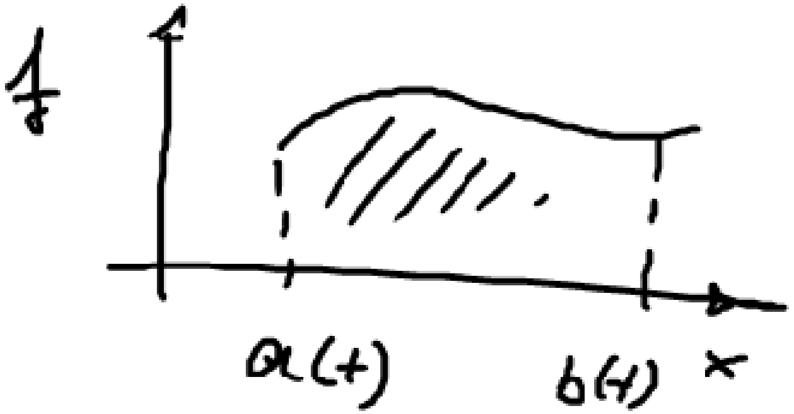
Literatur:

- Joseph H. Spurk
  Fluid Mechanics Springer Verlag 1997.
  Kap. 9.2 instationäre Gasdynamik
- Streeter and Wylie
  Fluid Transients McGraw Hill
  1D-Methoden → Charakteristiken methoden.
- Le ?

Rheologie •

Leibnitzsche Regel ≙ Reynoldsches Transporttheorem.

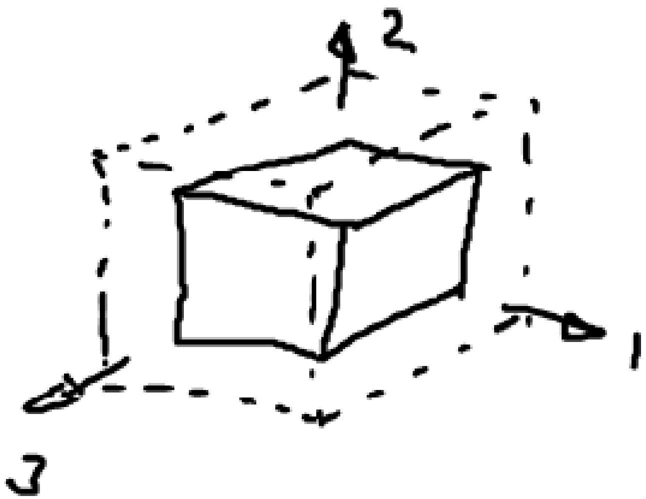
$$\frac{d}{dt}\int_{a(t)}^{b(t)} f(t,x)\,dx = \int_a^b \frac{\partial f}{\partial t}\,dx + \frac{db}{dt} f - \frac{da}{dt} f$$



Kontigleichung

$$\frac{Dm}{Dt} = \frac{D}{Dt}\int_{V(t)} \rho \, dV \overset{!}{=} 0$$

$$\Leftrightarrow \int_V \frac{D\rho}{Dt}\, dV + \rho \frac{D\, dV}{Dt} \overset{!}{=} 0$$



$$\Leftrightarrow \int_V \left(\frac{D\rho}{Dt} + \rho \underbrace{\frac{1}{dV}\frac{D\, dV}{Dt}}_{\text{Volumenänderungsrate}} \right) dV \overset{!}{=} 0$$

