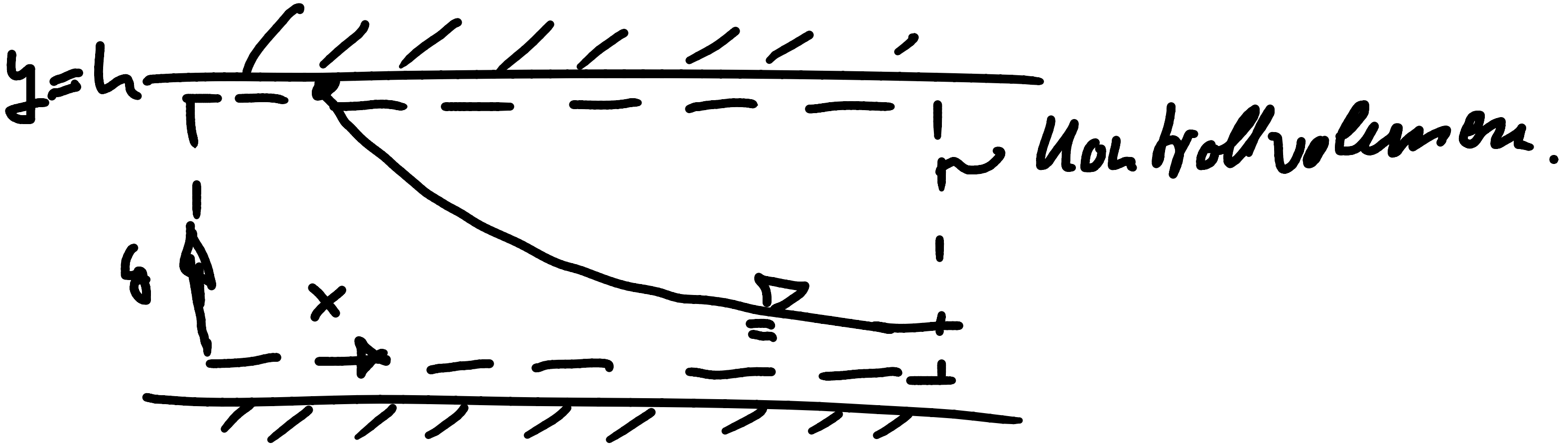
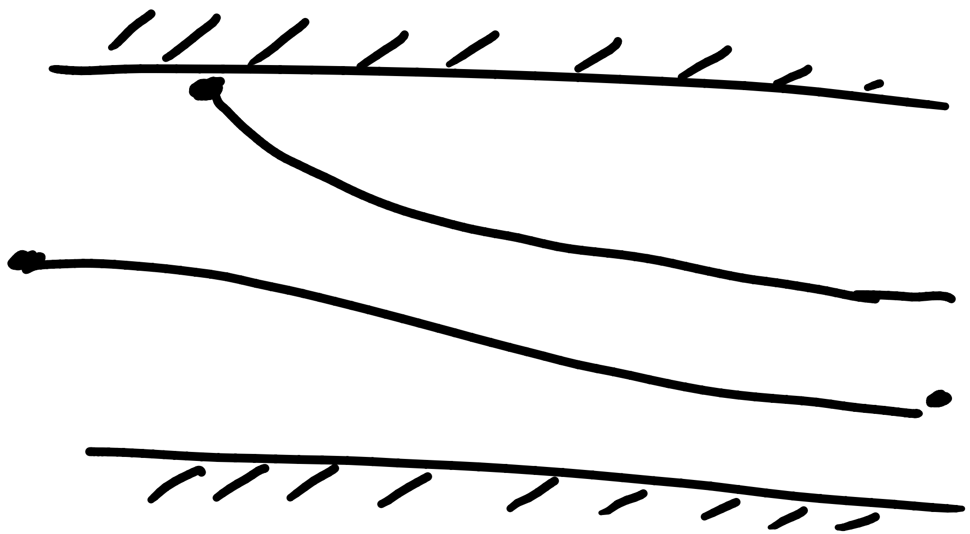
# Reynoldsches Transporttheorem; Konti-Gleichung





## Kontigleichung

$$\left.\begin{array}{l}\dfrac{Dm}{Dt}=0 \\[2em] m=\displaystyle\int_{V(t)} \rho\, dV\end{array}\right\} \quad \int_V \underbrace{\left(\frac{D\rho}{Dt}+\rho\,\frac{1}{dV}\,\frac{D(dV)}{Dt}\right)}_{\equiv\, 0} dV=0$$

Hinweis: Volumenänderungsrate eines materiellen Teilchens ist gleich der Divergenz des Geschwindigkeitsfeldes.

