# Towards the Equation of State of Neutron Stars

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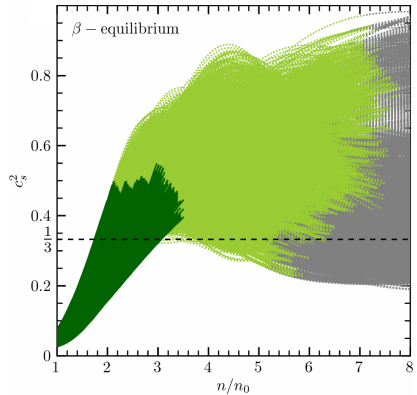
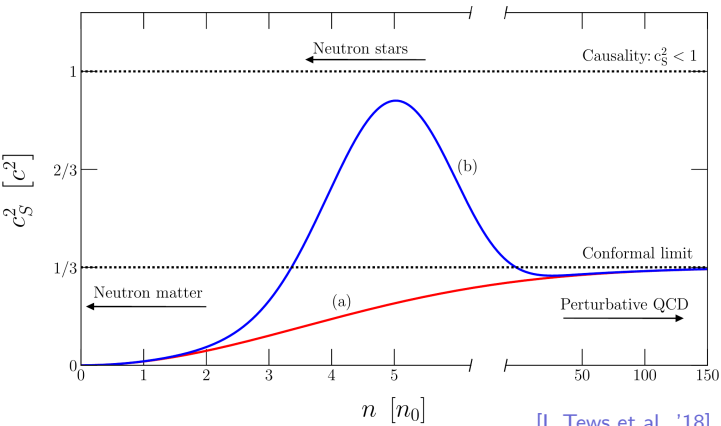
# Speed of Sound

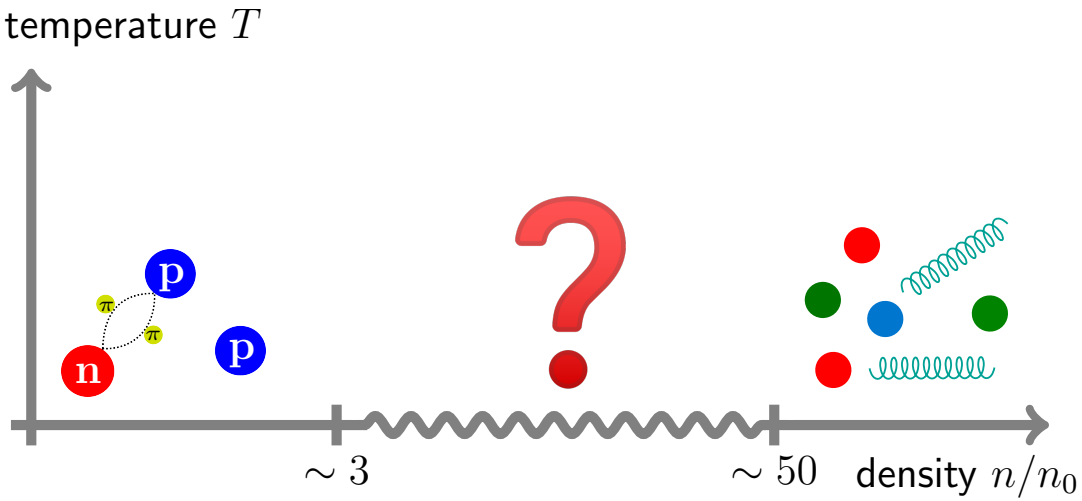



[S. Huth, C. Wellenhofer, and A. Schwenk '20]




[I. Tews et al. '18]

