Resonances, currents, and medium mass nuclei

Sebastian König

SFB 1245 Workshop 2017

Mainz

July 4, 2018

European Research Council
Established by the European Commission

TECHNISCHE UNIVERSITÄT DARMSTADT

EMMI

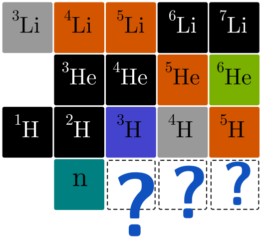
SFB 1245

Signatures of few-body resonances in finite volume

P. Klos, SK, J. Lynn, H.-W. Hammer, and A. Schwenk, arXiv:1805.02029 [nucl-th]

Motivation

terra incognita at the doorstep. . .



- bound dineutron state not excluded by pionless EFT

  Hammer + SK, PLB **736** 208 (2014)

- recent indications for a three-neutron resonance state. . .

  Gandolfi *et al.*, PRL **118** 232501 (2017)

- . . . although excluded by previous theoretical work

  Offermann + Glöckle, NPA **318**, 138 (1979); Lazauskas + Carbonell, PRC **71** 044004 (2005)

- **possible evidence for tetraneutron resonance**

  Kisamori *et al.*, PRL **116** 052501 (2016)

- **conflicting theoretical results!**

  Hiyama *et al.*, PRC **93** 044004 (2016); Deltuva, PLB **782** 238 (2018)
  Shirokov *et al.* PRL **117** 182502(2016); Gandolfi *et al.*, PRL **118** 232501 (2017); Fossez *et al.*, PRL **119** 032501 (2017)

Finite-volume resonance signatures

Lüscher formalism: phase shift ↔ box energy levels

$$p \cot \delta_0(p) = \frac{1}{\pi L} S(\eta) \quad , \quad \eta = \left(\frac{Lp}{2\pi}\right)^2 \quad , \quad p = p(E(L))$$

Lüscher, Nucl. Phys. B **354** 531 (1991); ...

resonance contribution ⇝ **avoided level crossing**

Wiese, Nucl. Phys. B (Proc. Suppl.) 9, 609 (1989); ...

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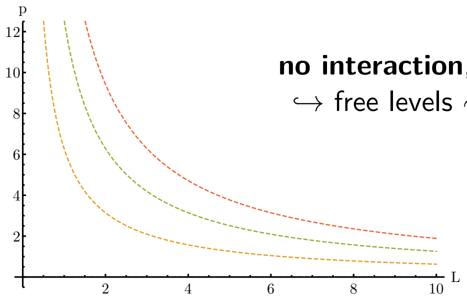

no interaction, $\delta(p) = 0$

↪ free levels $\sim 1/L$

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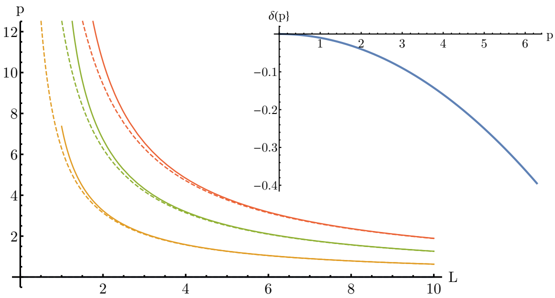

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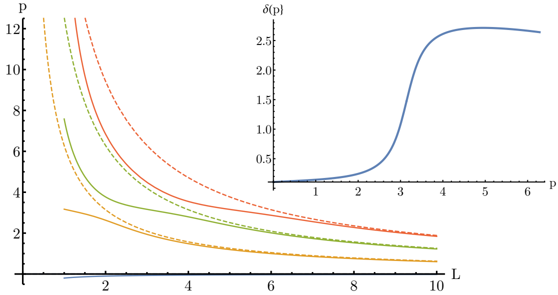


Discrete variable representation

Needed: calculation of several few-body energy levels

- difficult to achieve with QMC methods — Klos *et al.*, PRC **94** 054005 (2016)
- direct discretization possible, but not very efficient

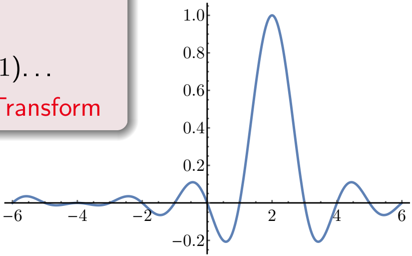
$\hookrightarrow$ **use a Discrete Variable Representation (DVR)**

well established in quantum chemistry, suggested for nuclear physics by Bulgac+Forbes, PRC **87** 87, 051301 (2013)

