TECHNISCHE UNIVERSITÄT DARMSTADT

FLUID SYSTEM TECHNIK

Evaluation von FEM

http://evaluation.tu-darmstadt.de/evasys/online/

Tutorium FEM: 19.07. – 22.07.
(Mo) (Do.)

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Sommersemester 2010
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Zusammenhang zw. Zirkulation und Auftrieb

0. Wiederholung:

Drallsatz: zeitliche Änderung des Dralls ist gleich dem Moment

$$\vec{M} = \frac{D\vec{D}}{Dt} \qquad \vec{D} = \int_V \vec{x} \times \rho \vec{c}\, dV$$

für Turbomaschinen => axiale Komponente

Eulersche Turbinengleichung:

$$dM_z = d\dot{m}\left(c_{u2} r_2 - c_{u1} r_1\right)$$

$$dM_z = d\dot{m}\left(\frac{\Gamma_2}{2\pi} - \frac{\Gamma_1}{2\pi}\right)$$

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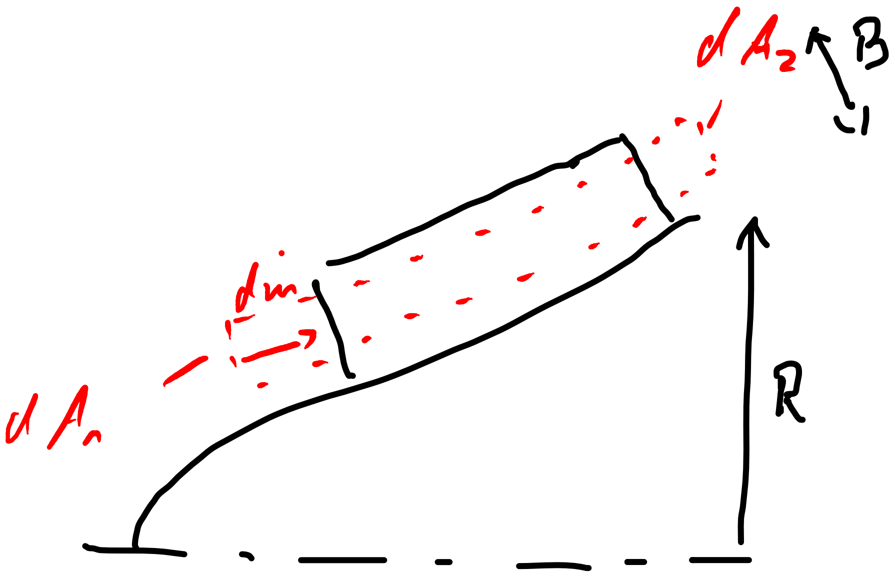


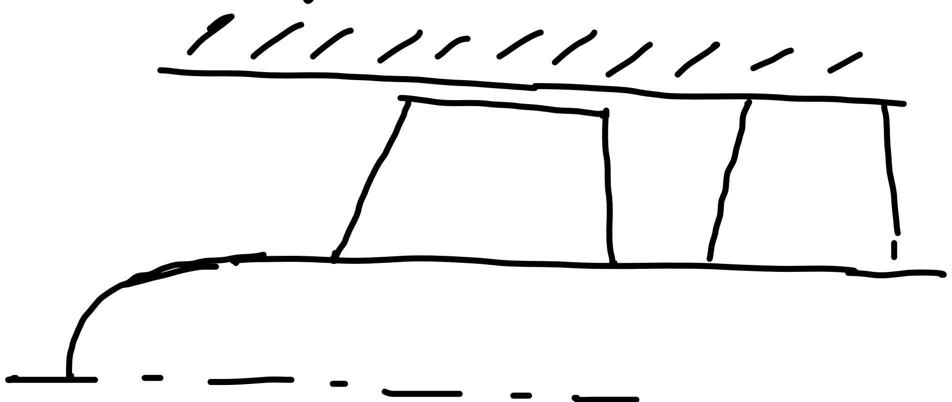
$B \gg R$: $dM_z = \frac{d\dot m}{2\pi}(\Gamma_2 - \Gamma_1)$

- Windkraftmaschine
- Propeller
- i.d.R. bei Axialmasch.