# Degrees of Freedom



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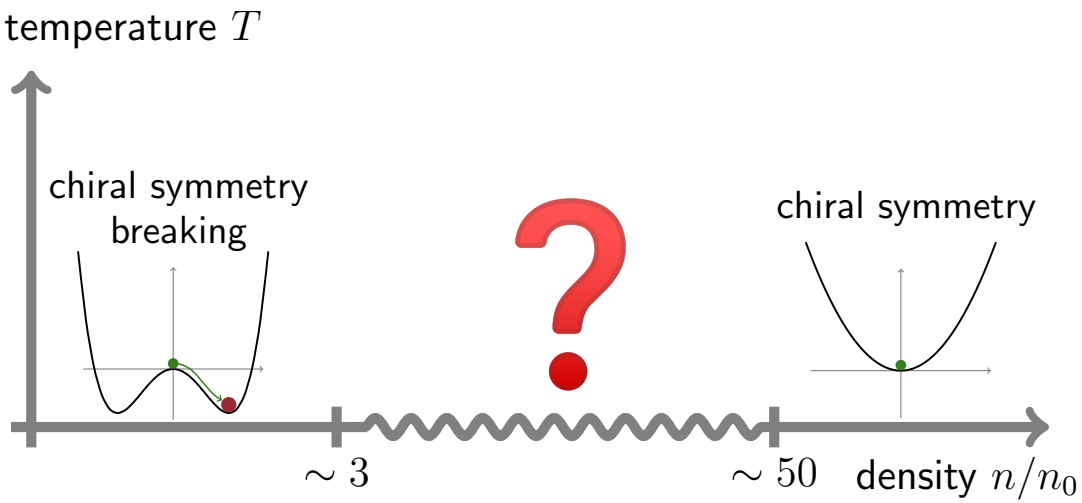
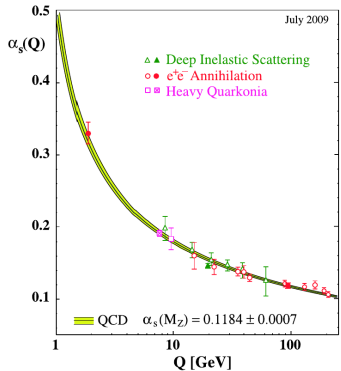


# Degrees of Freedom

temperature $T$




[Bethke '09]

$\chi$EFT

[K. Hebeler '20; E. Epelbaum, H.W. Hammer, and Ulf-G. Meißner '08; ...]

Functional Renormalization Group

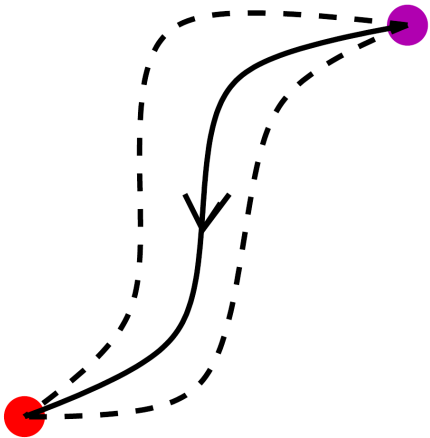
[A. Kurkela et al. '14; T. Gorda et al. '18; ...]

pQCD

$\sim 3$ $\sim 50$ density $n/n_0$

# Functional Renormalization Group

[Wetterich '93; (figure taken from) Gies '06]



$\Gamma_{k=\Lambda} = S$

$\Gamma_{k=0} = \Gamma \rightarrow$ pressure, expectation values, ...

# Functional Renormalization Group

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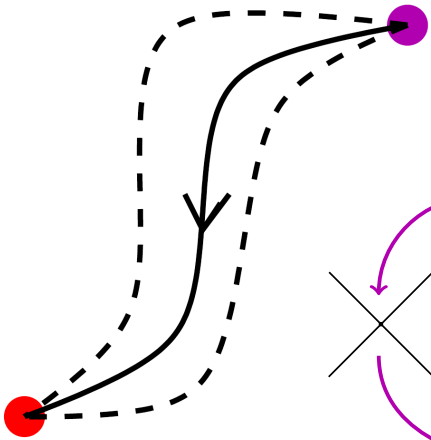
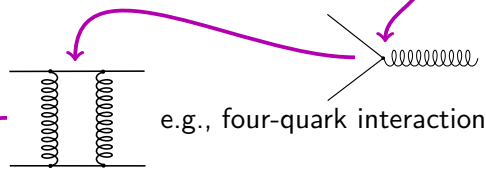


$$\Gamma_{k=\Lambda} = S = \int_x \bar{\psi}\left(i\partial\!\!\!/ - i\gamma_0\mu\right)\psi + g_s\bar{\psi}A\!\!\!/\psi + \frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} \qquad (N_\mathrm{f} = 2)$$