Main features

- basis functions localized at grid points
- potential energy matrix diagonal
- kinetic energy matrix sparse (in $d > 1$)...
- ...or implemented via Fast Fourier Transform

periodic boundary condistions
$\leftrightarrow$ **plane waves as starting point**



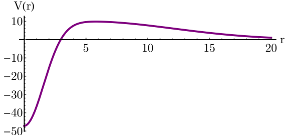
Three-body check

Take established three-body resonance from literature:

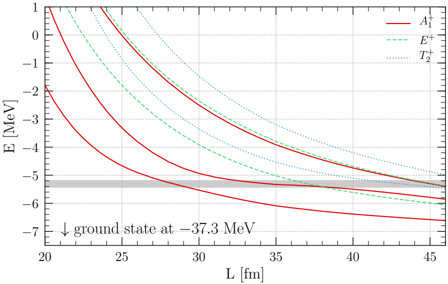
Fedorov *et al.*, Few-Body Syst. P **33** 153 (2003); Blandon *et al.*, PRA **75** 042508 (2007)

$$V(r) = V_0 \exp\left(-\left(\frac{r}{R_0}\right)^2\right) + V_1 \exp\left(-\left(\frac{r-a}{R_1}\right)^2\right)$$

$V_0 = -55$ MeV, $V_1 = 1.5$ MeV, $R_0 = \sqrt{5}$ fm, $R_1 = 10$ fm, $a = 5$ fm



- three spinless bosons with mass $m = 939.0$ MeV
- two- and three-body bound states at -6.76 MeV and -37.22 MeV
- three-body resonance at $-5.31 - i0.12$ MeV (Blandon *et al.*), $-5.96 - i0.40$ MeV (Fedorov *et al.*)

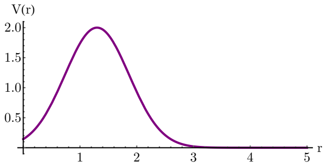


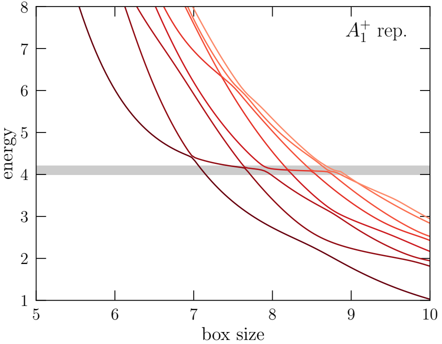
- **fit inflection point(s)** to extract resonance energy ⇝ $E_R = -5.32(1)$ MeV

Three bosons with shifted Gaussian interaction

three-boson system

- shifted Gaussian 2-body potential
- **note:** no 2-body bound state!





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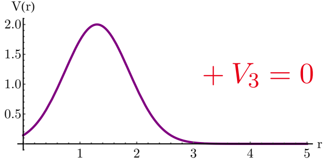
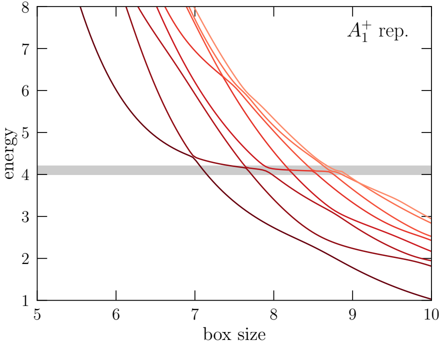


$+V_3 = 0$



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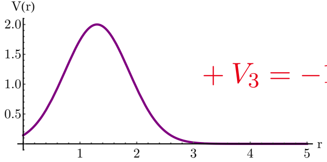
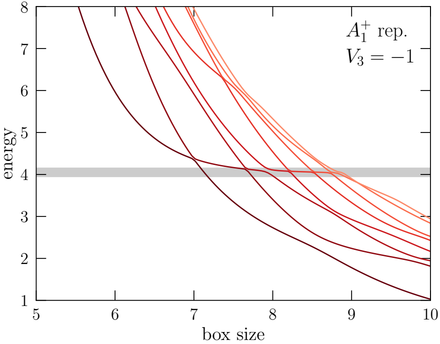