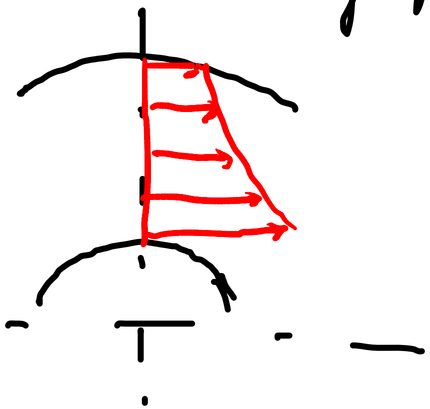
$B \ll R$: $M_z = \frac{\dot m}{2\pi}(\Gamma_2 - \Gamma_1)$

- i.d.R. bei Radialmaschinen

Wirbelflußmaschine:



Γ = const: Wirbelflußmaschine



$c_u = \frac{\Gamma}{2\pi r}$

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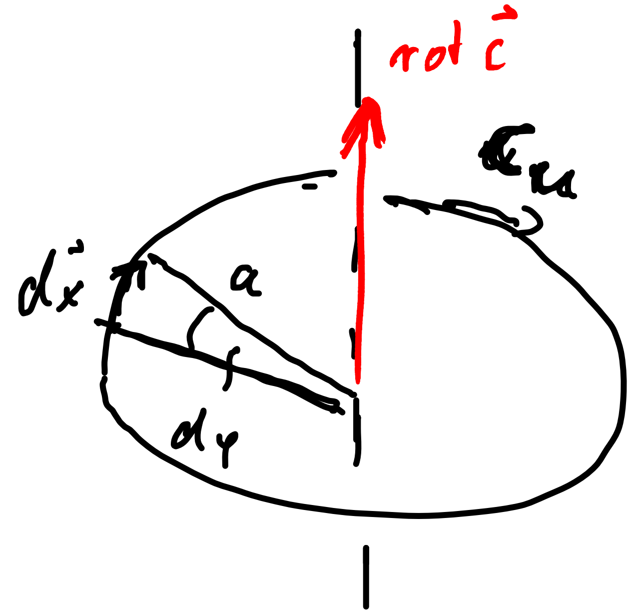
1. Zirkulation und 1. Helmholtzsche Wirbelsatz

$$\Gamma = \oint \vec{c} \cdot d\vec{x}$$

$$= \int_0^{2\pi} c_u \, a \, d\varphi$$



Satz von Stokes:

$$\Gamma = \oint \vec{c} \cdot d\vec{x} = \int_S (\text{rot}\,\vec{c}) \cdot \vec{n} \, dS$$

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Vorlesung 8

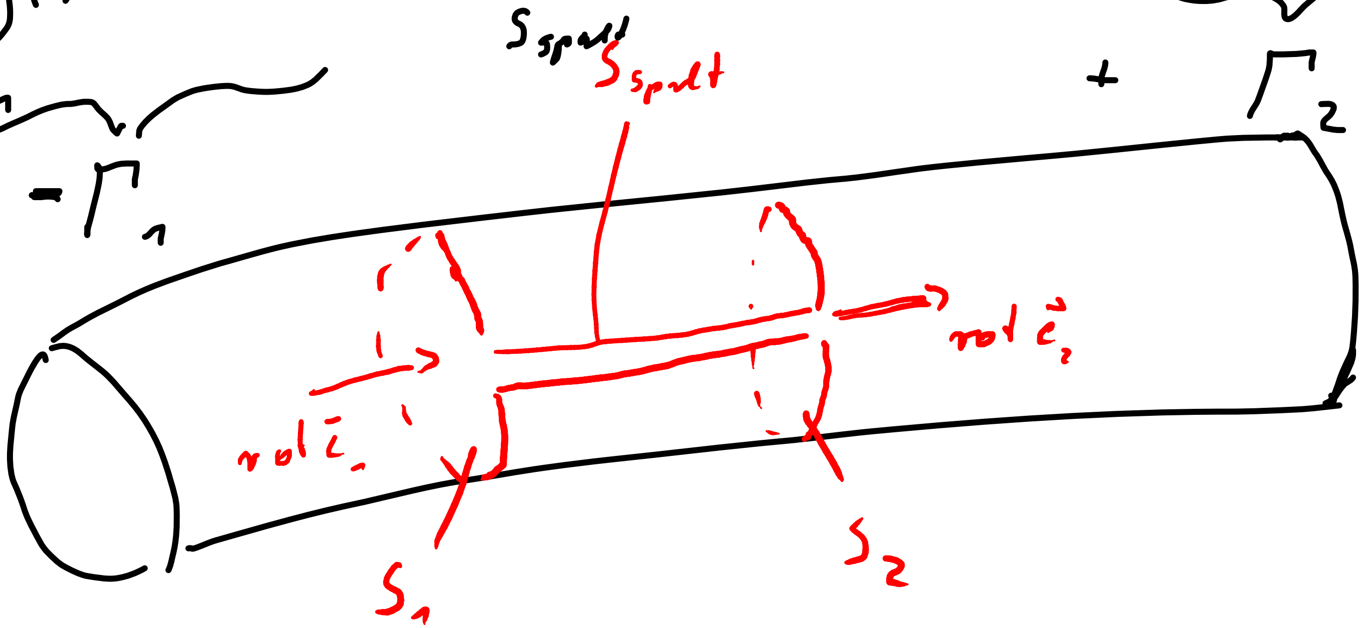
1. Helmholtzscher Wirbelsatz:

