^{205}Tl, the pp neutrino flux and ^{205}Pb/^{205}Tl s-process chronometry

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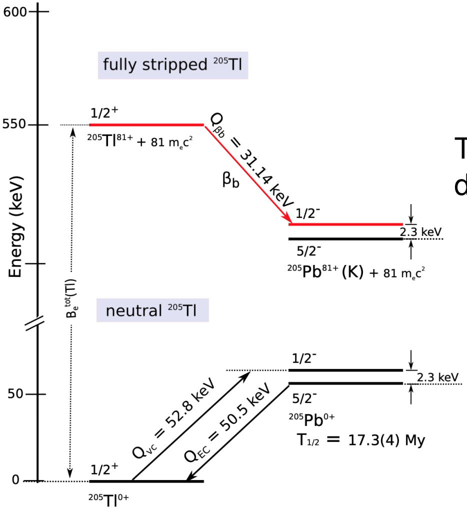
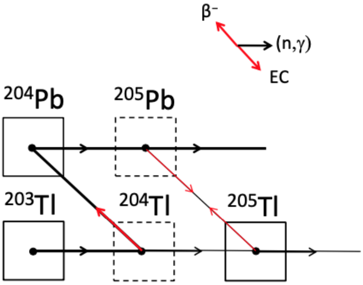
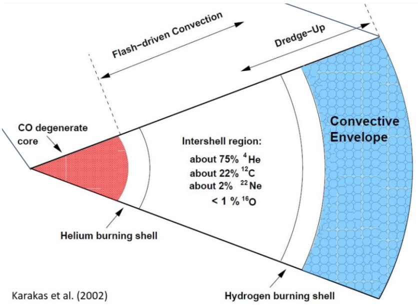


The half-life of ^{205}Tl can determine

- the ν-capture rate of ^{205}Tl
- the survival of ^{205}Pb in s-processes scenarios



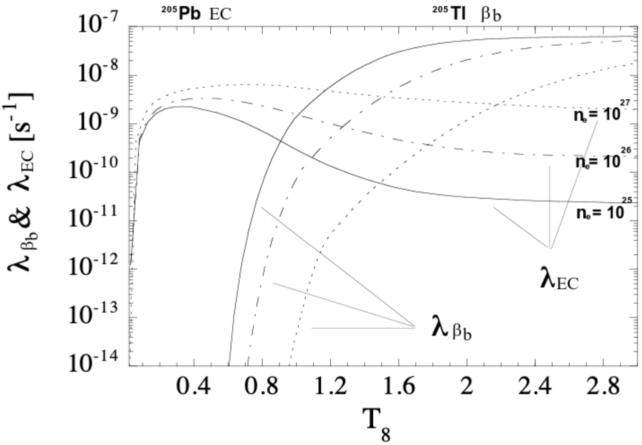


Karakas et al. (2002)

Region of interest

- $T < 5 \times 10^8$ K
- $\rho < 10^6 \mathrm{g/cm}^3$
- typically $10^7 \mathrm{cm}^{-3} < n_n < 10^9 \mathrm{cm}^{-3}$

Weak processes rates

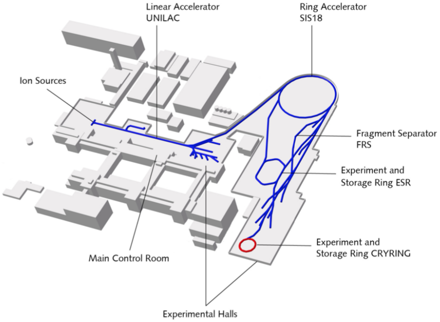


Yokoi, Takahashi, Arnould 1985

Experimental measurement

ASTRUm, HGF-CAS Joint Research group and E121 Collaboration at GSI have measured $T_{1/2}$ for the ^{205}Tl bound-state β-decay.
More details in the talk of Y. Litvinov in the P2 session.

- $^{205}\mathrm{Tl}^{81+}$ produced and stored.
- $T_{1/2}^{exp}(^{205}\mathrm{Tl})_{\beta_b} = 295 \pm 58$ d.
- $T_{1/2}^{exp}(^{205}\mathrm{Pb})_{EC} = (1.73 \pm 0.04) \times 10^7$ y.