$$\frac{1}{dV}\,\frac{D(dV)}{Dt}=\operatorname{div}\vec{v}$$

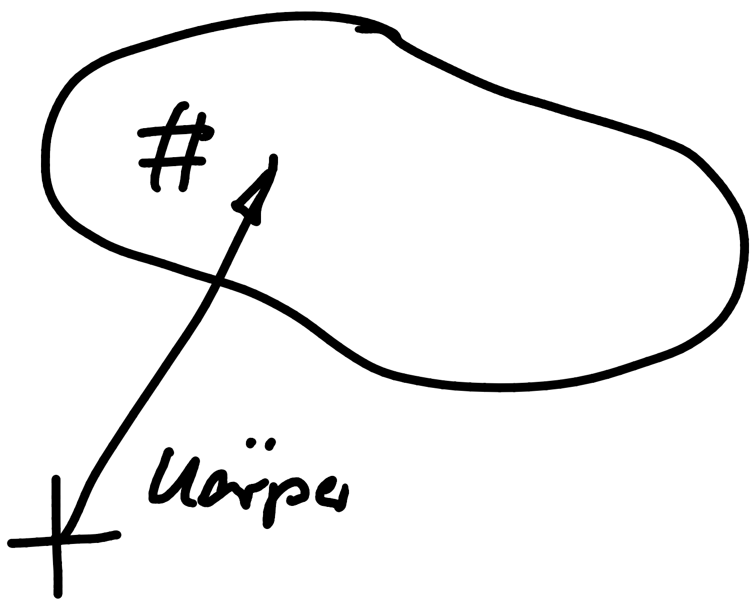
TECHNISCHE UNIVERSITÄT DARMSTADT

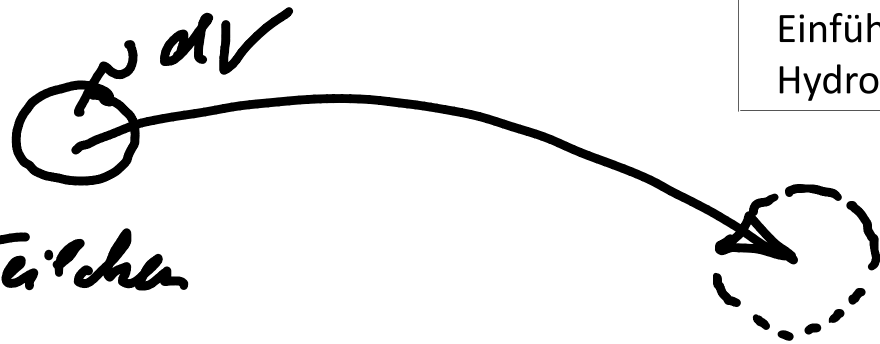
FLUID SYSTEM TECHNIK

Einfühung in die Hydrodynamik

Erhaltungsgleichung in integraler Form

Erhaltungsgleichung in differentieller Form




$$\frac{D}{Dt}\int_{V(t)} \rho \, dV = 0$$

$$\boxed{\frac{D\rho}{Dt} + \rho \, \mathrm{div} \, \vec{u} = 0}$$

inkompressible Strömung

$$\mathrm{div} \, \vec{u} = 0 \iff \frac{D\rho}{Dt} = 0$$

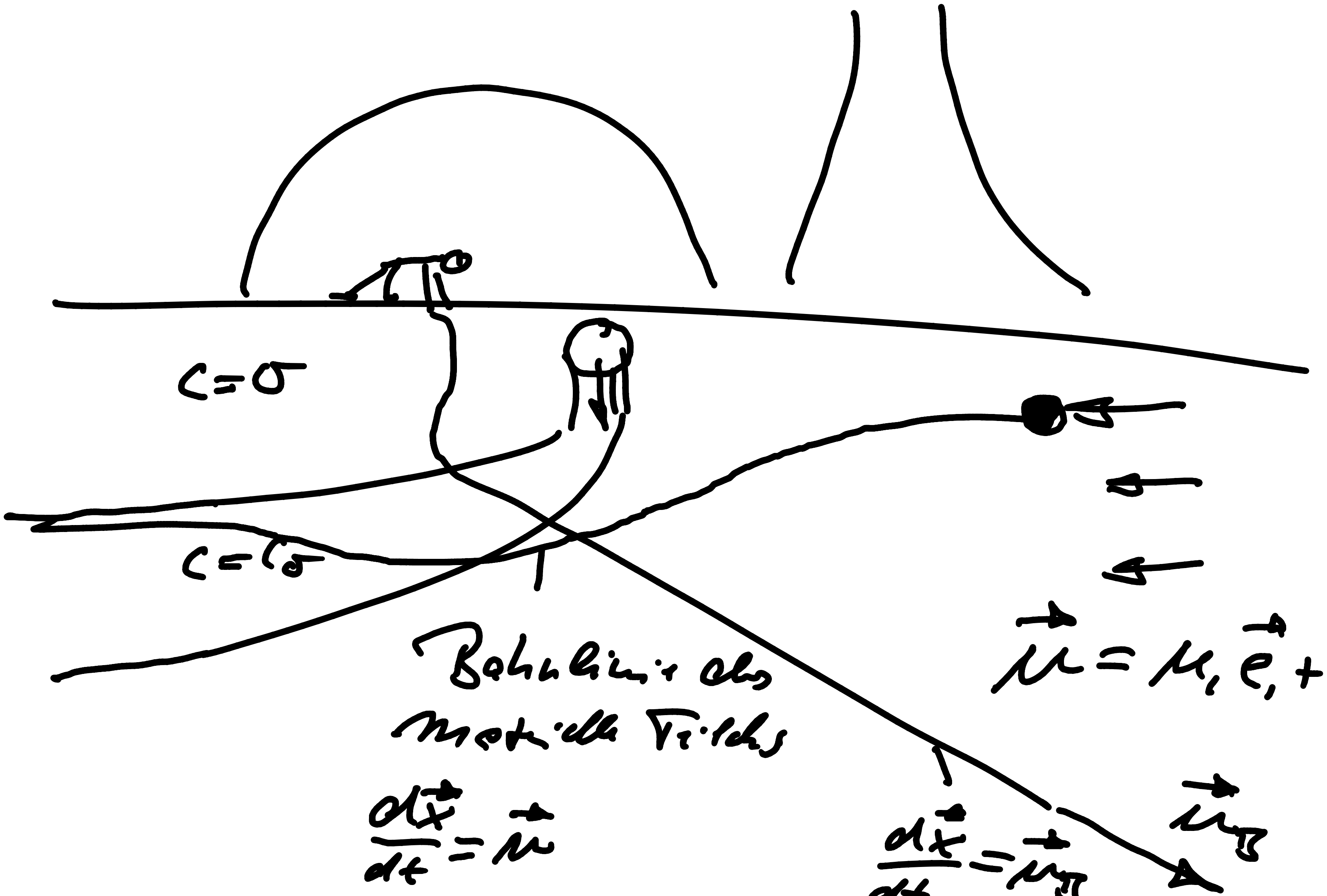
Zur materiellen Zeitableitung $\frac{D\phi}{Dt}$