Zirkulation einer Wirbelröhre ist längs dieser Röhre konstant.

$$\int (\mathrm{rot}\,\vec{c})\cdot\vec{n}\,dS = 0$$

$$0 = \underbrace{\int_{S_1} (\mathrm{rot}\,\vec{c})\,\vec{n}\,dS_1}_{-\Gamma_1} + \int_{S_{spalt}} (\mathrm{rot}\,\vec{c})\,\vec{n}\,dS_{spalt} + \underbrace{\int (\mathrm{rot}\,\vec{c})\,\vec{n}\,dS_2}_{\Gamma_2}$$

$$0 = -\Gamma_1 + \Gamma_2$$



$$\Gamma_1 = \Gamma_2$$

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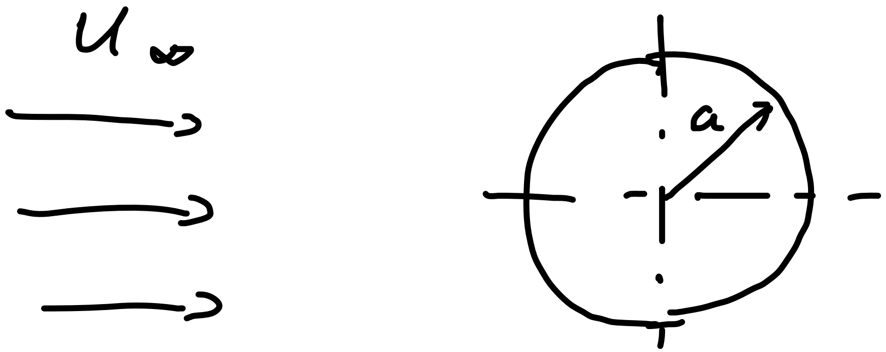
2. Zirkulation und Auftrieb am Kreiszylinder



I) ebene Potentialströmung

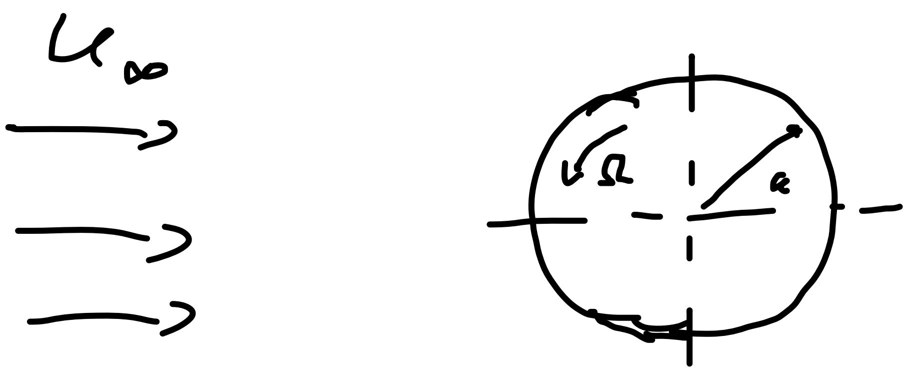
$$\frac{\partial \phi}{\partial x_i} = u_i$$