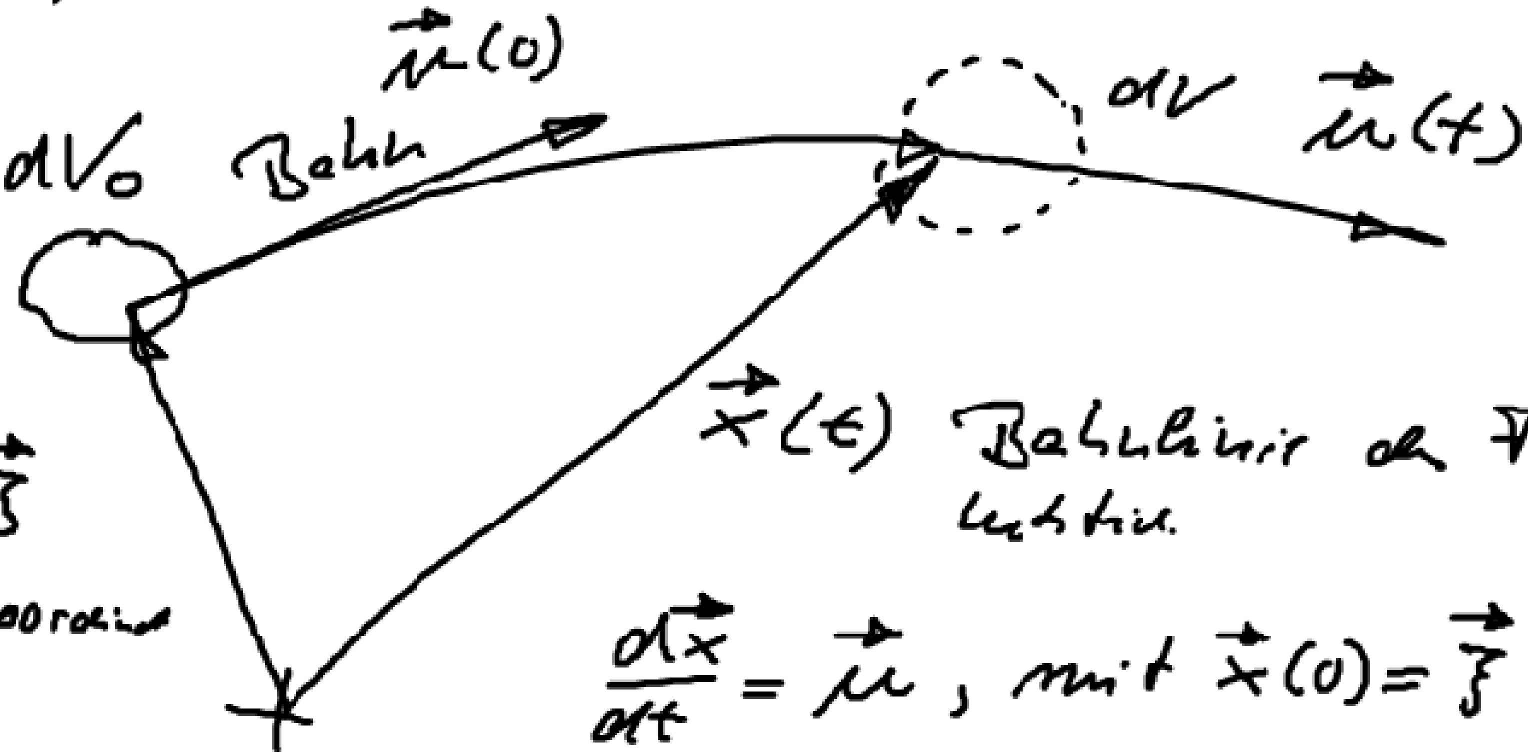
$$\frac{1}{dV}\frac{D\, dV}{Dt} = \operatorname{div} \vec{u}$$

vgl. Spur

$$\operatorname{div} \vec{u} = \frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2} + \frac{\partial u_3}{\partial x_3} = \frac{1}{dV} \frac{D\, dV}{Dt}$$

Volumenänderungsrate eines Flüssigkeitsteilchens.