$\phi$ ist ein Platzhalter für Temperatur, $T$
Konzentration, $c$
Druck $p$
⋮
Geschwindigkeit $\vec{u}$
⋮

allgemeine Zeitableitung $\frac{d\phi}{dt}$ längs einer Beobachterbahn.

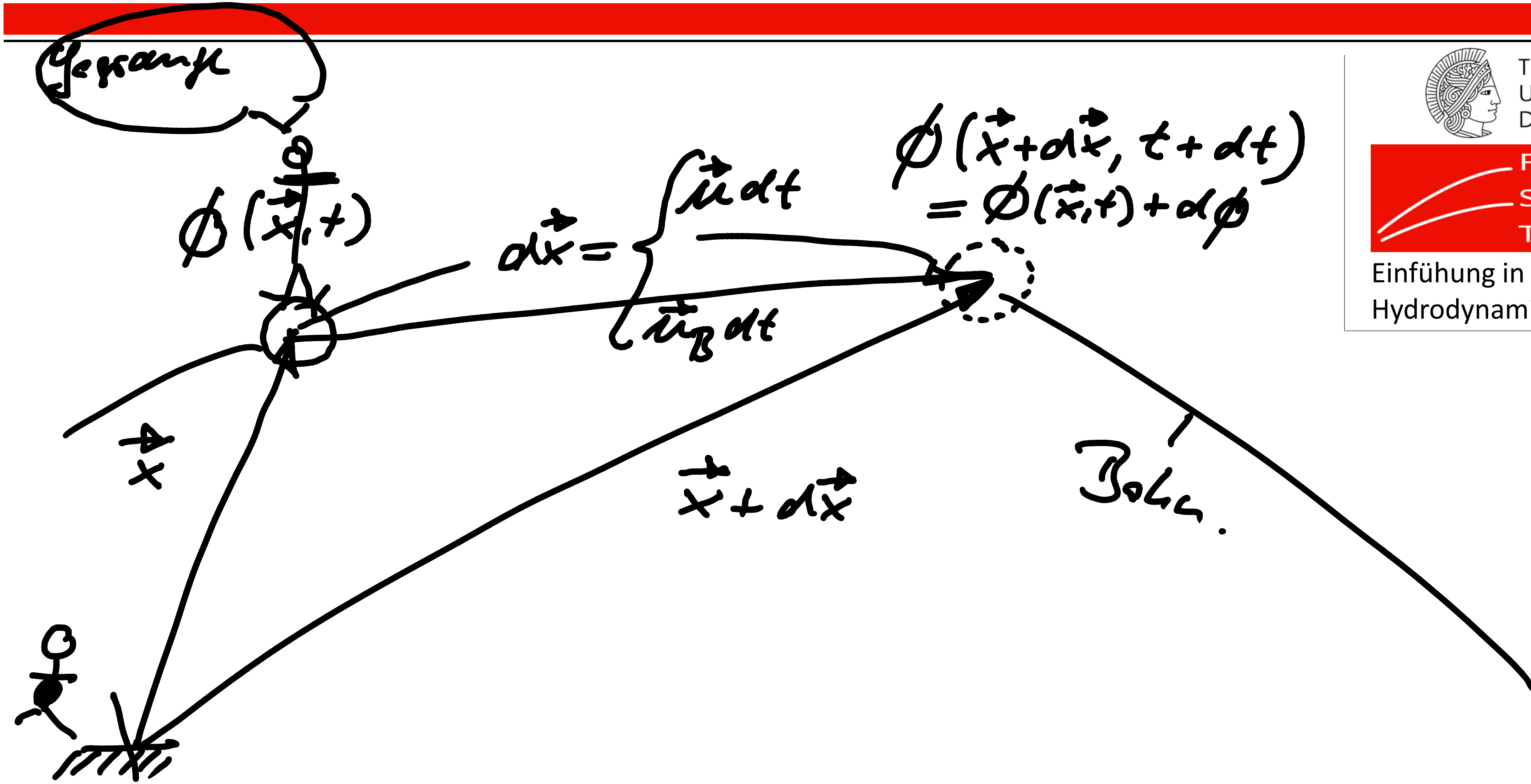
| Zeit | c |
|---|---|
| 0 | 0 |
| 1 sec | 10 $\frac{mol}{m^3}$ |
| ⋮ | ⋮ |