$$\phi = U_\infty x$$

$$\frac{\partial \phi}{\partial x} = U_\infty$$

II) Geschwindigkeiten

III) Auftriebskraft



$$\phi = \phi_1 + \phi_2 + \phi_3 + \phi_4$$

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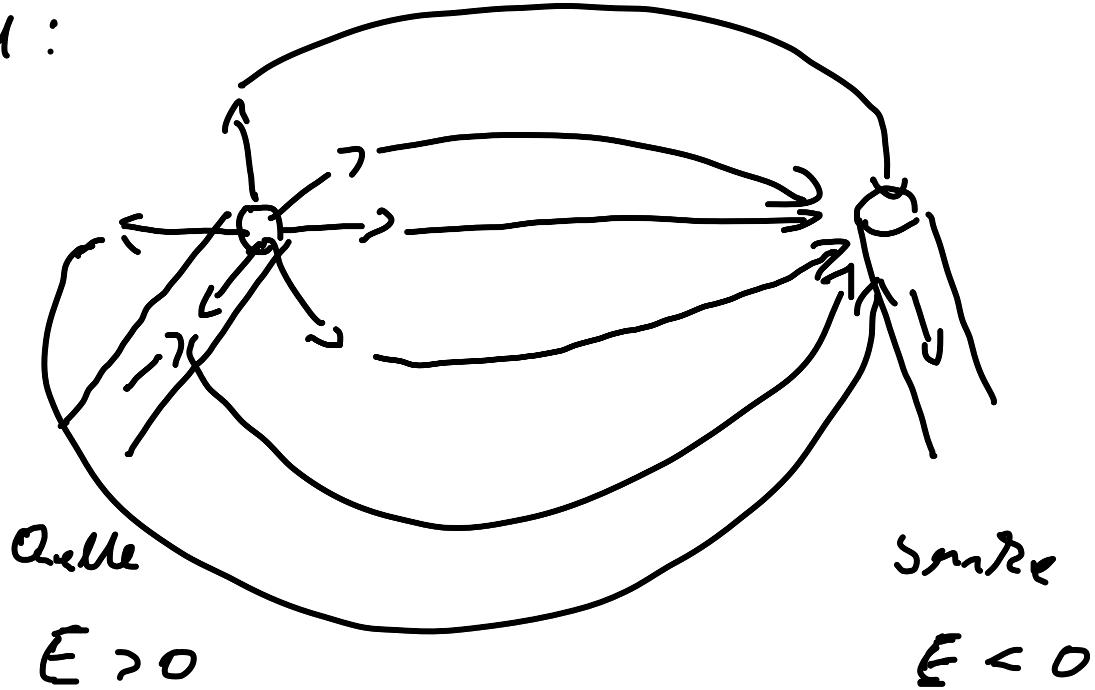
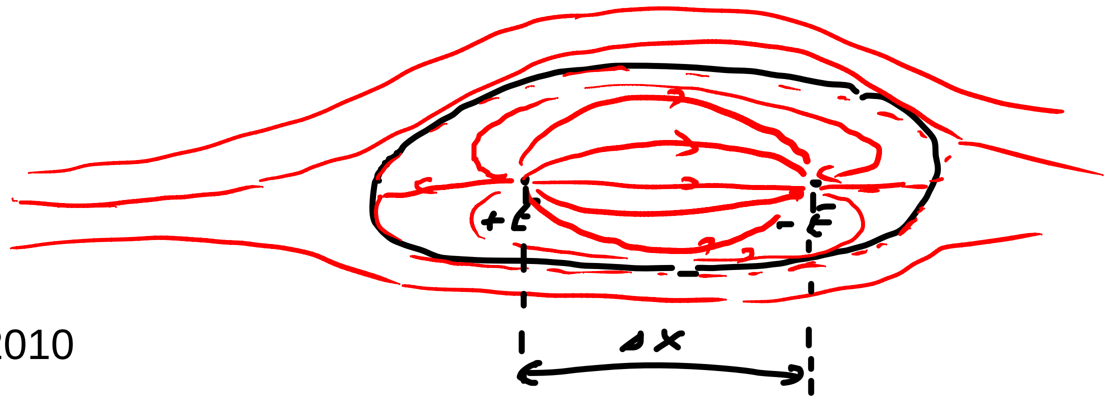
Parallele Strömung: $\Phi = u_\infty x$