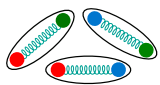
e.g., four-quark interactions



dominant four-quark interaction (at high densities)

[M. Leonhardt et al. '19; J. Braun and B. Schallmo '21]

$\Gamma_{k=0} = \Gamma \rightarrow$ pressure, expectation values, ...

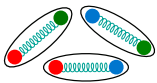


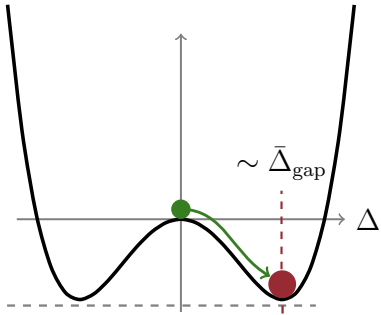
color-superconducting gap
diquark condensate

# Color Superconductivity

$\mathrm{U}(1)_V$ symmetry breaking

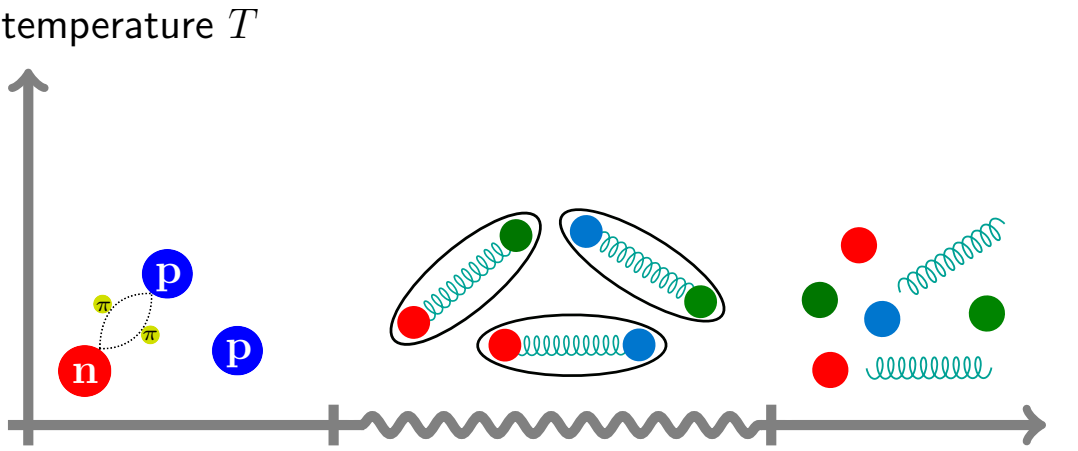
quark cooper pairs
$\sim$ diquarks






2-flavor diquark condensate is chirally symmetric

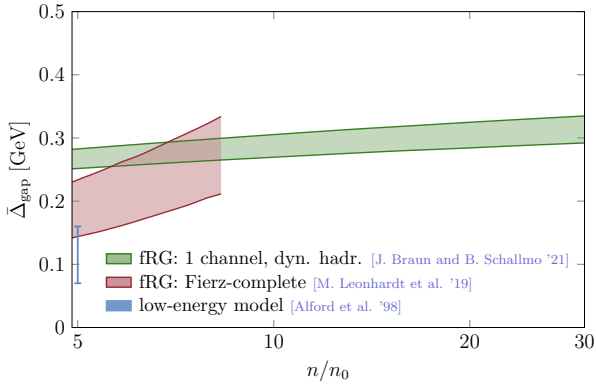
# Degrees of Freedom



# Diquark Gap

[M. Leonhardt et al. '19;
J. Braun and B. Schallmo '21;
J. Braun and B. Schallmo '22]

no Taylor expansion about $g_s = 0$




$$\bar{\Delta}_{\rm gap} \sim \exp\left(-\frac{c}{g_s^4 n^{2/3}}\right), \; c > 0$$