$\operatorname{div} \vec{u} = 0 \rightsquigarrow$ inkompressible Flüssigkeit,



$\vec{x}(0) = \vec{\xi}$

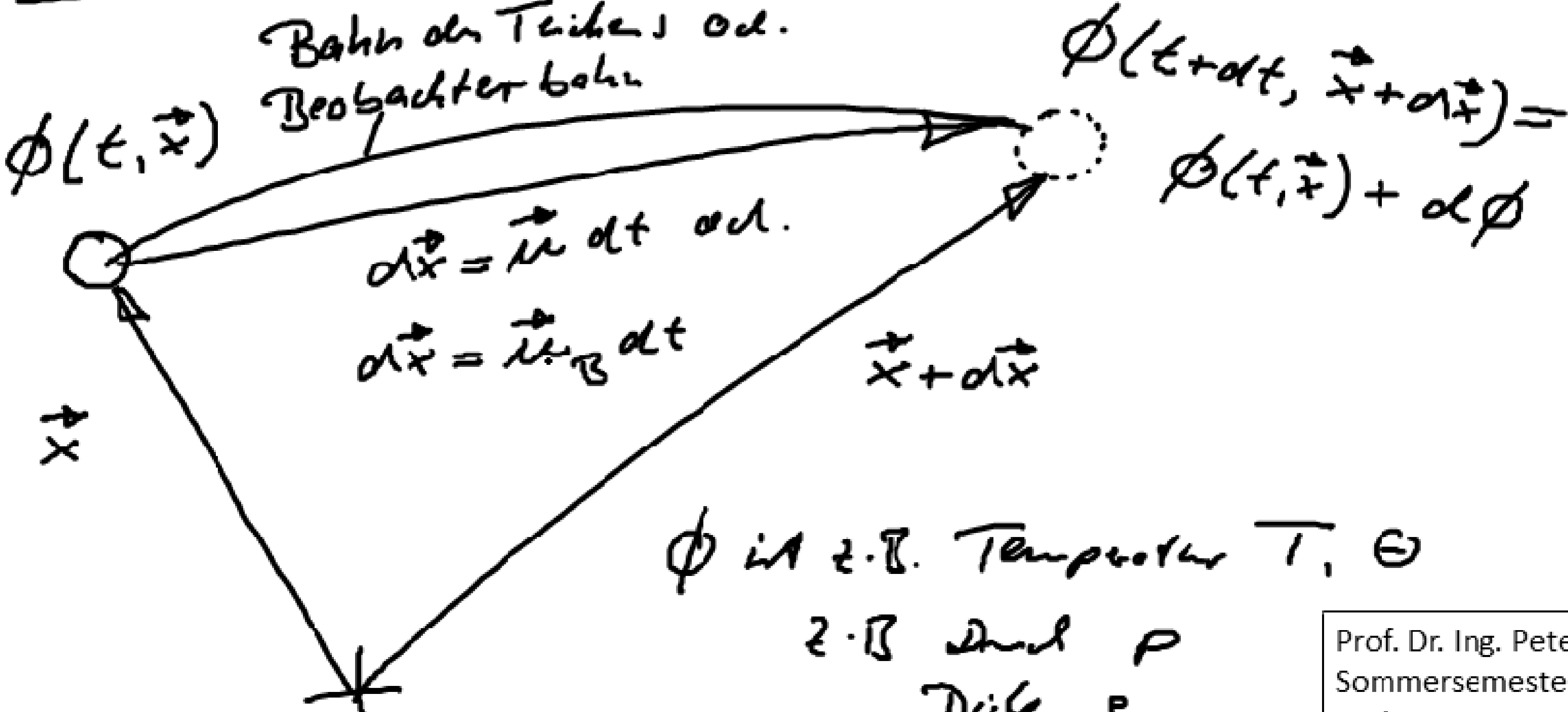
materielle Koordinaten

$$\frac{d\vec{x}}{dt} = \vec{u}, \quad \text{mit } \vec{x}(0) = \vec{\xi}$$

Für $dV = dV_0$ für alle Zeiten:

„inkompressible Flüssigkeit"

<u>besser inkompressible Strömung</u>



ϕ ist z.B. Temperatur T, Θ

z.B. Druck p

Dichte ρ ....