$$\vec{u}_B = \frac{d\vec{x}}{dt}$$




$$\frac{d\vec{x}}{dt} = \vec{u}$$

$$\frac{d\vec{x}}{dt} = \vec{u}_{II}$$




$$d\phi = \nabla\phi \cdot d\vec{x} + \frac{\partial \phi}{\partial t}\,dt \quad \Big| \ \frac{1}{dt}$$

$$\frac{d\phi}{dt} = \frac{\partial \phi}{\partial t} + \frac{d\vec{x}}{dt} \cdot \nabla\phi$$

allgemeine Zeitableitung

$$\frac{d\phi}{dt} = \frac{\partial \phi}{\partial t} + \vec{u}_B \cdot \nabla \phi$$

längs der Beobachter

$$\frac{d\phi}{dt} = \frac{\partial \phi}{\partial t} + \underbrace{u_{iB} \frac{\partial \phi}{\partial x_i}}$$

materielle Zeitableitung

$$\frac{D\phi}{Dt} = \underbrace{\frac{\partial \phi}{\partial t}}_{\text{lokale zeitliche Änderung}} + \underbrace{\vec{u} \cdot \nabla \phi}_{\text{konvektive zeitliche Änderung}}$$

längs der Bahnlinie

Kontigleichung in differentieller Form

$$\boxed{\frac{D\rho}{Dt} + \rho \, \mathrm{div} \, \vec{u} = 0}$$

$$\frac{\partial \rho}{\partial t} + \vec{u} \cdot \nabla \rho + \underbrace{\rho \nabla \cdot \vec{u}}_{} = 0$$

$$\boxed{\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{u}) = 0}$$

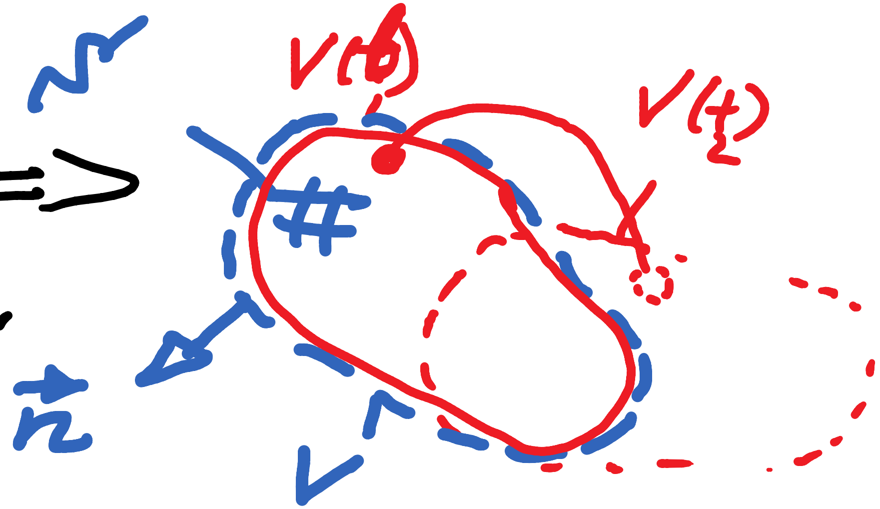
Zurück zur Erhaltungsgleichung in integraler Form



Prof. Dr.-Ing. Peter Pelz

$$\frac{D}{Dt}\int_{V(t)} \rho \, dV = 0 \iff$$

$V(t)$ materielles Volumen




$$\int_V \frac{\partial \rho}{\partial t}\, dV + \int_V \nabla\cdot(\rho\vec{u})\, dV = 0$$

Kontrollvolumen.

