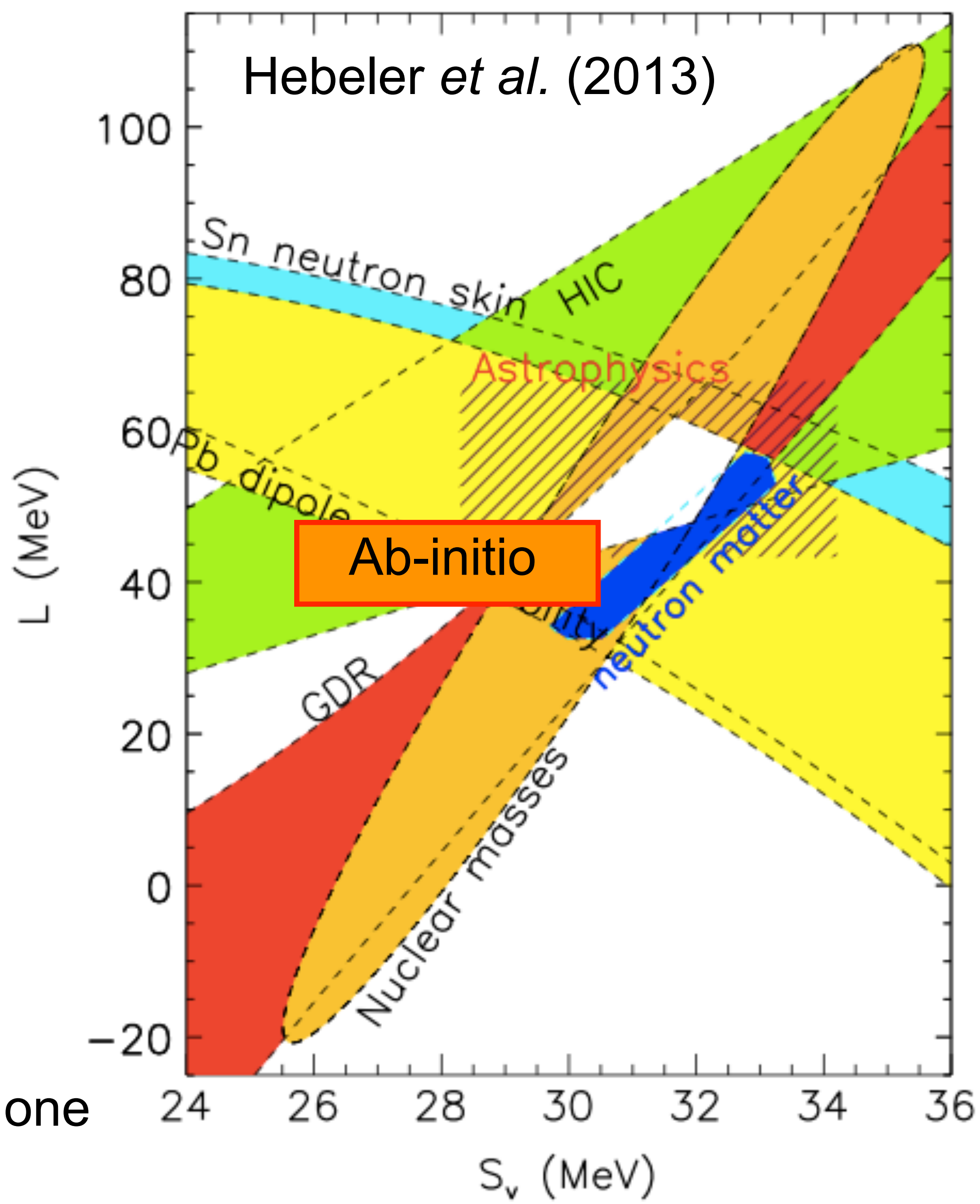

$$25.2 \leq S_v \leq 30.4 \quad \text{MeV}$$

$$37.8 \leq L \leq 47.7 \quad \text{MeV}$$