$+V_3 = -1.0$



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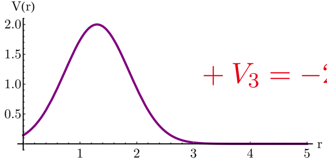
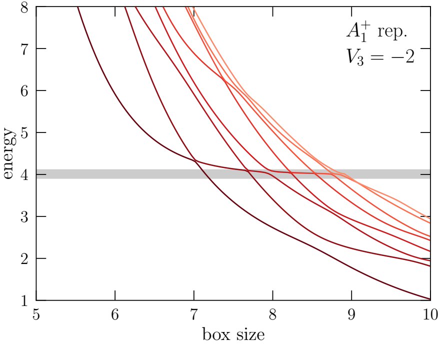


$+\,V_3 = -2.0$



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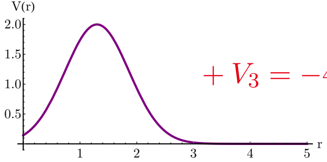
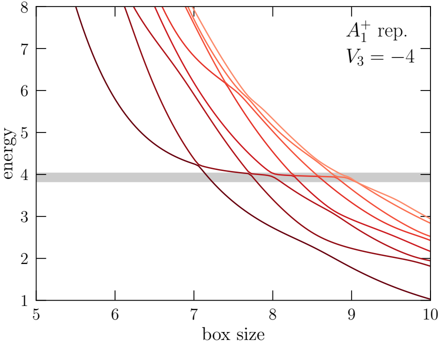


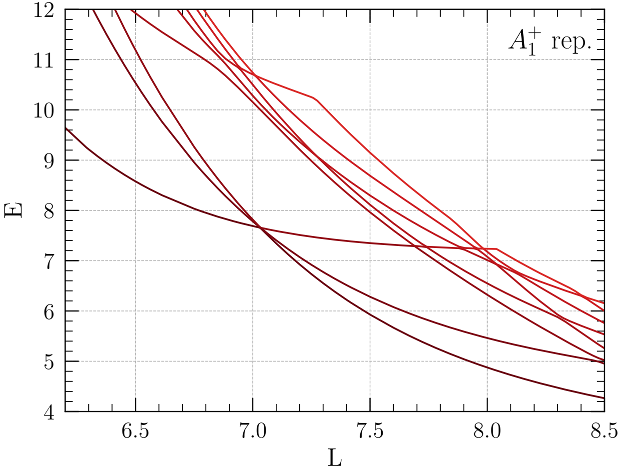
$+ V_3 = -4.0$



$\hookrightarrow$ **possible to move three-body state $\leftrightarrow$ spatially localized wf.**

Four-boson resonance

Still same potential, look at four bosons. . .



$\hookrightarrow$ (supposedly) narrow resonance at $E_R = 7.31(8)$

Summary and outlook

- ✓ **method established** for up to four particles
- ✓ handle **large N_{DVR} for three-body systems** (current record: 32)
- ✓ efficient **symmetrization and antisymmetrization**
- ✓ projection onto **cubic irreps.** ($H \to H + \lambda(1 - P_\Gamma)$, λ large)

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Work in progress

- ✓ **chiral interactions** (non-diagonal due to spin dependence!)
- application to **few-neutron systems**
- **further optimization** (especially for spin-dep. potentials)
  - $\hookrightarrow$ need to reach decent N_{DVR} for four-neutron calculation!
- isospin degrees of freedom $\rightsquigarrow$ **treat general nuclear systems**
- **different boundary conditions** (e.g., antiperiodic)

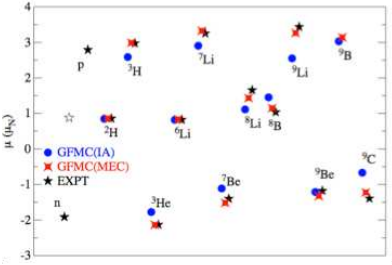
Electroweak currents from chiral EFT in few-nucleon systems

R. Seutin, SK, K. Hebeler, A. Schwenk *et al.*, work in progress

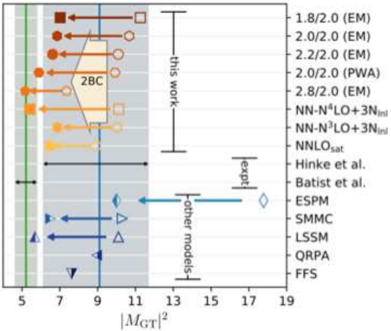
Chiral EFT currents

Chiral EFT predicts consistent electroweak 1+2-body currents

magnetic moments in light nuclei
Pastore et al. (2012-)



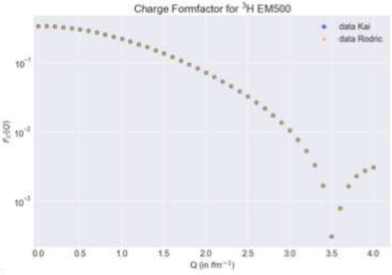
Gamow-Teller beta decay of ^{100}Sn
Gysbers, Hagen et al.



contributions from 2-body currents are key!