Decay rate of β_b-decay

The decay rate for the bound state β-decay to the K-shell is given by

$$\lambda_b = \frac{ln(2)}{\kappa} n_K C_K f_K$$

where n_K is the relative vacancy of the shell,
$f_K = (\pi/2) q_K^2 \beta_K^2 B_K^2$ and $q_K = Q_b + E_K = 31.14$ keV.
β_K^2 is the amplitude of the electron wave function in the nucleus and B_K^2 is a parameter for the effects of electron exchange and overlap.

$$\sqrt{C_K^{exp}} = 34 \pm 5\,\mathrm{fm}$$

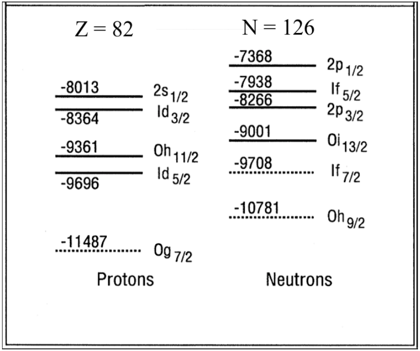
Nuclear shape factor

Different weak processes involving the same nucleus can have differences in their respective nuclear shape factor. In the case of bound β-decay and ν_e-capture

$$C_K = \overline{M} + M(q_k)$$
$$C(W_\nu) = \overline{M} + M(W_\nu)$$

To convert the experimental result to other observables theoretical calculations are needed and are performed within the framework of the shell model.

Model space and interaction



- Poppelier-Kuo-Herling (PKH) model space.
- KH_h interaction.
- shell model code NATHAN.

First forbidden β-decays from ^{205}Au, ^{205}Hg, ^{206}Hg, ^{206}Tl and ^{207}Tl.

Warburton 1999
Zhi et al. 2013

FF decay operators

There are 8 linearly independent operators involved in the description of the first-forbidden β-decay that can be collected in five groups:

- w and w' scalar-axial $\quad g_A\,[r \times \sigma]^0$
- u and u' vector-axial $\quad g_A\,[r \times \sigma]^1$
- x and x' vector-vector $\quad g_V\,r$
- z tensor-axial $\quad g_A\,[r \times \sigma]^2$
- $\xi' v$ recoil-axial $\quad g_A\,\gamma_5$

Nuclear shape factor C(W)

The decay rate for the β decay can be expressed as

$$\lambda_{if} = \frac{ln(2)}{\kappa} f_0 \overline{C_\alpha(W)}$$
$$C_{\text{F}}(W) = \text{B(F)}$$
$$C_{\text{GT}}(W) = \text{B(GT)}$$
$$C_{\text{FF}}(W) = C(w, w', u, u', x'x', \xi'v, W)$$

where W is the total energy of the e^- in m_e units.

Averaged nuclear shape factor $\overline{C(W)}$

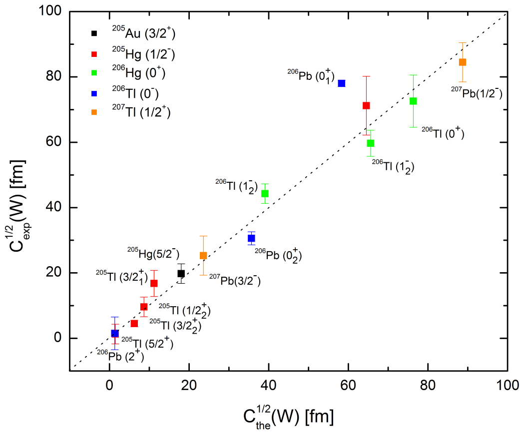
$$f = \int_1^{W_0} C(W) F(Z, W)(W^2 - 1)^{1/2} W (W_0 - W)^2 dW$$

$$f_0 = \int_1^{W_0} F(Z, W)(W^2 - 1)^{1/2} W (W_0 - W)^2 dW$$

$$\overline{C(W)} = f / f_0$$

where $\mathrm{W}_0 = (M_i - M_f)/m_e$ and $F(Z, W)$ is the Fermi function.

