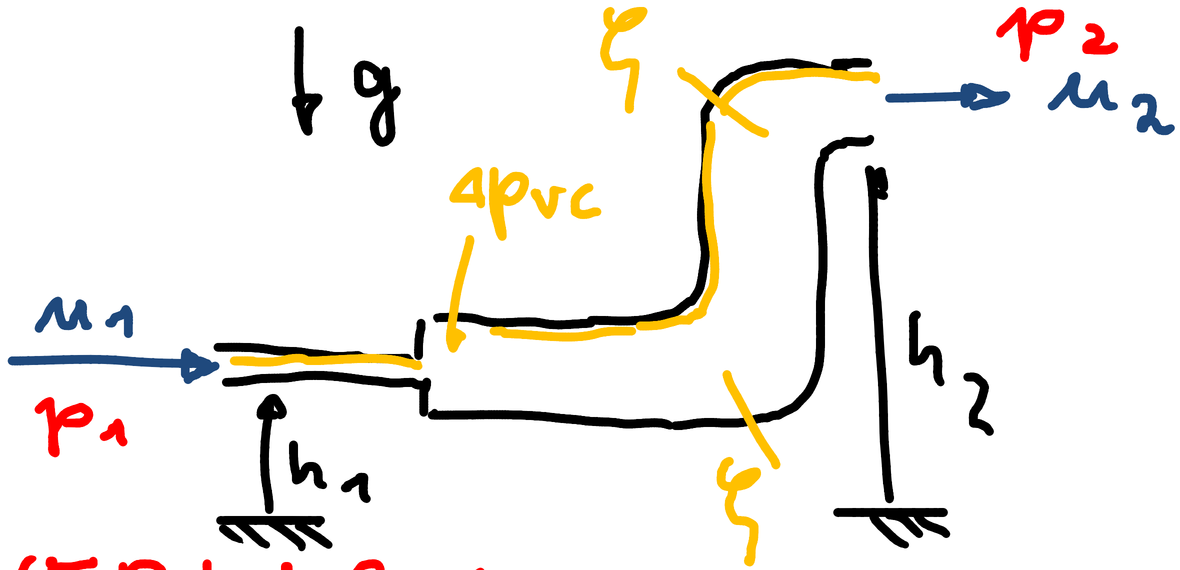
Die Sprechstunde am 2 Juli 2012 fällt aus!



TECHNISCHE UNIVERSITÄT DARMSTADT

FLUID SYSTEM TECHNIK

Einfühung in die Hydrodynamik

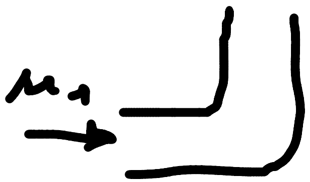
VERLUSTE IN ROHRSYSTEMEN

$$p_1 + \frac{\varrho}{2} u_1^2 + \varrho g h_1 = p_2 + \frac{\varrho}{2} u_2^2 + \varrho g h_2 + \Delta p_v$$

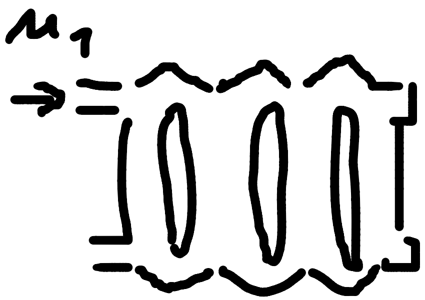
Bernoulli mit Verlusten

$$\Delta p_{vc} = \frac{1}{2}\rho(u_1 - u_2)^2 = \frac{1}{2}\rho u_1^2 \underbrace{\left(1 - \frac{A_1}{A_2}\right)^2}_{\zeta}$$

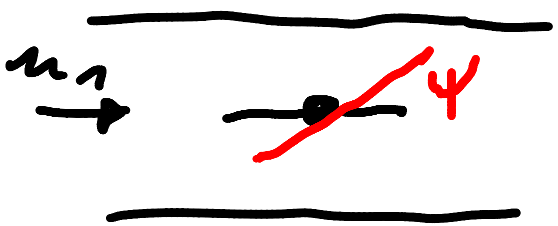
$$\Delta p_v = \frac{1}{2}\rho\,\zeta\,u_1^2$$