Dipolpotential:





$$\lim_{\substack{\Delta x \to 0 \\ E \to \infty}} E \, \Delta x = M$$

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$$\Phi_{Quell} = \frac{E}{2\pi} \ln r \qquad \frac{\partial \Phi_{Quell}}{\partial r} = u_r = \frac{E}{2\pi r}$$

$$\Phi_{Dipol} = \nabla \Phi_{Quell} \cdot \vec{e}_x$$

$$= \frac{E}{2\pi r} \nabla r \cdot \vec{e}_x$$

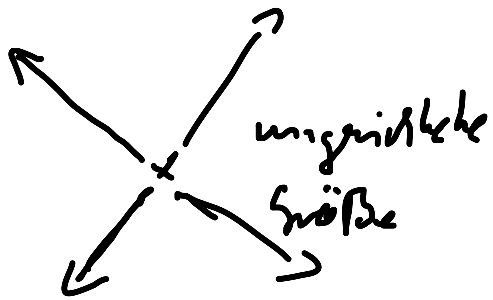
$$\downarrow$$

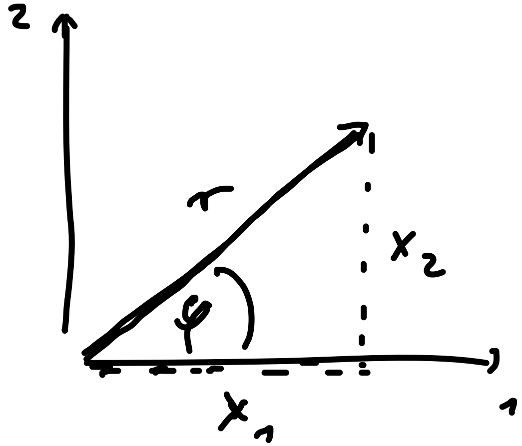
$$r = \sqrt{x_i x_i}$$

$$\frac{\partial r}{\partial x_i} = \frac{1}{2} \frac{x_j + x_j}{\sqrt{x_i x_i}}$$

$$= \frac{2 x_j}{2\sqrt{x_i x_i}} = \frac{x}{r} = \cos\varphi$$

$$= \frac{E}{2\pi r} \cos\varphi$$





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Wirbelpotential:

$$\phi_{Wirbel} = \frac{\Gamma}{2\pi}\varphi \qquad u_\varphi = \frac{1}{r}\frac{\partial \phi_{Wirbel}}{\partial\varphi} = \frac{\Gamma}{2\pi r}$$

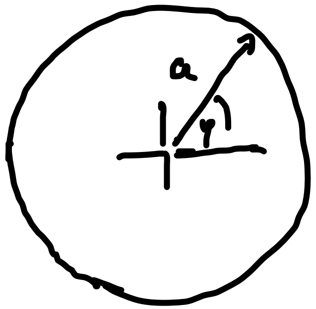
$$\phi = \phi_{Parallel} + \phi_{Dipols} + \phi_{Wirbels} \qquad \Big| \; x = r\cos\varphi$$

$$= U_\infty x + \frac{E}{2\pi r}\cos\varphi + \frac{\Gamma}{2\pi}\varphi$$

?

$$= u_\infty r\cos\varphi + \frac{E}{2\pi r}\cos\varphi + \frac{\Gamma}{2\pi}\varphi$$

$u_r = 0$



$$u_r = \frac{\partial\phi}{\partial r} = \left(u_\infty \cos\varphi - \frac{E}{2\pi r^2}\cos\varphi\right)\Bigg|_{r=a} = 0$$

$$\underline{E = 2\pi a^2 u_\infty}$$

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II Geschwindigkeit:

$$\boxed{\Phi = U_\infty r \cos\varphi + U_\infty \frac{a^2}{r}\cos\varphi + \frac{\Gamma}{2\pi}\varphi}$$

$$u_\varphi = \frac{1}{r}\frac{\partial \Phi}{\partial \varphi}\bigg|_{r=a} = -U_\infty \sin\varphi - U_\infty \frac{a^2}{r^2}\sin\varphi + \frac{\Gamma}{2\pi r}\bigg|_{r=a}$$