Normalization of the FF decay operators

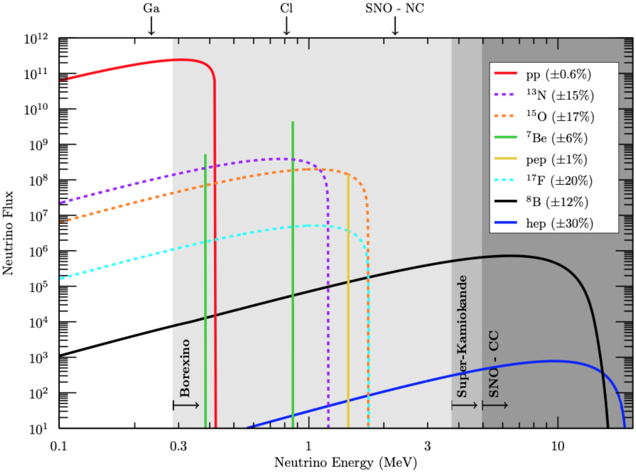


$$q(w) = 0.64$$
$$q(u) = 0.40$$
$$q(x) = 0.53$$
$$q(z) = 0.45$$
$$q(\xi' v) = 1.27$$

$$T^{the}_{1/2}(^{205}\mathrm{Tl})_{\beta_b} = 138 \text{ d}$$
$$T^{exp}_{1/2}(^{205}\mathrm{Tl})_{\beta_b} = 295 \text{ d}$$

Solar ν absorption

The neutrino capture ${}^{205}Tl + \nu_e \rightarrow {}^{205}Pb + e^-$ has an energy threshold of $E_\nu \geq 52$ keV, by far the smallest threshold for any known neutrino-induced nuclear reaction.



Solar ν absorption

The ν_e capture rate R in SNU (10^{-36} captures per target atom per second) is obtained from the cross-section $\sigma_{if}(W_\nu)$:

$$\sigma_{if}(W_\nu) = \frac{G_F^2 V_{ud}^2 (m_e c^2)^2}{\pi(\hbar c)^4} p_e W\, F(Z, W) C(W_\nu)$$

$$R_i = 10^{36} \sum_f \int \sigma_{if}(W_\nu)\phi_i(W_\nu) dW_\nu$$

Where ϕ_i are the electron neutrino fluxes for $i =$ pp, ^{7}Be, ^{8}B
Assuming an electron-neutrino survival probability of 0.54 for pp and ^{7}Be and 0.36 for ^{8}B this leads to
$\lambda_{\nu_e}(^{205}\text{Tl}) = 8.5 \times 10^{-35} s^{-1}$.
$\lambda_{EC}(^{205}\text{Pb}) = 1.40 \times 10^{-15} s^{-1}$

Agostini et al. 2018

Solar ν absorption

The contributions to the capture rate in SNU for the individual fluxes without correction for the oscillation of neutrinos are

Flux	$\overline{M} + M$	$\overline{M}$	BU	KSZ
R_{pp}	121 $\pm$21	118 $\pm$21	173	-
$R_{^7Be}$	37 $\pm$4	33 $\pm$4	34	-
$R_{^8B}$	2.6 $\pm$0.5	2.5 $\pm$0.5	46	-
R_{tot}	161 $\pm$25	154 $\pm$25	263	[100.2, 132.4]

Bahcall and Ulrich 1988
Kostensalo, Suhonen and Zuber 2020

S-process chronometry

The s-process production ratio of ^{205}Pb to ^{204}Pb is given by

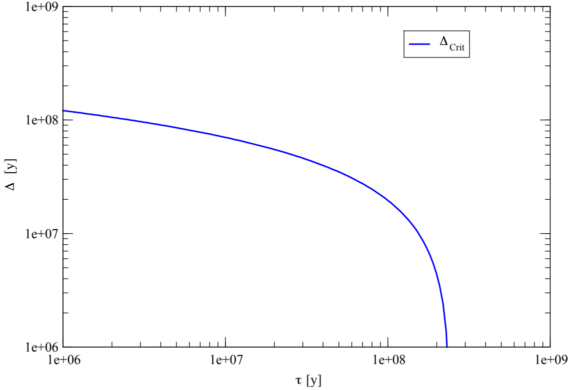
$$\left.\frac{N_{^{205}\mathrm{Pb}}}{N_{^{204}\mathrm{Pb}}}\right|_{t>500y} = (\lambda_{EC}/\lambda)[N_{205}(0)/N_{^{204}\mathrm{Pb}}(0)]$$