Die Änderung der allgemeinen Größe ϕ

$$d\phi = \phi(t+dt, \vec{x}+d\vec{x}) - \phi(t, \vec{x})$$

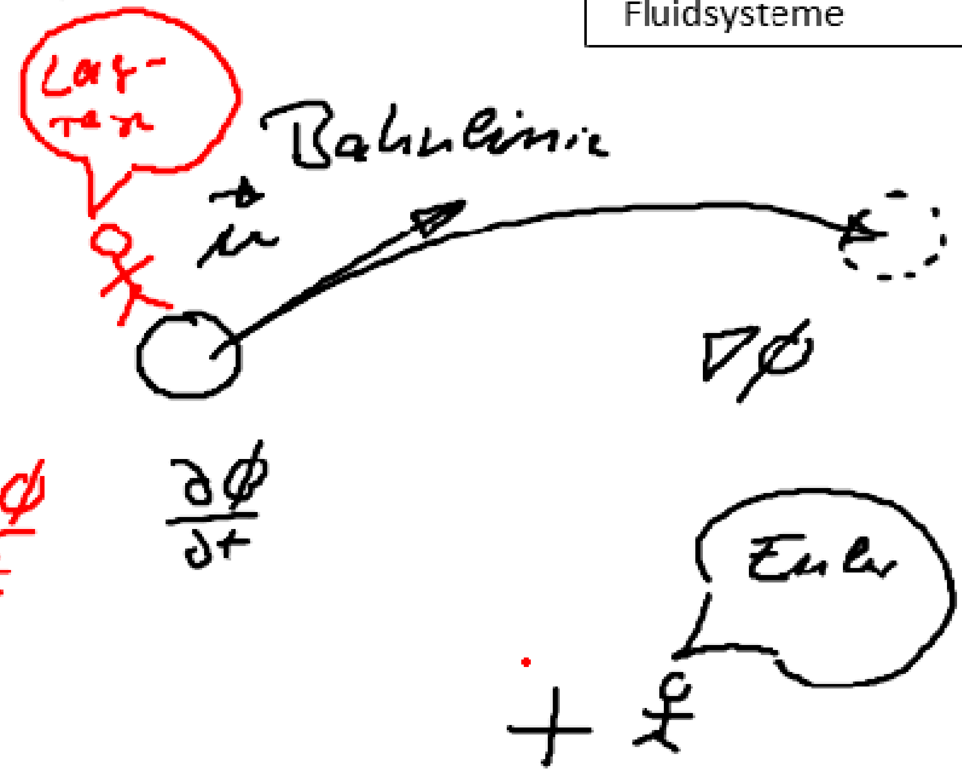
$$= \frac{\partial \phi}{\partial t} dt + \nabla \phi \cdot d\vec{x} \quad \Big| \; \frac{1}{dt}$$

$\hat{=}$ Taylorentwicklung in allen 3 Raumrichtungen + Zeit.