Gauß.

$$\int_V \nabla\cdot(\,)\, dV = \oint_S (\,)\cdot\vec{n}\, dS$$

$$\boxed{\frac{\partial}{\partial t}\int_V \rho\, dV + \oint_S \rho\,\vec{u}\cdot\vec{n}\, dS = 0}$$

$$\boxed{\frac{D\rho}{Dt} + \rho\,\mathrm{div}\,\vec{u} = 0}$$

lokale Änder.  konvektive Änd.

$$\boxed{\frac{\partial}{\partial t}\int_V \phi \, dV + \oint_S \phi \, \vec{u}\cdot\vec{n} \, dS = \frac{D}{Dt}\Phi}$$

Reynoldsche-Transporttheorem (kinematischer Satz)

$$\Phi = \int_{V(t)} \phi \, dV$$

$$\phi \overset{z.B.}{=} \rho$$

$$= T$$

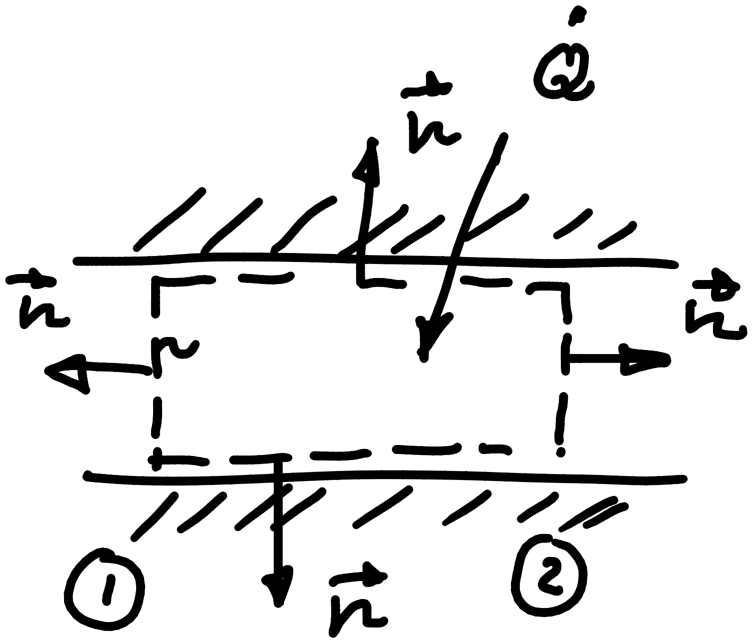
$$= c$$

$$= e$$

$$\vdots$$

Zur konvektiven Änderung $\hat{=}$ Flußintegrale

$$P_S + \dot{Q} = \dot{m}\,(h_{t2} - h_{t1})$$




$\dot{m}\, h_{t2}$ ist ein Flußintegral über die Austrittsfläche.

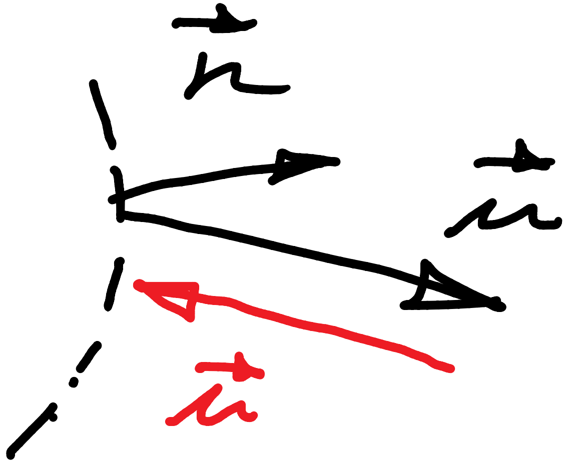
$$\int_{S_2} \underbrace{h_t\, \rho}_{\phi}\, \vec{u}\cdot\vec{n}\, dS = h_t\, \dot{m} \quad \text{für}$$

homogene Totalenthalpie über die Austrittsfläche

$$S = S_1 + S_2 + S_{Wand}.$$

$$\dot{m} := \pm \int \rho \, \vec{u} \cdot \vec{n} \, dS$$

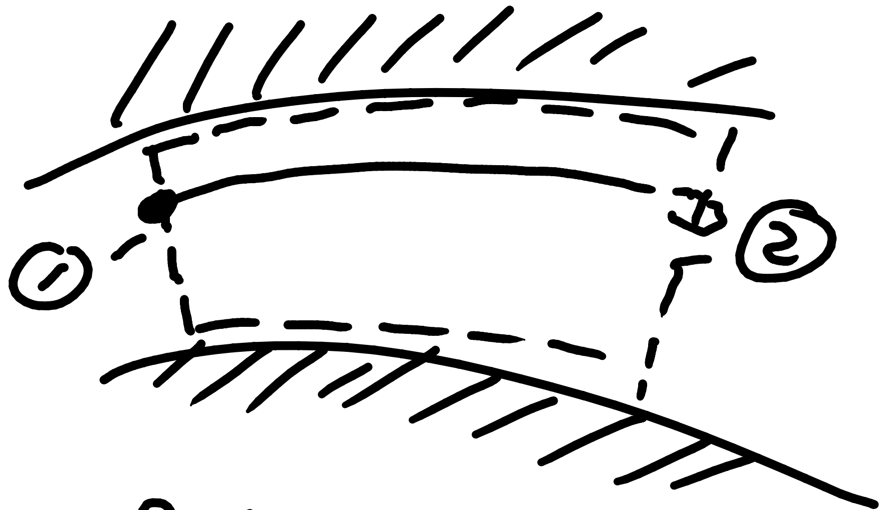
+, wenn $\vec{u}$ und $\vec{n}$ einen spitzen Winkel bildet




− wenn $\vec{u}$ und $\vec{n}$ einen stumpfen Winkel bildet.