Chiral EFT currents

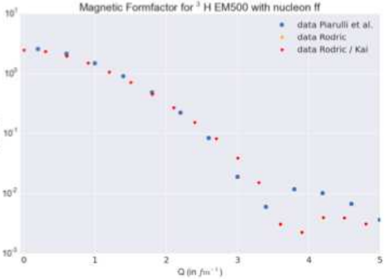
developing framework to include 1+2-body currents at finite q
in partial-wave basis, momentum basis, HO matrix elements



charge form factor for A=3
pw basis vs. mom basis check

magnetic form factor benchmarks
with literature not perfect

2-body current tests ongoing,
especially for cm-dependent part



Probing next-generation nuclear forces in medium-mass nuclei

J. Hoppe, J. Simonis, K. Hebeler, A. Schwenk *et al.*, work in progress

Shell-model interactions from chiral EFT: L. Huth, V. Durant, J. Simonis, A. Schwenk, arXiv:1804.04990

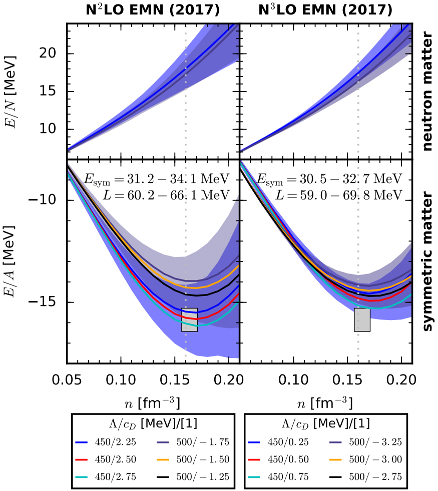
Nuclear forces and nuclear matter

Monte-Carlo calculation of all energy diagrams
up to 4th order in MBPT
Drischler, Hebeler, AS, arXiv:1710.08220

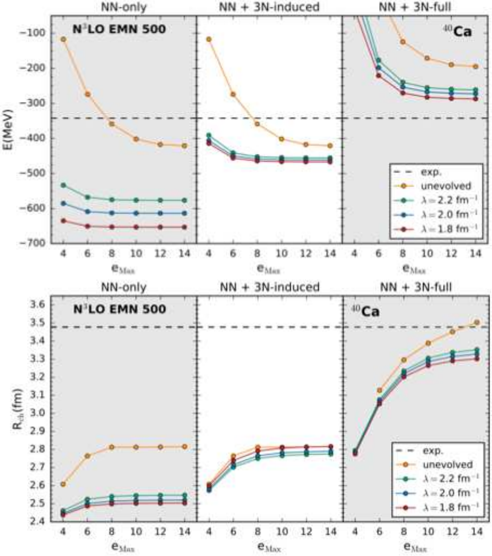
including NN, 3N, 4N
3N fit to saturation region

systematic improvement
from N^2LO to N^3LO

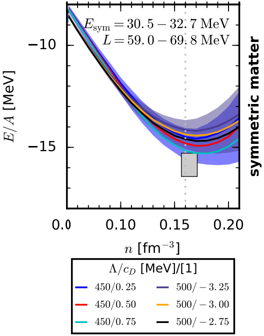
first full N^3LO Hamiltonians
for use in nuclear structure!



First (preliminary) N^{3}LO results for nuclei



energy and radius trends as expected from nuclear matter saturation, but more studies needed!

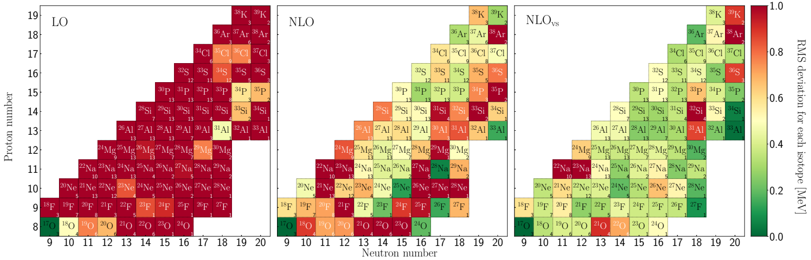


use chiral EFT interactions as basis and fit in sd shell directly

Huth, Durant et al., arXiv:1804.04990

includes new valence-space (vs) operators

all LECs turn out natural



The end

Thank you!

