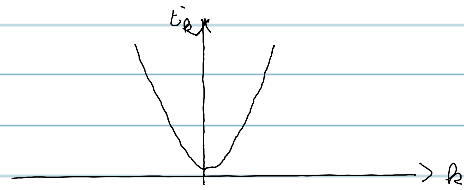
13.6.14

MFE:

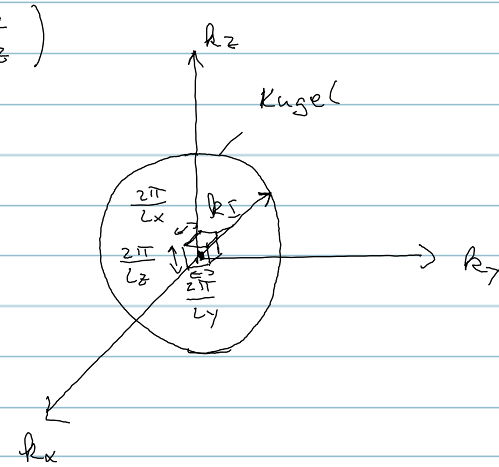
1 dim $\quad \Psi = e^{ikx} \qquad k = \frac{p}{\hbar}$

$\hookrightarrow \quad E_k = \frac{\hbar^2 k^2}{2m} \qquad k = \pm \frac{2\pi}{L} n \qquad n = 0,1,2 \ldots$



→ nicht kontinuierlich → diskret

3 dim: $\quad \hookrightarrow E_k = \frac{\hbar^2}{2m} \left(k_x^2 + k_y^2 + k_z^2 \right)$



$E_F = \frac{\hbar^2 k_F^2}{2m}$

Vol. Fermi-Kugel: $V = \frac{4}{3} \pi k_F^3$

Anzahl N: $\quad N = 2 \frac{\frac{4}{3}\pi k_F^3}{\left(\frac{2\pi}{L}\right)^3}$

$L^3 = V$

$k_F^3 = \frac{N \cdot \pi^2}{\cancel{2} \cdot V} \cdot \frac{1}{\cancel{4}} \cdot \cancel{8} \cdot 3$

$k_F^3 = \frac{3\pi^2 N}{V}$

$k_F = \left(\frac{3\pi^2 N}{V} \right)^{1/3}$ $\quad$ abhängig von Anzahl der e^- pro Volumen

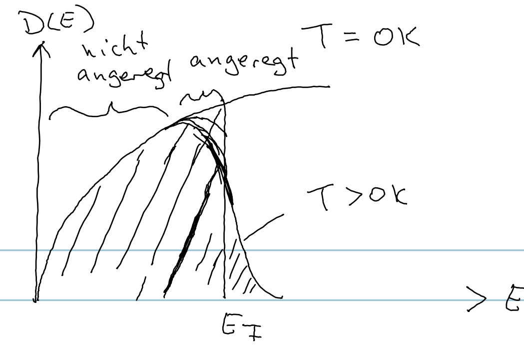
$E_F = \frac{\hbar^2}{2m} \left(\frac{3\pi^2 N}{V} \right)^{\frac{2}{3}}$

Bsp:

Metall	E_F (eV)
Li	4,72
Na	3,23
Cu	7,0
Al	11,63

Zustandsdichte:

Def: $\quad D(E) = \frac{dN}{dE}$



$$\boxed{D(E) \propto E^{1/2}}$$

Wärmekapazität

z.B. Na: klassisch: $C'_{el} = \frac{3}{2} R$

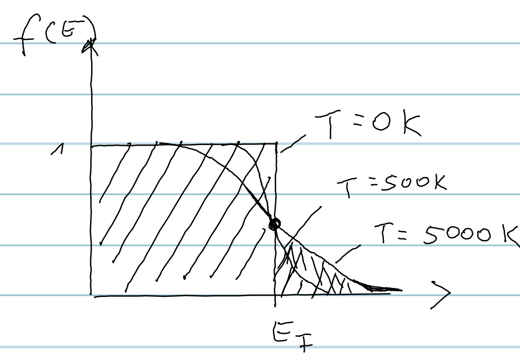
experimentell: $C_{el} \simeq 10^{-2} \, C'_{el}$?

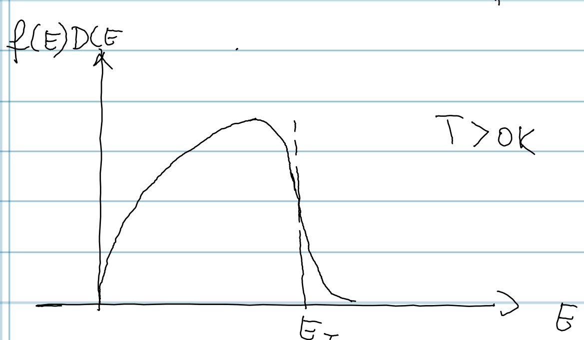
T - Abhängig. Besetzungswahrscheinlichkeit ? → Fermi-Dirac-Statistic

Besetzungswahrsch.: $f(E) = \frac{1}{e^{E-E_F/kT} + 1}$

$\to e^{-\infty} \to 0$

T = 0K	$f(E) = 0$	$E > E_F$
	$f(E) = 1$	$E < E_F$
T > 0K	$f(E) = 1/2$	$E = E_F$





Anteil e^-: $\simeq \frac{kT}{E_F}$

N_{e^-} mit kT: $C_{el} \to \Delta E \simeq N kT \left(\frac{kT}{E_F}\right)$

$\simeq N \frac{k^2 T^2}{E_F}$

$$C_{el} = \frac{\partial \Delta E}{\partial T} = 2N \frac{k^2}{E_F} T$$

↳ gesamte Wärmekapazität $C_V = C_{el} + C_{Gitter}$

$= aT + bT^3$

↑ Debye T^3-Gesetz

4.2. e^- im periodischen Potential

Freie e^- Modell (FEM): Probleme:

→ unendliche Anzahl Energiezustände
→ elektr. Leitfähigkeit ~~z~~ nimmt mit $\frac{N}{V}$ zu
Li → C aber Diamant Isolator

periodische Potential!