Bei stationären Strömungen $\left(\frac{\partial}{\partial t} \equiv 0\right)$
nimmt die Kontinuität
die Form

$$\oint \rho \vec{u} \cdot \vec{n} \, dA = 0$$




$$-\dot{m}_1 + \dot{m}_2 = 0$$

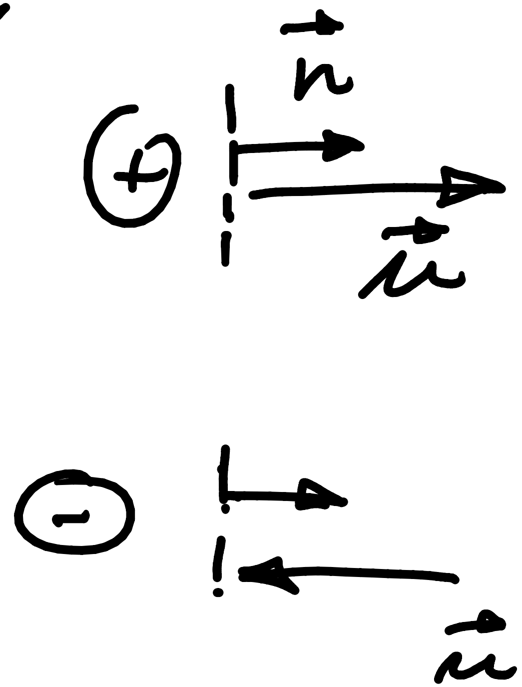
Bei inkompressibler Strömung und homogener Dichteverteilung

$$\nabla \rho \equiv 0 \rightsquigarrow \rho = \text{const.}$$

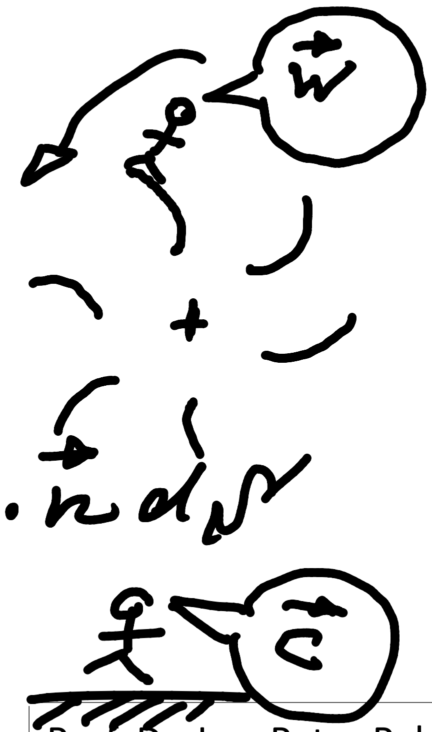
$$-\dot{V}_1 + \dot{V}_2 = 0$$

Volumenstrom = Flußintegral

$$\dot V = \pm \int_S \vec u \cdot \vec n \, dS$$




Reynoldsche Transporttheorem im bewegten System (B)




$$\left[\frac{D}{Dt}\Phi\right]_B = \left[\frac{\partial}{\partial t}\int \phi \, dV\right]_B + \oint_S \phi \, \vec w \cdot \vec n \, dS$$

$\vec w$ relative Geschwindigkeit.