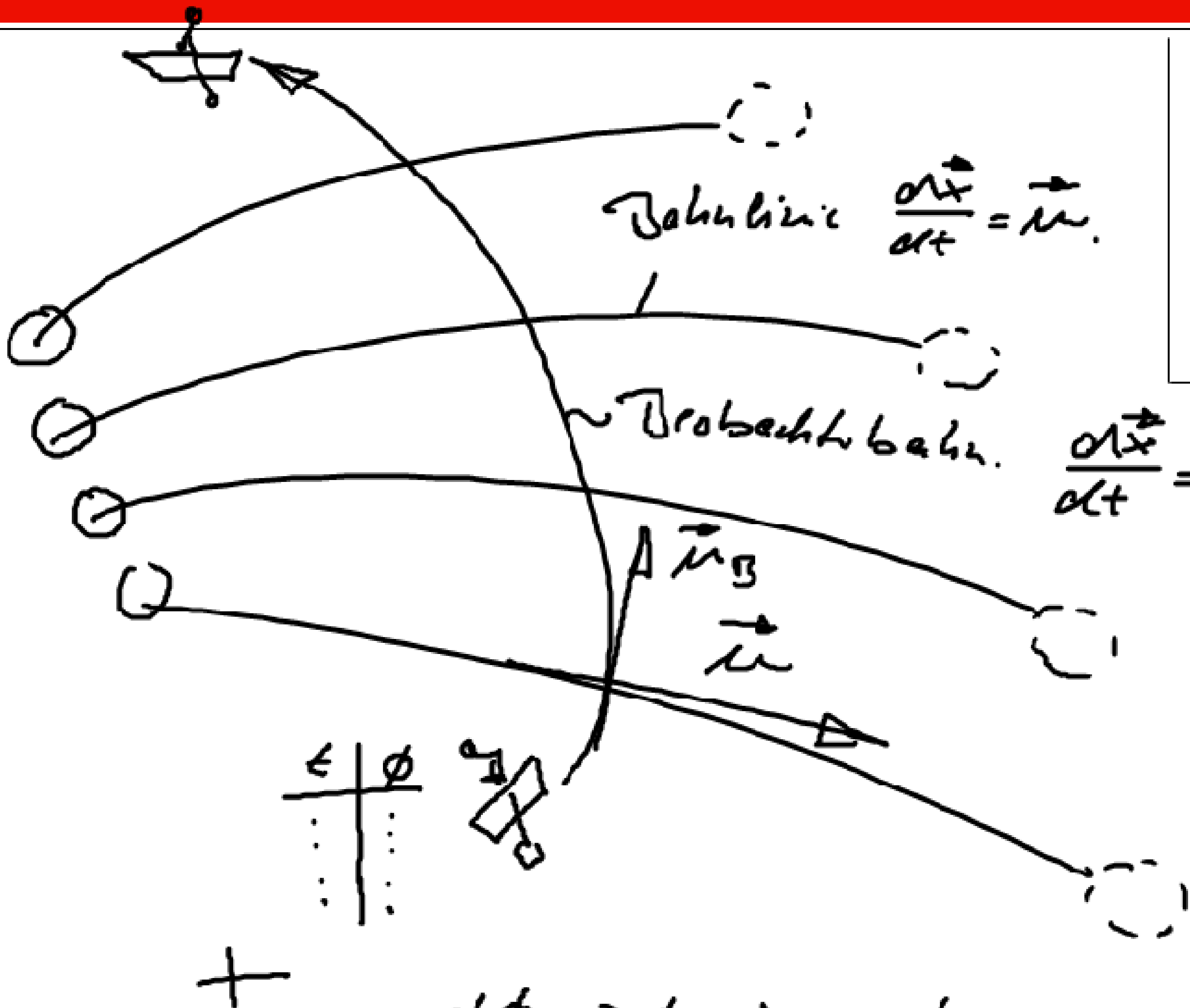
$$\frac{d\phi}{dt} = \frac{\partial \phi}{\partial t} + \frac{d\vec{x}}{dt} \cdot \nabla \phi$$

$\vec{u}$ oder $\vec{u}_B$

Wenn $\frac{d\vec{x}}{dt} = \vec{u}$ (Geschwindigkeit der materiellen Teilchen), dann wird die Änderung längs der Bahnlinie der Teilchen betrachtet. $\rightsquigarrow$ $\frac{d}{dt} \to \frac{D}{Dt}$



$$\frac{D\phi}{Dt} = \underbrace{\frac{\partial\phi}{\partial t}}_{\text{lokale Änderung}} + \underbrace{\vec{u}\cdot\nabla\phi}_{\text{konvektive Änderung}} = \frac{d\phi}{dt}$$



Bahnlinie $\frac{d\vec{x}}{dt} = \vec{u}$

Beobachterbahn $\frac{d\vec{x}}{dt} = \vec{u}_B$

$$\frac{d\phi}{dt} = \frac{\partial \phi}{\partial t} + \vec{u}_B \cdot \nabla \phi$$

allgemeine Zeitableitung

$\vec{u}_B \neq \vec{u}$

$$\int_V \underbrace{\left(\frac{D\rho}{Dt} + \rho \operatorname{div} \vec{u}\right)}_{\equiv 0} dV = 0$$

Nebenergebnis Konti-Gleichung in differentieller Form für ein Flüssigkeitsteilchen

$$\frac{D\rho}{Dt} + \rho \operatorname{div} \vec{u} = 0$$

inkompressibel $\operatorname{div} \vec{u} = 0 \Longleftrightarrow \frac{D\rho}{Dt} = 0.$

$$\frac{\partial \rho}{\partial t} + \vec{u}\cdot\nabla\rho + \rho\,\nabla\cdot\vec{u} = 0$$

$\parallel$ Produktregel.

$$\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\vec{u}) = 0$$

↑ differentiell

↓ integral Form

$$\nabla\cdot(\;)\,dV \mathrel{\hat{=}} (\;)\cdot\vec{n}\,dA$$

Gaußsche Integralsatz.