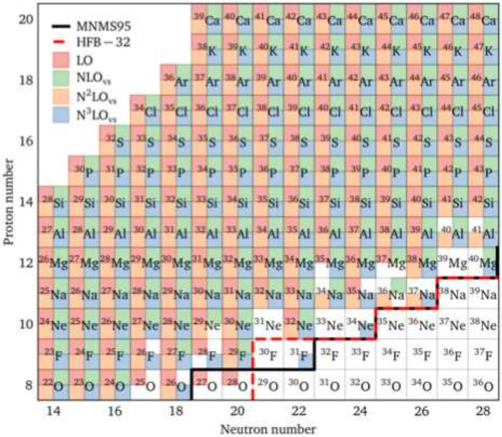
Backup slides

Chiral shell model interactions

use chiral EFT interactions as basis and fit in sd shell directly
Huth, Durant et al., arXiv:1804.04990

includes new valence-space (vs) operators
all LECs turn out natural

explore dripline
in sdf$_{7/2}$ space
Huth et al., in prep.



Tetraneutron evidence


Physics ABOUT BROWSE PRESS COLLECTIONS

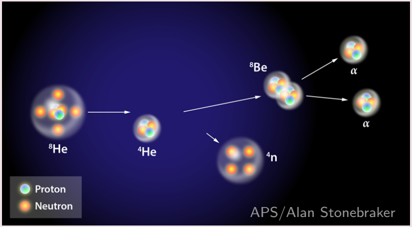
Viewpoint: Can Four Neutrons Tango?

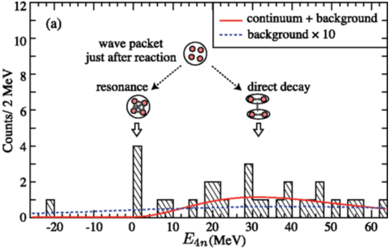
Nigel Orr, Laboratoire de Physique Corpusculaire de Caen, ENSICAEN, IN2P3/CNRS et Université de Caen Normandie, 14050 Caen cedex, France

February 3, 2016 • *Physics* 9, 14

Evidence that the four-neutron system known as the tetraneutron exists as a resonance has been uncovered in an experiment at the RIKEN Radioactive Ion Beam Factory.

Experimental setup





Kisamori *et al.*, PRL **116** 052501 (2016)

Short (recent) history of tetraneutron states

1. **2002:** experimental claim of bound tetraneutron Marques *et al.*, PRC **65** 044006
2. **2003:** several studies indicate unbound four-neutron system
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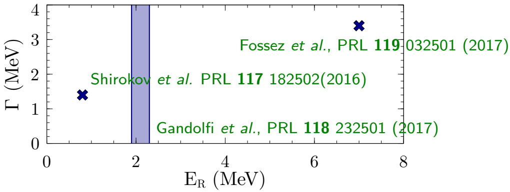
   $E_R = (0.83 \pm 0.65_{\text{stat.}} \pm 1.25_{\text{syst.}})$ MeV, $\Gamma \lesssim 2.6$ MeV — Kisamori *et al.*, PRL **116** 052501
5. **following this:** several new theoretical investigations
   - complex scaling → need unphys. $T = {}^3/_2$ 3N force or strong rescaling
     Hiyama *et al.*, PRC **93** 044004 (2016); Deltuva, PLB **782** 238 (2018)
   - incompatible predictions:



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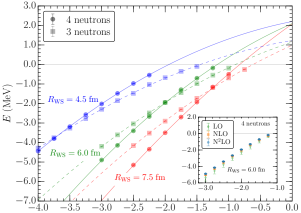
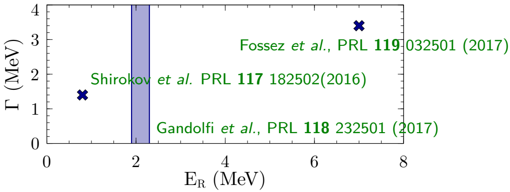


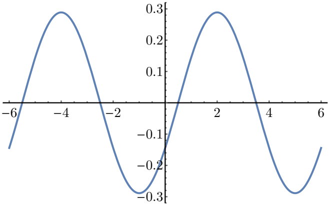


- **indications for three-neutron resonance. . .**
- **. . . lower in energy than tetraneutron state**

Gandolfi *et al.*, PRL **118** 232501 (2017)

DVR construction

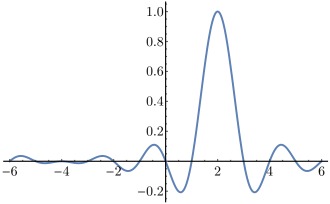
- start with some initial basis; here: $\phi_i(x) = \frac{1}{\sqrt{L}} \exp\left(\mathrm{i}\frac{2\pi i}{L}x\right)$
- consider (x_k, w_k) such that $\sum_{k=-N/2}^{N/2-1} w_k\, \phi_i^*(x_k)\phi_j(x_k) = \delta_{ij}$



unitary trans.