$$\boxed{u_\varphi = -2U_\infty \sin\varphi + \frac{\Gamma}{2\pi a}}$$

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FLUID SYSTEM TECHNIK

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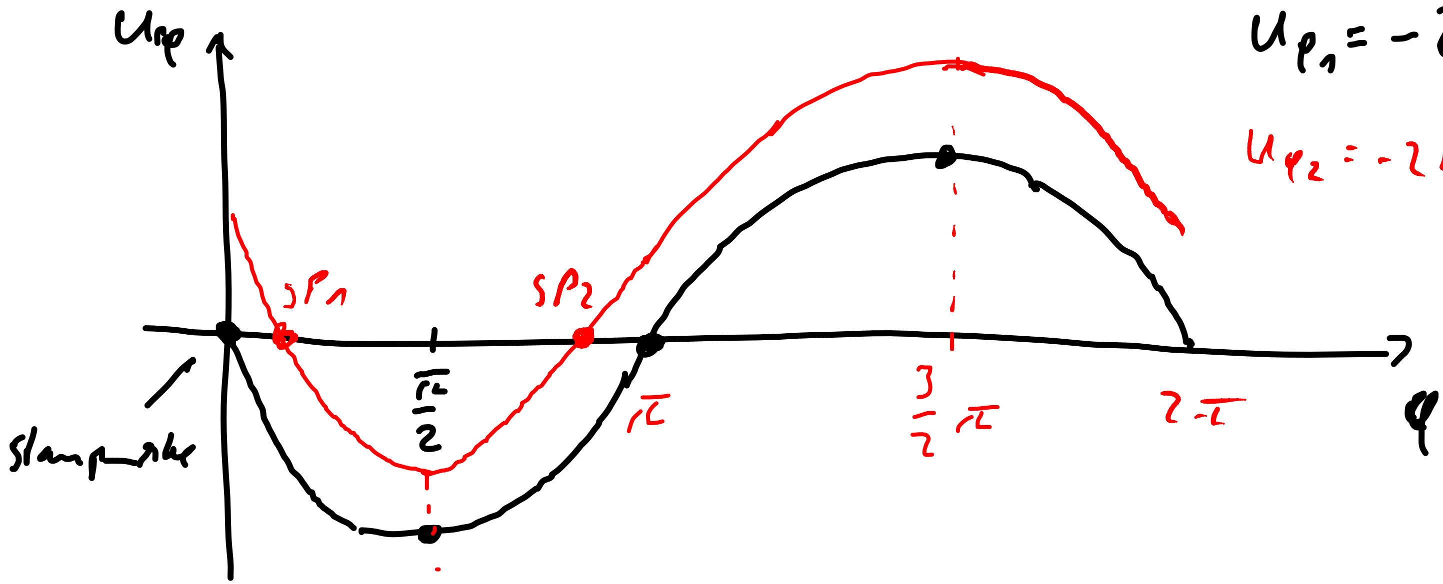
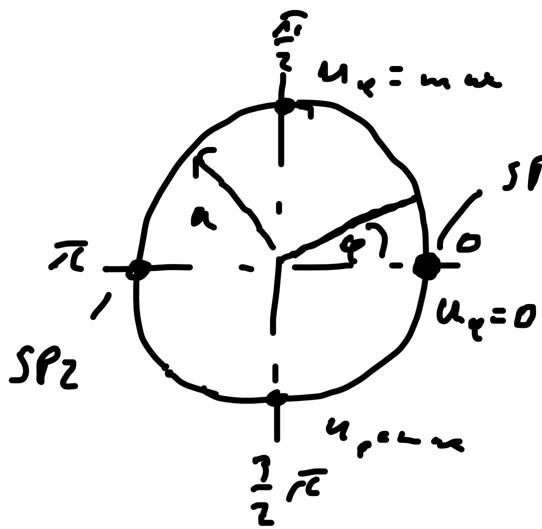
$$u_\varphi = -2U_\infty \sin\varphi + \frac{\Gamma}{2\pi a}$$