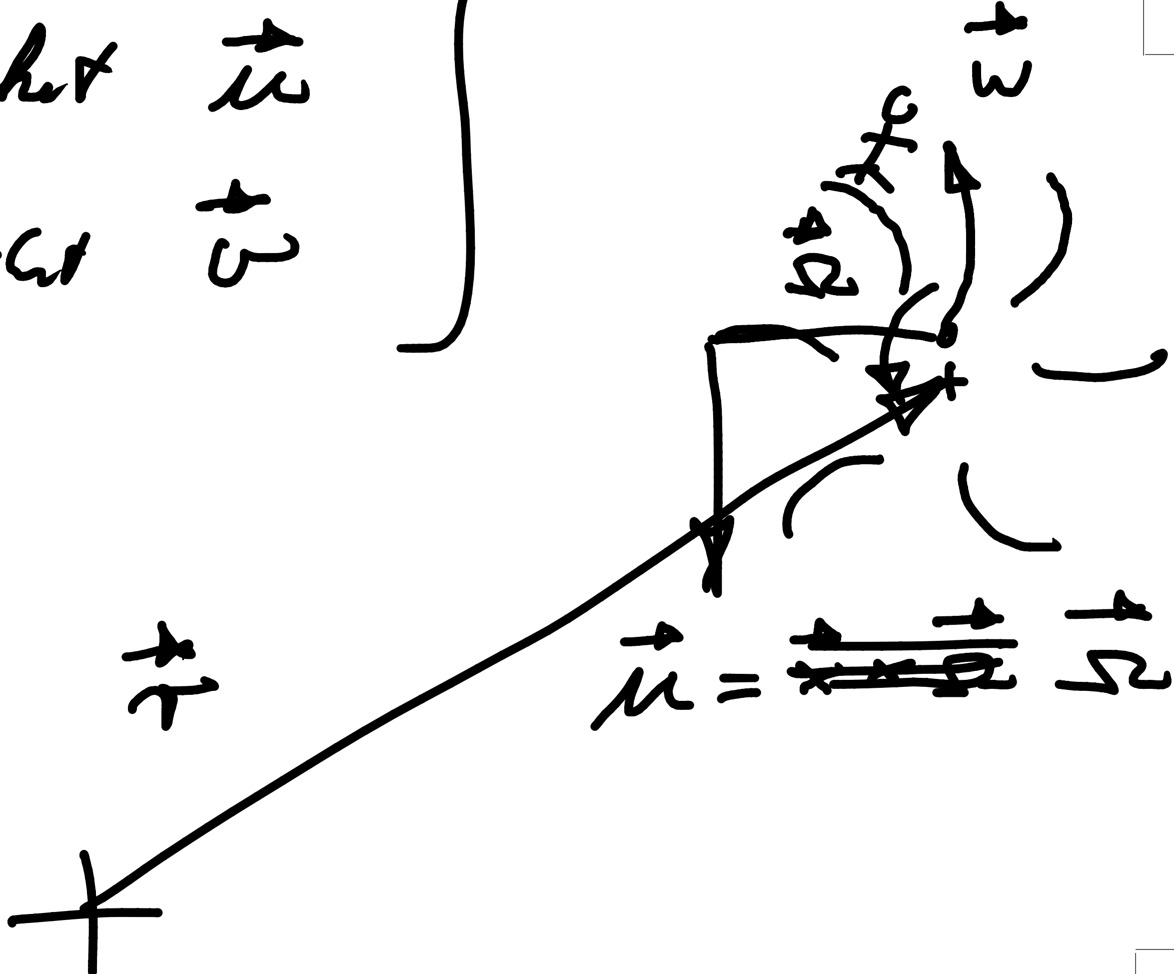
relativ Geschwindigkeit $\vec{w}$

absolut Geschwindigkeit $\vec{c}$

Umfangsgeschwindigkeit $\vec{u}$

Führungsgeschwindigkeit $\vec{v}$

$$\vec{c} = \vec{w} + \vec{u} + \vec{v}$$




$$\frac{d\vec{r}}{dt} = \vec{v}$$

$\vec{r}$

$$\vec{u} = \cancel{\vec{x} \times \vec{\Omega}} \; \vec{\Omega} \times \vec{x}$$