$\longrightarrow$

$\mathcal{U}_{ki} = \sqrt{w_k}\phi_i(x_k)$



DVR states

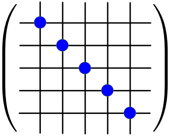
- $\psi_k(x)$ localized at x_k, $\psi_k(x_j) = \delta_{kj}/\sqrt{w_k}$
- **note:** momentum mode $\phi_i \leftrightarrow$ spatial mode ψ_k

DVR features

1. **potential energy is diagonal!**

$$\langle \psi_k | V | \psi_l \rangle = \int \mathrm{d}x \, \psi_k(x) \, V(x) \, \psi_l(x)$$

$$\approx \sum_{n=-N/2}^{N/2-1} w_n \, \psi_k(x_n) \, V(x_n) \, \psi_l(x_n) = V(x_k)\delta_{kl}$$



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- number N of DVR states controls quadrature approximation

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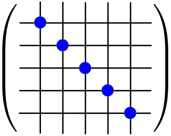

- no need to evaluate integrals
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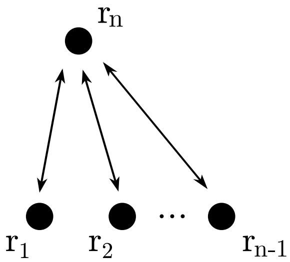
2. **kinetic energy is simple (via FFT) or sparse (in $d > 1$)!**

- plane waves ϕ_i are momentum eigenstates $\rightsquigarrow \hat{T} \, |\psi_k\rangle \sim \mathcal{F}^{-1} \otimes \hat{p}^2 \otimes \mathcal{F} \, |\psi_k\rangle$
- $\langle \psi_k | \hat{T} | \psi_l \rangle$ = known in closed form

  $\hookrightarrow$ replicated for each coordinate, with Kronecker deltas for the rest

General DVR basis states

- construct DVR basis in simple relative coordinates…
- …because Jacobi coord. would complicate the boundary conditions
- separate center-of-mass energy (choose $\mathbf{P} = \mathbf{0}$)
- mixed derivatives in kinetic energy operator



$$\mathbf{x}_i = \sum_{i=1}^{n} U_{ij} \mathbf{r}_i$$

$$U_{ij} = \begin{cases} \delta_{ij} & \text{for} \quad i, j < n \\ -1 & \text{for} \quad i < n,\, j = n \\ 1/n & \text{for} \quad i = n \end{cases}$$

General DVR state

$$|s\rangle = |(k_{1,1}, \cdots, k_{1,d}), \cdots, (k_{n-1,1}, \cdots); \text{spins}\rangle \in B$$

basis size: $\dim B = (2S+1)^n \times N^{d \times (n-1)}$

(Anti-)symmetrization and parity

Permutation symmetry

- for each $|s\rangle \in B$, construct $|s\rangle_{\mathcal{A}} = \mathcal{N} \sum_{p \in S_n} \operatorname{sgn}(p)\, D_n(p)\, |s\rangle$
- then $|s\rangle_{\mathcal{A}}$ is antisymmetric: $\mathcal{A}\,|s\rangle_{\mathcal{A}} = |s\rangle_{\mathcal{A}}$
- for bosons, leave out $\operatorname{sgn}(p) \rightsquigarrow$ symmetric state
- $D_n(p)\,|s\rangle =$ some other $|s'\rangle \in B$ — modulo PBC

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This operation partitions the orginal basis, *i.e.*, each state appears in at most one (anti-)symmetric combination.

- efficient reduction to (anti-)symmetrized orthonormal basis

  $\hookrightarrow$ no need for numerically expensive diagonalization!
- $B \to B_{\text{reduced}}$, significantly smaller: $N \to N_{\text{reduced}} \approx N/n!$

Note: parity (with projector $\mathcal{P}_{\pm} = 1 \pm \mathcal{P}$) can be handled analogously.