$$u_{\varphi_1} = -2U_\infty \sin\varphi$$

$$u_{\varphi_2} = -2U_\infty \sin\varphi + \frac{\Gamma}{2\pi a}$$



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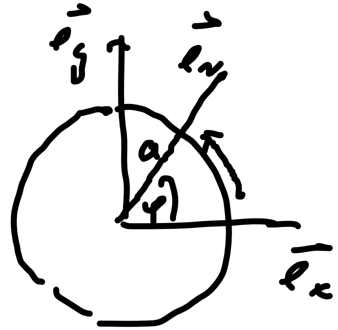
III Auftriebskraft

$$u_\varphi = -2\,U_\infty \sin\varphi + \frac{\Gamma}{2\pi a}$$

$$\vec{F} = \oint -p\big|_a\, \vec{n}\, ds \qquad ds = a\, d\varphi$$

$$= \int_0^{2\pi} -p\big|_a\, \vec{e}_r\, a\, d\varphi$$



$$p_\infty + \frac{\rho}{2}U_\infty^2 = p + \frac{\rho}{2}u_\varphi^2$$

$$c_p = \frac{p - p_\infty}{\frac{\rho}{2}U_\infty^2} = -\left(\frac{u_\varphi}{U_\infty}\right)^2$$

$$= -\left(-2\sin\varphi + \frac{\Gamma}{2\pi U_\infty a}\right)^2$$

$$c_p = -4\sin^2\varphi + 2\frac{\Gamma}{\pi U_\infty a}\sin\varphi - \left(\frac{\Gamma}{a\,2\pi U_\infty}\right)^2$$

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Fluidenergiemaschinen
Vorlesung 8

$$\vec{F} = \int_0^{2\pi} -p\Big|_a \vec{e}_r \, a \, d\varphi$$

$$F_x = \int_0^{2\pi} -p \cos\varphi \, a \, d\varphi \qquad F_y = \int_0^{2\pi} -p \sin\varphi \, a \, d\varphi$$

$$\frac{p - p_\infty}{\frac{\varrho}{2} u_\infty^2} = c_p = \underbrace{-4\sin^2\varphi + \frac{2\Gamma}{\pi a u_\infty} - \left(\frac{\Gamma}{2\pi a u_\infty}\right)^2}$$

$$F_x = \int_0^{2\pi} -c_p \cos\varphi \, d\varphi \, \frac{\varrho}{2} a U_\infty^2 \qquad F_y = \int_0^{2\pi} -c_p \sin\varphi \, \frac{\varrho}{2} a U_\infty^2$$

$$\frac{F_x}{\frac{\varrho}{2} a U_\infty^2} = \int_0^{2\pi} -c_p \cos\varphi \, d\varphi = 0 \qquad \frac{F_y}{\frac{\varrho}{2} a U_\infty^2} = \int_0^{2\pi} -c_p \sin\varphi \, d\varphi$$

$$\int_0^{2\pi} \sin\varphi \, d\varphi = 0 \qquad \int_0^{2\pi} \sin^2\varphi \, d\varphi = \pi \qquad \int_0^{2\pi} \sin^3\varphi \, d\varphi = 0$$

$$\int_0^{2\pi} \sin^n\varphi \cos\varphi \, d\varphi = \frac{1}{n+1} \sin^{n+1}\varphi \Big|_0^{2\pi} = 0$$

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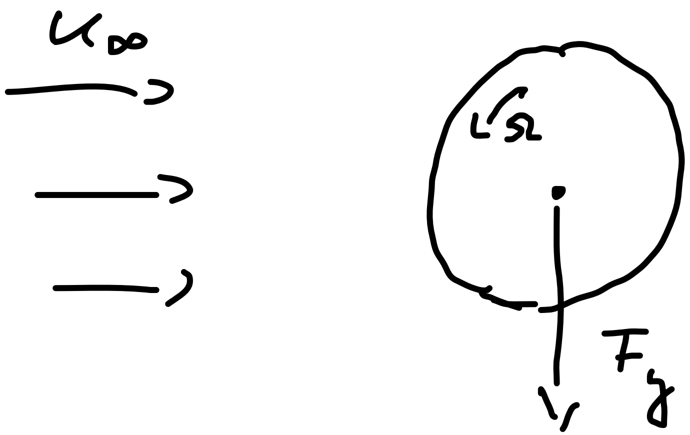
$$\boxed{F_y = -\rho\, u_\infty\, \Gamma \qquad \text{Kutta-Joukowski}}$$