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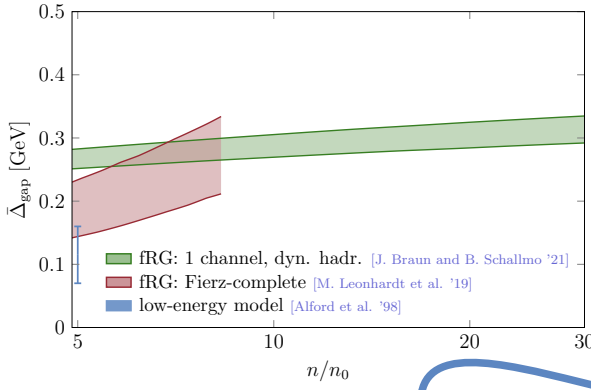



very high densities

$$\bar{\Delta}_{\rm gap} \sim \exp\left(-\frac{c}{g_s^4 n^{2/3}}\right), \; c > 0$$

$$\bar{\Delta}_{\rm gap} \sim \frac{1}{g_s^5} \exp\left(-\frac{c'}{g_s}\right), \; c' > 0$$

[D. T. Son '99; R. Pisarski and D. Rischke '00]

# Expansion of the Pressure

[J. Braun, AG, and B. Schallmo '22]

**expansion about vanishing gap**

$$P = P_{\mathrm{SB}} \left( \gamma_0(g_s) + \gamma_1(g_s) \left( \frac{|\bar{\Delta}_{\mathrm{gap}}|}{\mu} \right)^2 + \dots \right)$$

Stefan-Boltzmann pressure

$\gamma_0 = 1 - \frac{g_s^2}{2\pi^2}$ [T. Toimela '85]

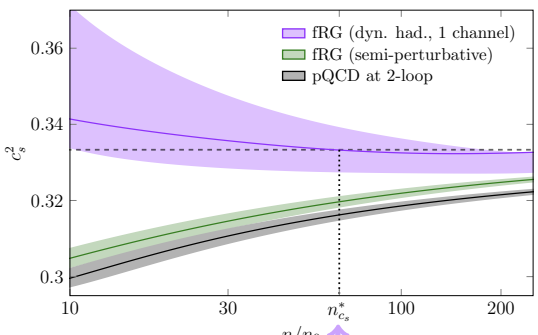
$\gamma_1 = 2$ [K. Rajagopal and F. Wilczek '01]

$\bar{\Delta}_{\mathrm{gap}}(g_s)$

# Speed of Sound

[J. Braun, AG, and B. Schallmo '22; J. Braun, AG, and B. Schallmo (in prep.); J. Braun and AG (in prep.)]

$$c_s^2 = \frac{\partial P}{\partial \epsilon} = \frac{1}{\mu}\left(\frac{\partial P}{\partial \mu}\right) \Big/ \left(\frac{\partial^2 P}{\partial \mu \partial \mu}\right)$$




"crossing" density

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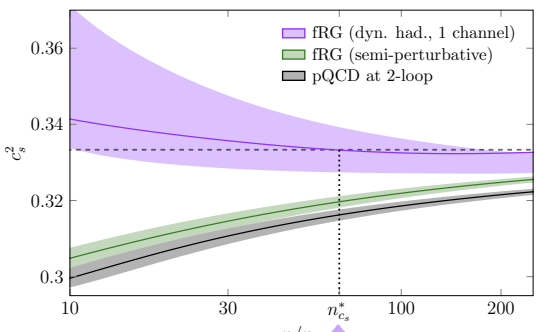
- $n \gg n^*_{c_s}$: gap contributions can be neglected
- $n > n^*_{c_s}$: gap contributions are sizable
- $n \gtrsim n^*_{c_s}$: gap contributions become significant