DVR computational aspects

DVR basis size $N = N_{\text{spin}} \,(\times N_{\text{isospin}}) \times N_{\text{DVR}}^{n_{\text{dim}} \times (n_{\text{body}} - 1)}$

- $N_{\text{spin}} = (2S+1)^{n_{\text{body}}}$, $N_{\text{isospin}} = 1$ for neutrons only
- $3n$: $8 \times N_{\text{DVR}}^6$, $4n$: $16 \times N_{\text{DVR}}^9$ $\rightsquigarrow$ **large-scale calculation**

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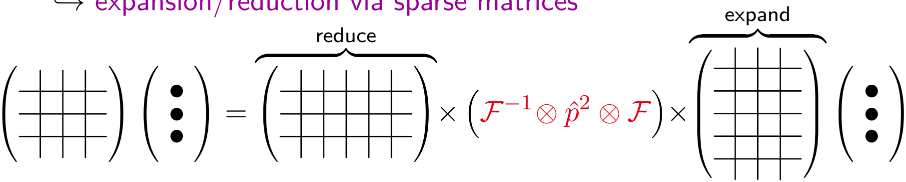
  $\rightsquigarrow$ large matrix-vector products
- kinetic part (via FFT) in original basis (before reduction)
  $\hookrightarrow$ expansion/reduction via sparse matrices



(note: kinetic matrix diagonal in spin-configurations space)

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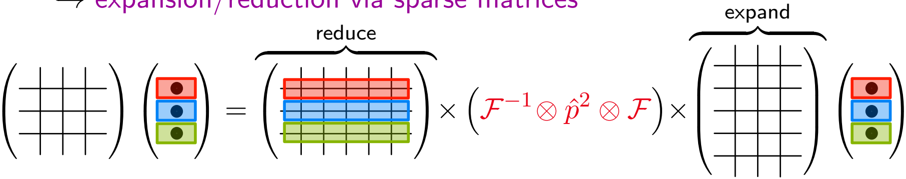

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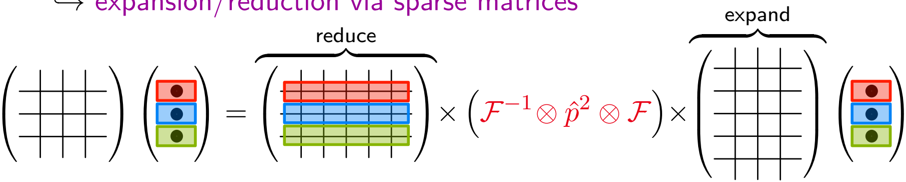

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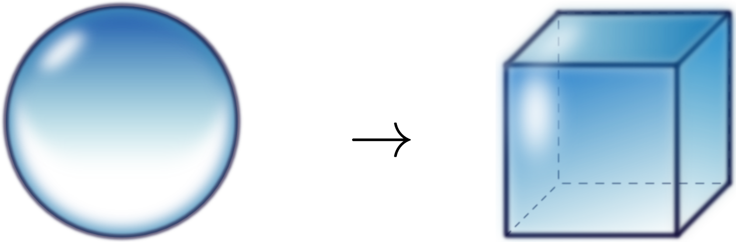
- potential part still diagonal in symmetry-reduced basis

Broken symmetry

The finite volume breaks the symmetry of the system:



rotation group $SO(3)$ **cubic group** O

Irreducible representations of $SO(3)$ are reducible with respect to O!

- finite subgroup of $SO(3)$
- number of elements $= 24$
- five irreducible representations

Γ	A_1	A_2	E	T_1	T_2
$\dim \Gamma$	1	1	2	3	3

Cubic projection

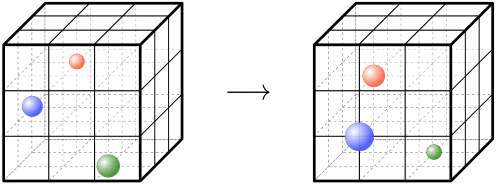
Cubic projector

$$\mathcal{P}_\Gamma = \frac{\dim\Gamma}{24}\sum_{R\in\mathcal{O}} \chi_\Gamma(R) D_n(R) \quad , \quad \chi_\Gamma(R) = \text{character}$$

Johnson, PLB **114** 147 (1982)

- $D_n(R)$ realizes a cubic rotation R on the n-body DVR basis
- $\rightsquigarrow$ permutation/inversion of relative coordinate components
- indices are wrappen back into range $-N/2, \ldots, N/2-1$

e.g.