Bsp: 1 dim

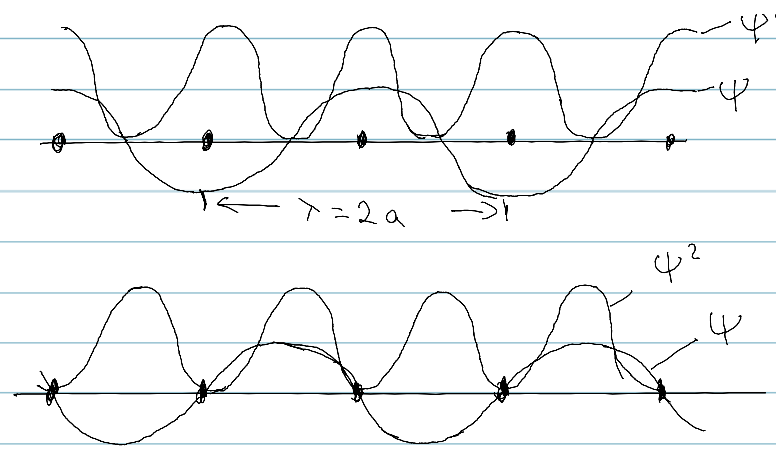
Bragg: $2d\sin\Theta = n\lambda$

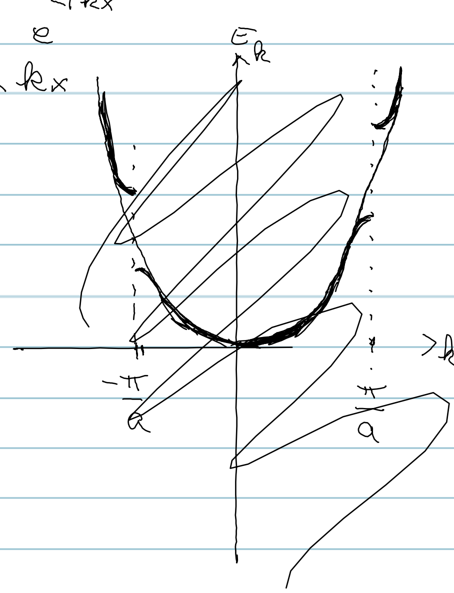
Rückstreuung $\Theta = \frac{\pi}{2}$

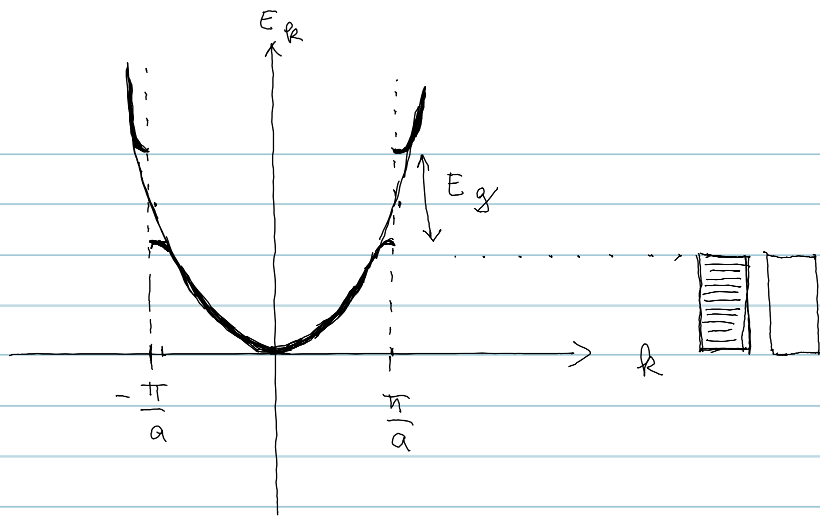
$2a \cdot 1 = n \cdot \lambda = n \cdot \frac{2\pi}{\underbrace{k}_{\text{Wellenvektor}}}$ $\qquad k = \frac{n \cdot 2\pi}{2a} = n\frac{\pi}{a}$

$\Psi \propto e^{ikx} + e^{-ikx}$ $\qquad \Psi \propto e^{ikx} - e^{-ikx}$

$\propto 2\cos kx$ $\qquad \propto 2\sin kx$







1dim SG $\quad -\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V\psi = E\psi$

$$V(x) = V(x+a)$$

$$\boxed{\psi(x) = e^{ikx}\, u_k(x)}$$ Bloch-Theorem

$u_k(x)$ beschreibt Periodizität
$u_k(x) = u_k(x+a)$

$$\boxed{\text{3 dim} \quad \psi(r) = e^{i\vec{k}\vec{r}}\, u_k(r)}$$

Beweis: Kreis Na $\quad L$

$$\psi(x+a) = c\,\psi(x)$$
$$\psi(x+Na) = \psi(x) = c^N\psi(x)$$
$$c^N = 1 \quad \rightarrow \quad c = e^{i2\pi n/N} \qquad (n = 0, 1, 2, \dots, N-1)$$

$$\psi(x+a) = u_k(x)\, e^{i2\pi n x/Na}$$

$$k = \frac{2\pi n}{Na} = n\frac{2\pi}{L}$$