Rohrecke $\Delta p_v = \frac{1}{2}\rho\,\zeta\,u_1^2$



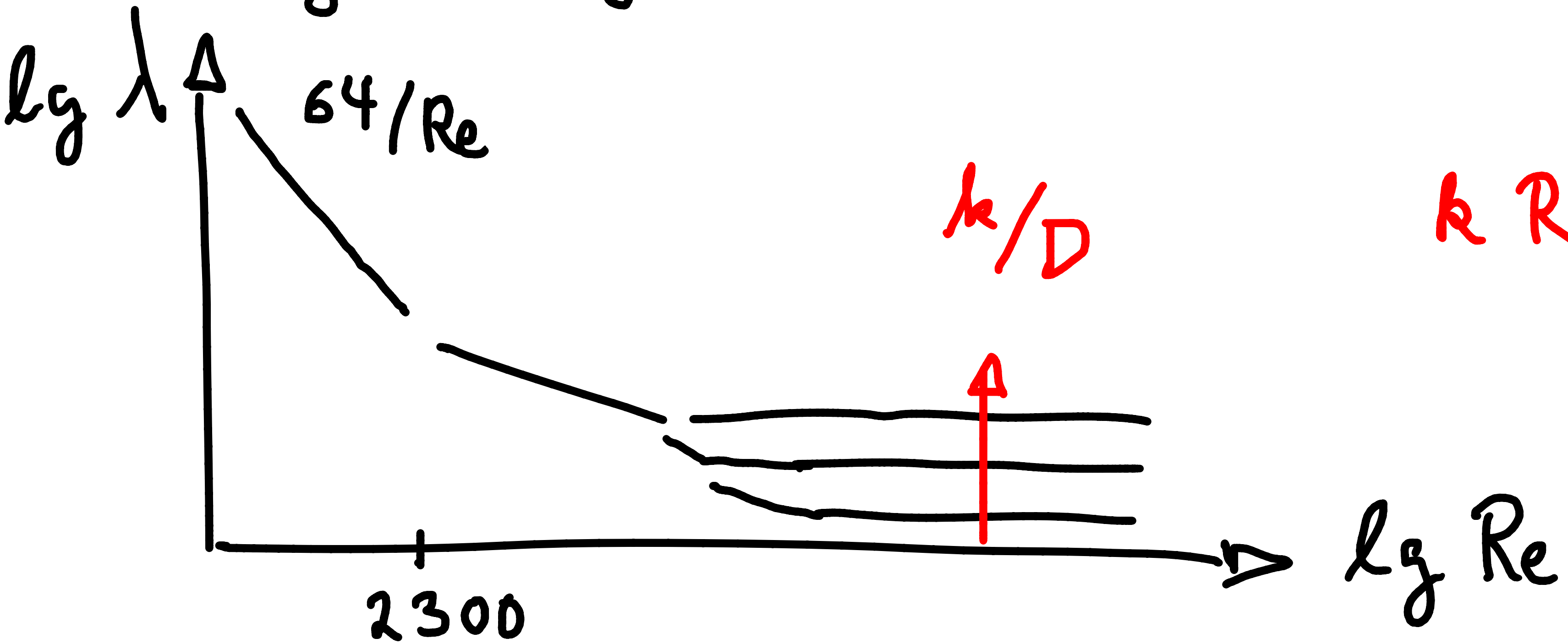
Heizkörper



$\zeta(\varphi)$

$$\zeta_{Rohr} = \frac{L}{D}\lambda$$

Moody-Diagramm



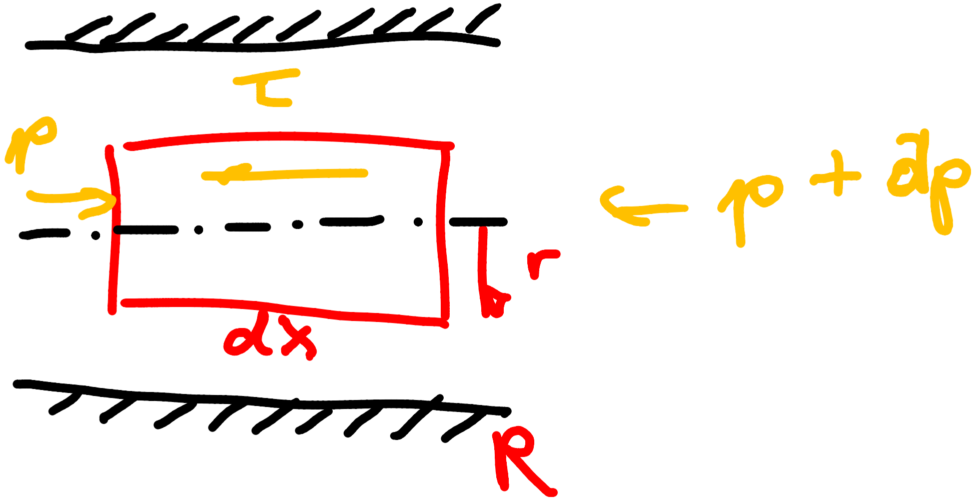
k Rautiefe

$$Re = \frac{UD}{\nu} \qquad \nu = \eta/\rho$$

Cauchy-Gleichung

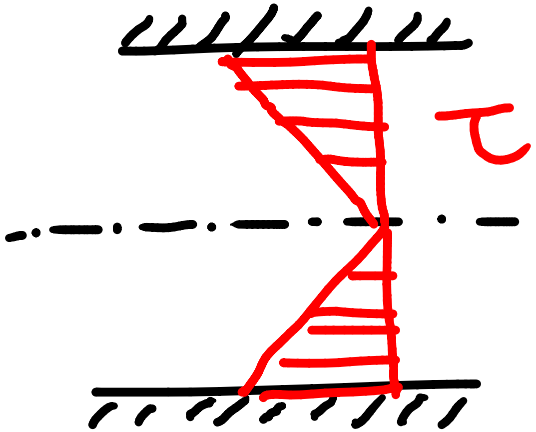
$$\cancel{\varrho \frac{D\vec{u}}{Dt}} = \cancel{\varrho \vec{k}} - \nabla p + \nabla \cdot \underset{\sim}{T}$$

$$\nabla p = \nabla \cdot \underset{\sim}{T} \qquad \frac{\partial p}{\partial x_i} = \frac{\partial \tau_{ij}}{\partial x_j}$$



$$GG \rightarrow: \; -dp\,\pi r^2 - \tau\, 2\pi r\, dx$$

$$\tau = -\frac{\partial p}{\partial x}\frac{r}{2} = K\,\frac{r}{2}$$



Materialgesetz

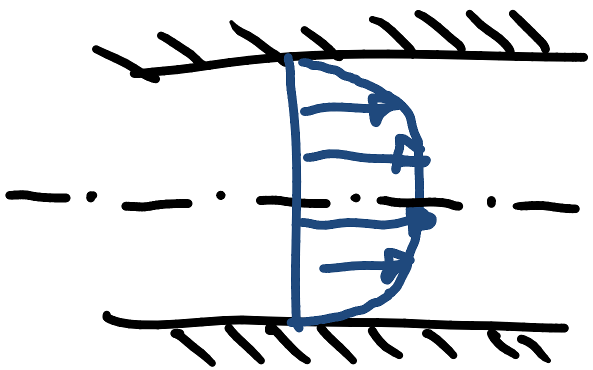
z.B $\tau = -\eta \dfrac{\partial u}{\partial r}$

$$\frac{\partial u}{\partial r} = \frac{\partial p}{\partial x}\frac{r}{2\eta}$$