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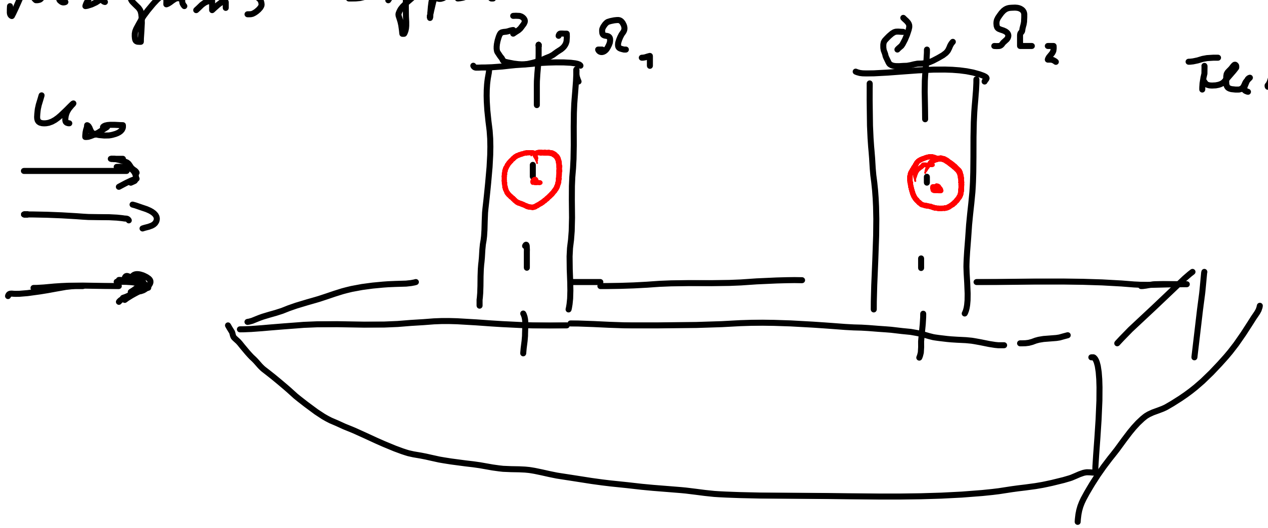
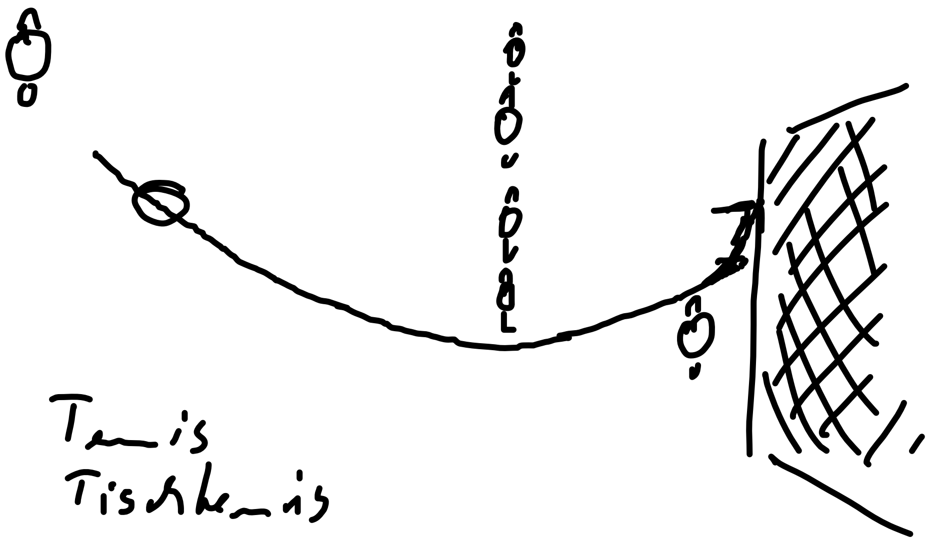
Anwendungsbeispiele Kutta-Joukowski:

Magnus-Effekt:



Flettner-Rotorschiff



Tennis
Tischtennis

Wilhelm Kutta:
* 3.11.1867 † 25.12.1944
→ München
→ Jena
→ RWTH Aachen
→ Stuttgart

- Kutta-Joukowski
- Runge-Kutta-Verfahren

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