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**scaling law**

$$c_s^2 \approx \frac{1}{3} + c_0\,(1+\sigma)\,n^{-\frac{2(1-\sigma)}{3}} - \frac{c_1}{\ln\left(c_2 n^{1/3}\right)}$$

$c_0, c_1 > 0$

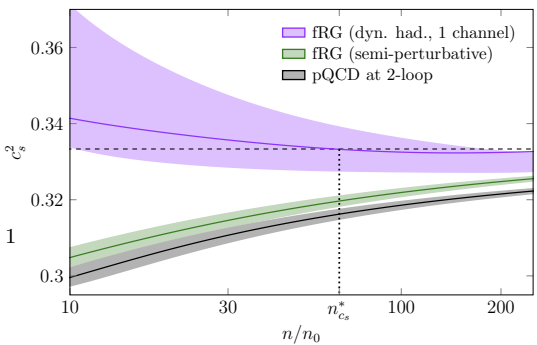
$|\sigma| < 1$

$c_2 n^{1/3} > 1$

free quark gas

gap contribution
($\sigma$ determined by scaling of gap)

perturbative contribution



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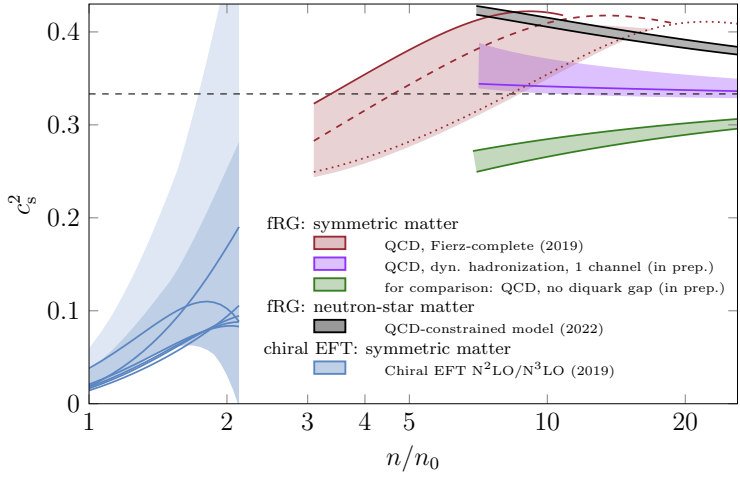

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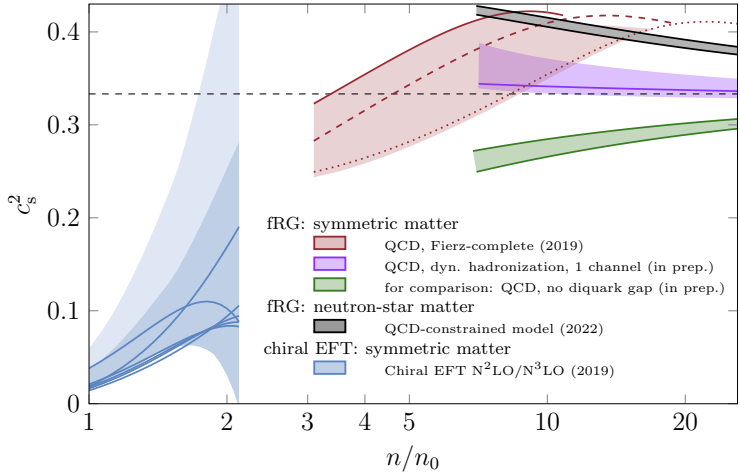

# Take Home Message

A color-superconducting **gap** suggests a **maximum** in the speed of sound at supranuclear densities.

At even higher densities, the speed of sound again **crosses** the conformal limit and approaches it from **below**.

# Thank you for your attention!