where $\lambda = \lambda_{\beta_b} + \lambda_{EC}$ and $N_{205} = N_{^{205}\mathrm{Pb}}(0) + N_{^{205}\mathrm{Tl}}(0)$ and $N_{205}(0)/N_{^{204}\mathrm{Pb}}(0)$ is almost independent on T and n_e.
At the time of solidification of the meteorite $\tau + \Delta$

$$\left.\frac{N_{^{205}\mathrm{Pb}}}{N_{^{204}\mathrm{Pb}}}\right|_{\tau+\Delta} = \left.\frac{N_{^{205}\mathrm{Pb}}}{N_{^{204}\mathrm{Pb}}}\right|_{t} \frac{exp(-\lambda_e^{ter}\Delta)}{\lambda_e^{ter}\tau}$$

with τ being the entire timespan of the nucleosynthesis.

Schramm and Wasserburg 1970
Yokoi, Takahashi and Arnould 1985



$$\lambda_{EC}/\lambda \geq 10^{-3}$$

$$\Delta > \Delta_{Crit} \equiv -2.2 \times \ln(4.1 \times 10^{-3}\tau)$$

$$\left.\frac{N_{^{205}Pb}}{N_{^{204}Pb}}\right|_{\tau+\Delta} < 9 \times 10^{-5}$$

Huey and Kohman 1972

Summary

- The measured decay rate of the bound β-decay of fully ionized ^{205}Tl has been used to determine the different weak processes involving ^{205}Tl and ^{205}Pb.
- It remains to compute the rates for a broad range of conditions and perform s-process simulations to predict the ^{205}Tl/^{205}Pb ratio.

SFB 1245

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GSI

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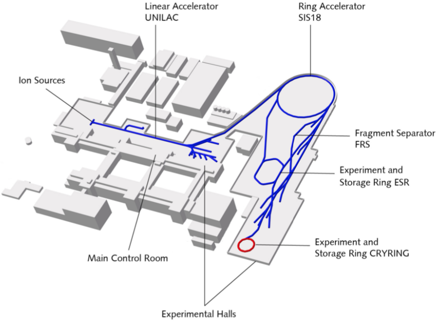
Experimental measurement

The experiment, conducted amidst the corona pandemic, put into use almost all of the experimental facilities at GSI:

- the ion-source,
- the UNILAC (UNIversal Linear ACcelerator),
- the Heavy Ion Synchrotron SIS-18,
- the FRagment Separator (FRS),
- the Experimental Storage Ring (ESR).



FF decay operators

While the allowed transitions require only one operator each, there are 9 operators involved in the description of the first-forbidden decay and they are:

$$w = -g_A\sqrt{3}\frac{\langle f||\sum_k r_k[C_1^k \times \sigma^k]^0 t_-^k||i\rangle}{\sqrt{2J_i+1}}$$

$$x = -\frac{\langle f||\sum_k r_k C_1^k t_-^k||i\rangle}{\sqrt{2J_i+1}}$$

$$u = -g_A\sqrt{2}\frac{\langle f||\sum_k r_k[C_1^k \times \sigma^k]^1 t_-^k||i\rangle}{\sqrt{2J_i+1}}$$

$$z = 2g_A\sqrt{2}\frac{\langle f||\sum_k r_k[C_1^k \times \sigma^k]^2 t_-^k||i\rangle}{\sqrt{2J_i+1}}$$

FF decay operators

$$w' = -g_A\sqrt{3}\frac{\langle f||\sum_k \frac{2}{3} r_k I(1,1,1,r_k)[C_1^k \times \sigma^k]^0 t_-^k||i\rangle}{\sqrt{2J_i+1}}$$

$$x' = -\frac{\langle f||\sum_k \frac{2}{3} r_k I(1,1,1,r_k) C_1^k t_-^k||i\rangle}{\sqrt{2J_i+1}}$$

$$u' = -g_A\sqrt{2}\frac{\langle f||\sum_k \frac{2}{3} r_k I(1,1,1,r_k)[C_1^k \times \sigma^k]^1 t_-^k||i\rangle}{\sqrt{2J_i+1}}$$

$$\xi' v = -\frac{g_A\sqrt{3}}{M}\frac{\langle f||\sum_k [\sigma^k \times \nabla^k]^0 t_-^k||i\rangle}{\sqrt{2J_i+1}}$$

$$\xi' y = -\frac{1}{M}\frac{\langle f||\sum_k \nabla^k t_-^k||i\rangle}{\sqrt{2J_i+1}}$$