Beispiel zur Kontigleichung in differentieller Form

$$\vec{u} = u_1 \vec{e}_1 + u_2 \vec{e}_2$$

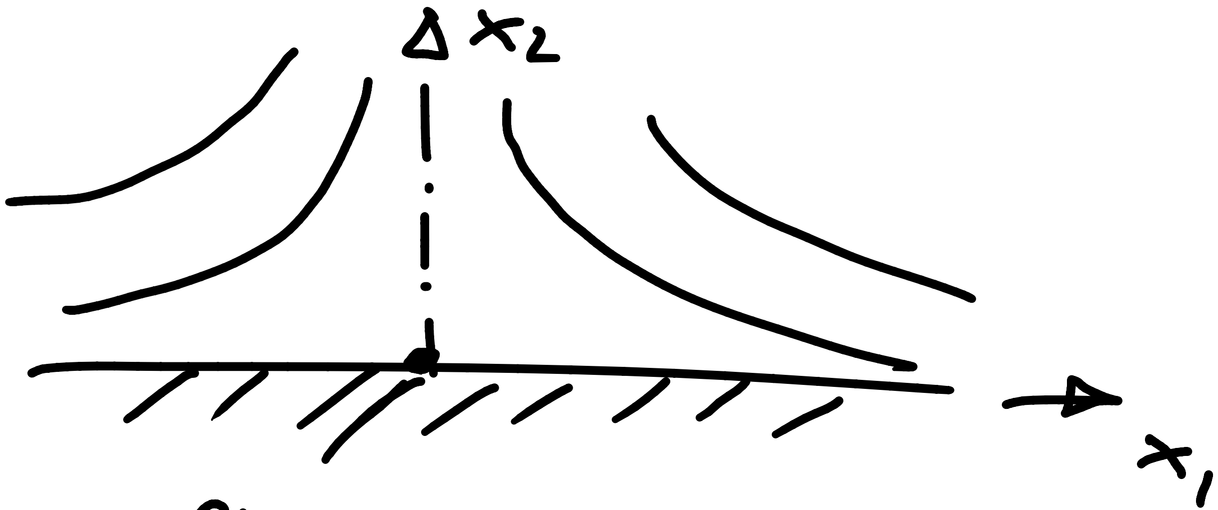
$$u_1 = a x_1$$

$$u_2 = b x_2$$

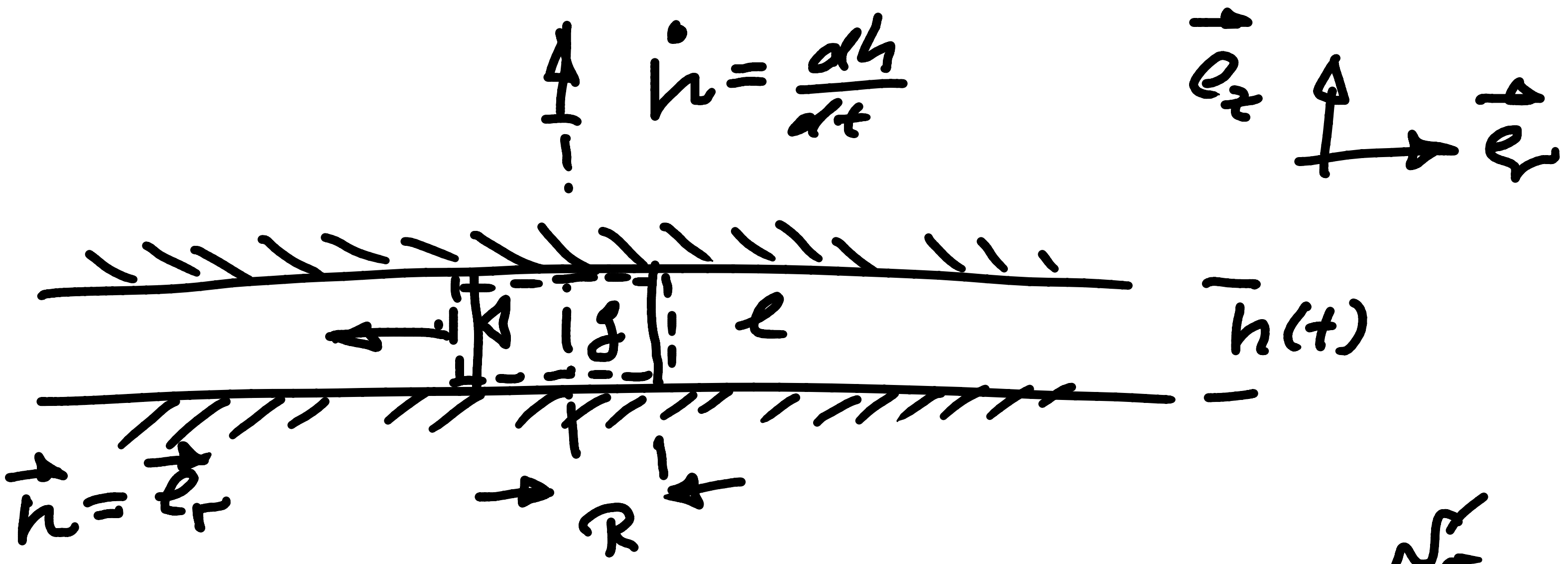
Frage: Wie hängt $b(a)$ ab, damit die Strömung inkompressibel ist

$$\frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2} \overset{!}{=} 0 \,, \quad \text{damit} \quad \frac{D\varrho}{Dt} \equiv 0 .$$

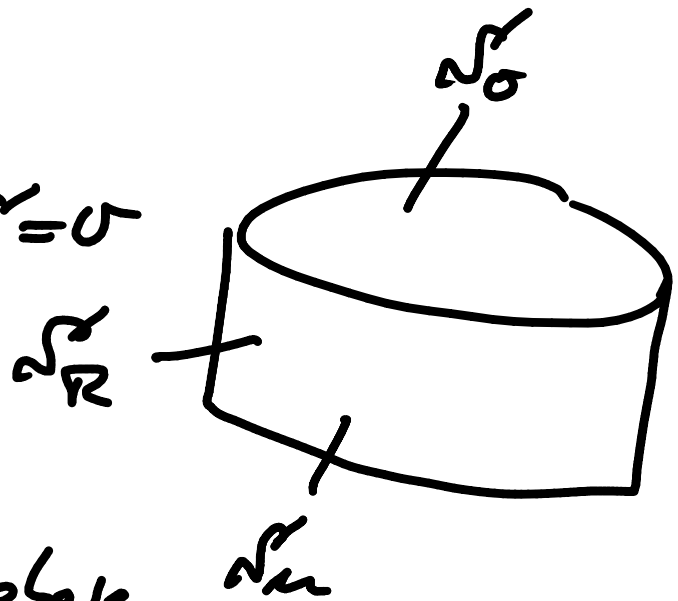
$$a + b = 0 \quad \rightsquigarrow \quad b = -a$$




Staupunkt.




$$\frac{\partial}{\partial t}\int_V \rho \, dV + \int_{S_R + S_O + S_M} \rho \, \vec{u}\cdot\vec{n} \, dS = 0$$




$\rho$ ist homogen in der Gasphase

$$\pi R^2 h \dot{\rho} + 2\pi R h \rho \dot{R} + \pi R^2 \rho \dot{h} = 0.$$