$$u = \frac{\partial p}{\partial x}\frac{1}{4\eta}\left[r^2 + C\right]$$

Randbedingung $u(R) = 0$

$$u = -\frac{\partial p}{\partial x}\frac{1}{4\eta}\left[R^2 - r^2\right] = K\frac{R^2}{4\eta}\left[1 - \frac{r^2}{R^2}\right]$$



$$\hat{u} = K\frac{R^2}{4\eta}$$

Hagen-Poiseulle-Strömung

$$\pi R^2 \bar{u} = \int_0^{2\pi}\int_0^R \hat{u}\left(1-\frac{r^2}{R^2}\right) r\,dr\,d\varphi$$

$$= 2\pi\hat{u}\left[\frac{r^2}{2}-\frac{r^4}{4R^2}\right]_0^R$$

$$= 2\pi\hat{u}\,\frac{R^2}{4}$$

$$\bar{u} = 1/2\,\hat{u} = \frac{K R^2}{8\eta}$$

$$\bar{u} = \underbrace{\frac{\Delta p_v}{L}}_{K} \frac{R^2}{8\eta}$$

$$\Delta p_v = \frac{L}{R^2} 8\eta \bar{u} = \frac{1}{2}\rho \zeta \bar{u}^2$$

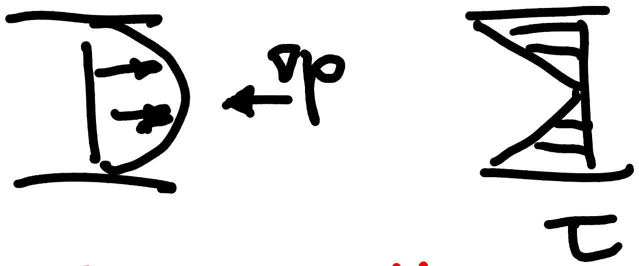
$$16 \frac{L}{R^2} \frac{\eta}{\rho} \frac{\cancel{u}}{\bar{u}^{\cancel{2}}} = \zeta = \lambda \frac{L}{D}$$

$$64 \frac{\cancel{L}}{D^{\cancel{2}}} \nu \frac{1}{\bar{u}} = \lambda \frac{\cancel{L}}{\cancel{D}}$$