FF decay operators

where

$$C_{lm} = \sqrt{\frac{4\pi}{2l+1}} Y_{lm}$$

with Y_{lm} being the spherical harmonics and

$$\begin{aligned} I(1,1,1,r) &= \frac{3}{2}\left[1 - \frac{1}{5}\left(\frac{r}{R}\right)^2\right] \quad \text{for} \quad 0 \le r \le R \\ &= \frac{3}{2}\left[\frac{R}{r} - \frac{1}{5}\left(\frac{R}{r}\right)^3\right] \quad \text{for} \quad r \ge R \end{aligned}$$

FF decay operators

It is however possible to obtain the matrix element of the operator $\xi' y$ with the following relation based on the conserved vector current theory:

$$\xi' y = E_\gamma x$$

where $E_\gamma x$ is the difference between the isobaric analog of the initial and the final state. As $E_\gamma > 1$ this has the effect of enhancing the matrix element of the operator x so that $\xi' y >> x$.

Operators quenching

While the shell model usually gives a good account of the strength distributions, it usually overestimates the total strength. For Gamow-Teller transitions, this can be accounted for introducing an effective operator $GT_{eff} = q\,GT$.
Since the FF decay involves operators of rank 0, 1, and 2, it is reasonable to expect a different behaviour of between of them.
To determine these different quenching factors, a least-squares fit was performed on the shell-model calculations for experimentally known β-decays in the proximity of ^{205}Tl.

Operators quenching

Initial	Final	$(\overline{C(W)})^{1/2}_{theo}[fm]$	$(\overline{C(W)})^{1/2}_{exp}[fm]$
^{205}Au($\frac{3}{2}^{+}$)	^{205}Hg($\frac{5}{2}^{-}_{1}$)	18.0	20(3)
^{205}Hg($\frac{1}{2}^{-}$)	^{205}Tl($\frac{1}{2}^{+}_{1}$)	64.5	71.3(9)
^{205}Hg($\frac{1}{2}^{-}$)	^{205}Tl($\frac{1}{2}^{+}_{2}$)	8.6	9(3)
^{205}Hg($\frac{1}{2}^{-}$)	^{205}Tl($\frac{3}{2}^{+}_{1}$)	11.2	17(4)
^{205}Hg($\frac{1}{2}^{-}$)	^{205}Tl($\frac{3}{2}^{+}_{2}$)	6.2	5(1)
^{205}Hg($\frac{1}{2}^{-}$)	^{205}Tl($\frac{5}{2}^{+}_{1}$)	1.5	1.3(3)
^{206}Hg(0^{+})	^{206}Tl(0^{-}_{1})	11.2	17(4)
^{206}Hg(0^{+})	^{206}Tl(1^{-}_{1})	6.2	5(1)
^{206}Hg(0^{+})	^{206}Tl(1^{-}_{2})	1.5	1.3(3)

Operators quenching

Initial	Final	$(\overline{C(W)})^{1/2}_{theo}[fm]$	$(\overline{C(W)})^{1/2}_{exp}[fm]$
^{206}Tl(0^-)	^{206}Pb(0^+_1)	76.3	78.0(1)
^{206}Tl(0^-)	^{206}Pb(0^+_2)	35	31(2)
^{206}Tl(0^-)	^{206}Pb(2^+_1)	1.31	1.52(5)
^{207}Tl($\frac{1}{2}^+$)	^{207}Pb($\frac{1}{2}^-_1$)	88.6	84.5(6)
^{207}Tl($\frac{1}{2}^+$)	^{207}Pb($\frac{3}{2}^-_1$)	23.6	25.3(6)

With these constraints the quenching factors for the operators are:

$$q(\xi' v) = 1.27 \qquad q(w) = q(w') = 0.64$$
$$q(x) = q(x') = 0.53 \qquad q(u) = q(u') = 0.40$$
$$q(z) = 0.45$$