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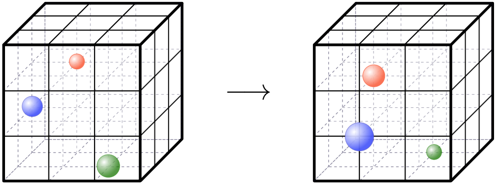
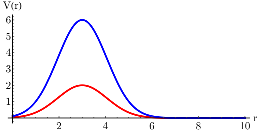


numerical implementation: $\hat{H} \to \hat{H} + \lambda(\mathbf{1} - \mathcal{P}_\Gamma) \quad , \quad \lambda \gg E$

Two-body check: anything goes

$$V(r) = V_0 \exp\left(-\left(\frac{r-a}{R_0}\right)^2\right)$$

$\hookrightarrow$ use barrier to produce S-wave resonance



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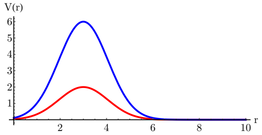

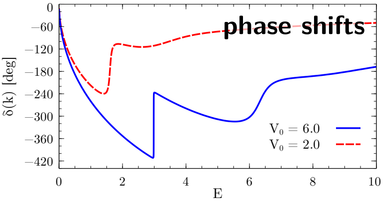

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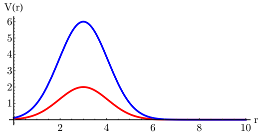

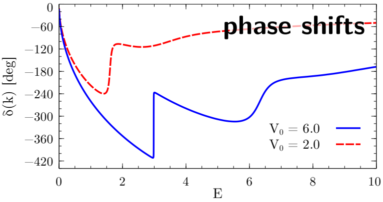

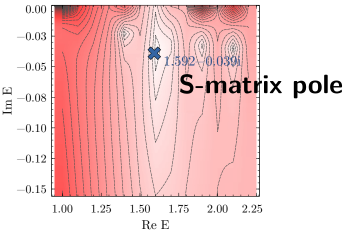

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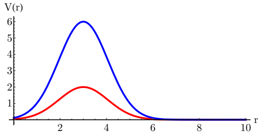

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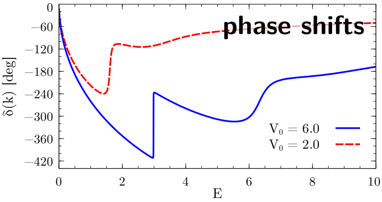
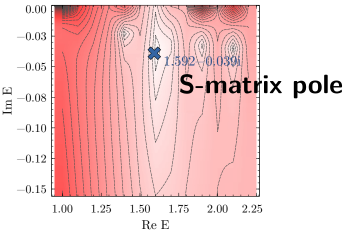
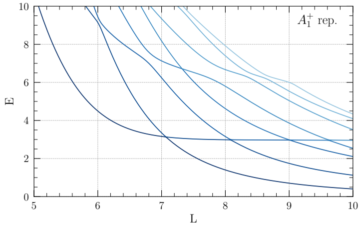
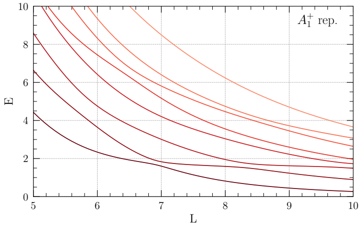




finite-volume spectra

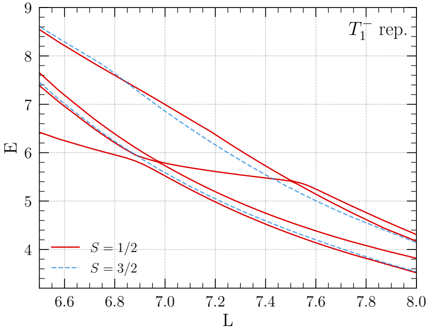


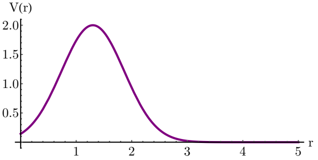


Three fermions

Consider same shifted Gaussian potential for three fermions. . .

- add spin d.o.f., but no spin dependence in potential
- ⇝ total spin S good quantum number (fix S_z to determine)
- also: can still consider simple cubic irreps.





$V_0 = 2.0$, $a = 3.0$, $R = 1.5$

- all lowest states found to be in T_1^- irrep. ($\sim$ P-wave state)
- some remaining volume dependence (box not very large)
- extracted $S = 1/2$ resonance energy: $E_R = 5.7(2)$