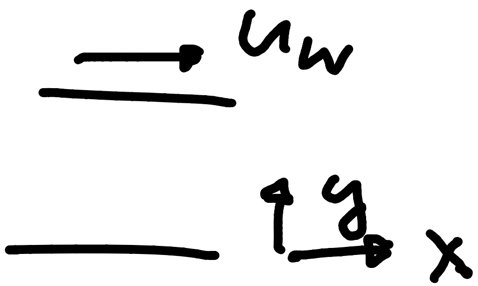
$$\lambda = \frac{64}{\left(\frac{uD}{\nu}\right)} = \frac{64}{Re}$$

$$\tau = \eta \frac{U_w}{h}$$

Couette -



Poiseuille - Strömung



Navier-Stokes-Gleichung

$$\cancel{\rho \frac{D\vec{u}}{Dt}} = \cancel{\rho \vec{k}} - \nabla p + \eta \Delta \vec{u}$$

$$\frac{\partial p}{\partial x} = \eta \left[\cancel{\frac{\partial^2}{\partial x^2}} + \frac{\partial^2}{\partial y^2} \right] u_x$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{1}{\eta} \frac{\partial p}{\partial x}$$

$$u = \frac{1}{2\eta}\frac{\partial p}{\partial x} y^2 + C_1 y + \cancel{C_2}$$

$$\text{RB} \quad u(y=0) = 0$$

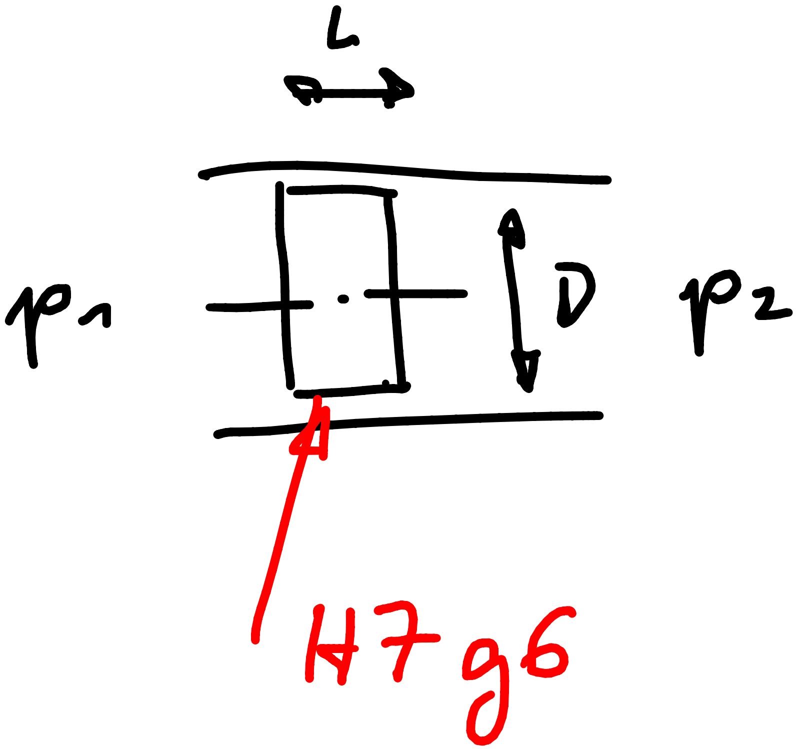
$$\text{RB} \quad u(y=h) = U_w$$

$$u = \frac{1}{2\eta}\frac{\partial p}{\partial x}\left(y^2 - hy\right) + U_w \frac{y}{h}$$

$$\underline{\bar{U}} = \frac{U_w}{2} + \frac{1}{h}\int^{h}(hy - y^2)\,dy \, \frac{K}{2\eta}$$

$$= \frac{U_w}{2} + \underline{K h^3/12}$$

$$\underline{\underline{Q}} = \bar{U}h = \frac{U_w h}{2} + \underline{\underline{\frac{K h^3}{12\eta}}}$$



$$Q_{Leck} = \frac{p_1 - p_2}{L} \frac{h^3}{12\eta}$$