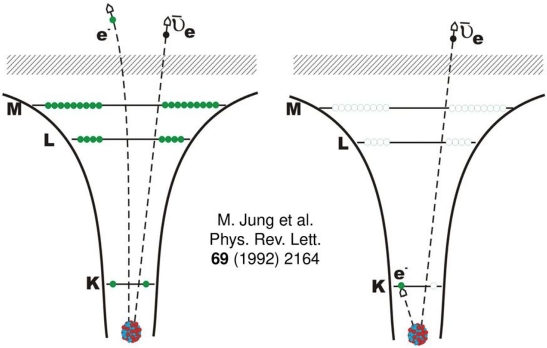
Bound-state EC and β_b-decay

$n \rightarrow p + e^- + \overline{\nu}_e.$

- Independent from atomic structure

$n \rightarrow p + e_b^- + \overline{\nu}_e.$
$p + e^- \rightarrow n + \overline{\nu}_e.$

- Interplay atomic-nuclear structure



Nuclear shape function

In the case of neutrino absorption the nuclear shape function takes the form of:

$$\begin{aligned} C_\nu(W) = {} & [M_0(1,1)]^2 + [m_0(1,1)]^2 - \frac{2\mu_1\gamma_1}{W} M_0(1,1) m_0(1,1) \\ & + [M_1(1,1)]^2 + [m_1(1,1)]^2 - \frac{2\mu_1\gamma_1}{W} M_1(1,1) m_1(1,1) \\ & + [M_1(1,2)]^2 + [M_2(1,2)]^2 + \lambda_2 [M_1(2,1)]^2 + \lambda_2 [M_2(2,1)]^2 \end{aligned}$$

where $\mu_1 \simeq 1$ and $\gamma_1 = \sqrt{1-(\alpha Z)^2}$.

Nuclear shape function

On the other hand, in the case of forbidden bound decay the nuclear shape function takes the form of:

$$C_x = [M_0(1,1) + \kappa_x m_0(1,1)]^2 + [M_1(1,1) + \kappa_x m_1(1,1)]^2 + [M_1(1,2)]^2 + [M_2(1,2)]^2$$

where, since the only energetically allowed decay of the electron can be to the K shell, $\kappa_x = -1$

Nuclear momenta for β_b

The nuclear matrix elements intervene in the nuclear shape factor C_x as:

$$M_0(1,1) = \xi' v + \xi w' + \frac{1}{3} W_0 w$$

$$m_0(1,1) = \frac{1}{3} m_e w$$

$$M_1(1,1) = -\xi' y + W_0 x + \xi(x' + u') + \frac{1}{3}(W_e - q_x) u$$

$$m_1(1,1) = \frac{1}{3} m_e (x + u)$$

Nuclear momenta for β_b

$$M_1(1,2) = -\frac{1}{3} q_x (\sqrt{2} x + \sqrt{\frac{1}{2}} u)$$

$$M_1(2,1) = -\frac{1}{3} p_e (\sqrt{2} x + \sqrt{\frac{1}{2}} u)$$

$$M_2(1,2) = -\frac{\sqrt{3}}{2} q_x z$$

$$M_2(2,1) = -\frac{1}{3} p_e z$$

where $\xi = \alpha Z/(2R)$, with R the radius of the nuclear charge distribution in units of λ_e.

Nuclear momenta for ν absorption

Due to the different energetics of the transition a slight adjustment of the nuclear momenta is needed.

$$M_0(1,1) = \xi' v + \xi w' + \frac{1}{3} W_0 w$$

$$m_0(1,1) = \frac{1}{3} m_e w$$

$$M_1(1,1) = -\xi' y + W_0 x + \xi(x' + u') + \frac{1}{3}(W_e + q)u$$

$$m_1(1,1) = \frac{1}{3} m_e (x + u)$$

Nuclear momenta for ν absorption

$$M_1(1,2) = +\frac{1}{3}q(\sqrt{2}x + \sqrt{\frac{1}{2}}u)$$
$$M_1(2,1) = -\frac{1}{3}p_e(\sqrt{2}x + \sqrt{\frac{1}{2}}u)$$
$$M_2(1,2) = +\frac{\sqrt{3}}{2}q\,z$$
$$M_2(2,1) = -\frac{1}{3}p_e z$$

where, in contrast with the case of β_b it holds that $q = W_e + W_0$.

Nuclear shape function

It is worth noting that C_x and $C_\nu(W)$ share a term C that is independent from W:

$$C = [M_0(1,1)]^2 + [m_0(1,1)]^2 + [M_1(1,1)]^2 + [m_1(1,1)]^2$$

And in particular both C_x and $C_\nu(W)$ are dominated by the combination of the terms $[M_0(1,1)]^2 + [M_1(1,1)]^2$