$$\int_V \left(\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\vec{u})\right) dV = 0$$

$$\boxed{\int_V \frac{\partial \rho}{\partial t}\,dV + \oint_A \rho\,\vec{u}\cdot\vec{n}\,dA \overset{!}{=} 0}$$

$$\Phi = \int_V \varphi \, dV$$

φ z.B. Dichte ρ

Impuls $\rho \vec{u}$

Drall $\vec{x} \times \rho \vec{u}$

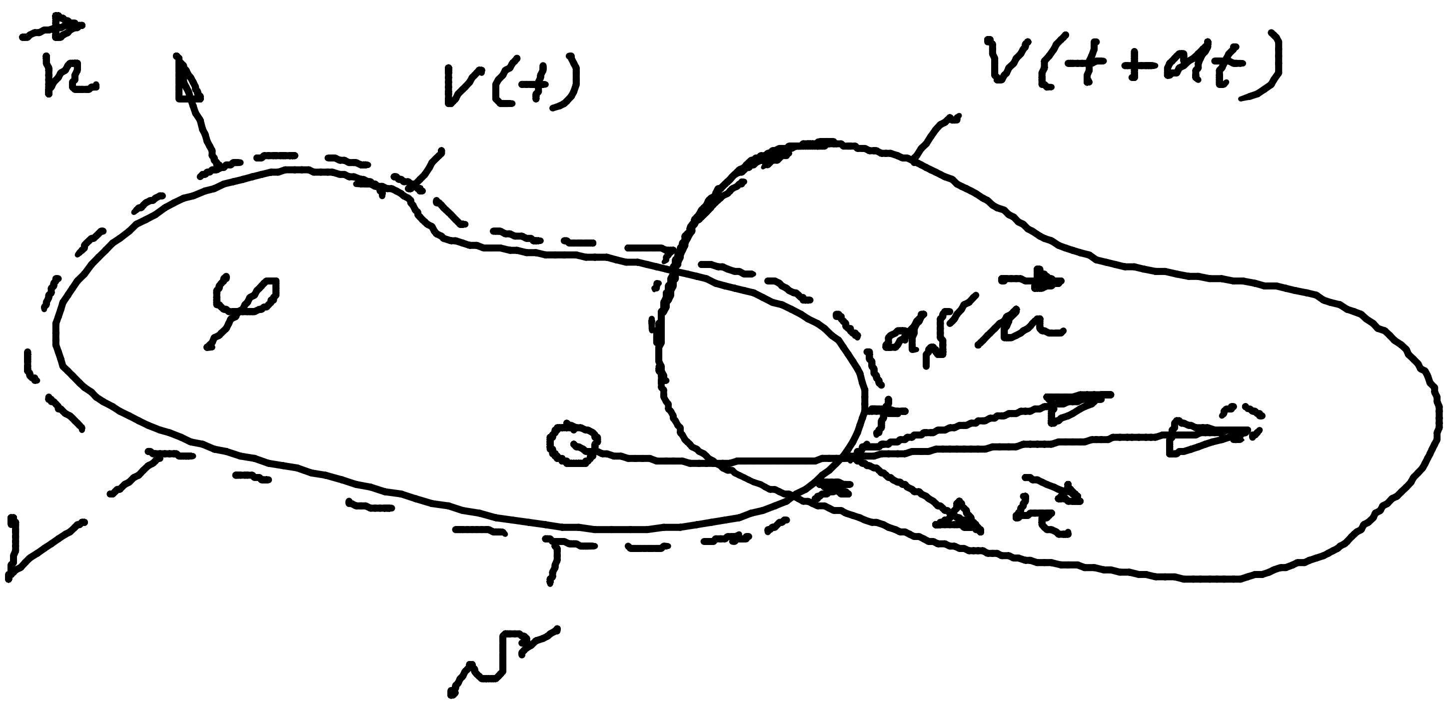
Energie $e + \frac{u^2}{2}$

$$\frac{D\Phi}{Dt} = \frac{D}{Dt}\int_{V(t)} \varphi \, dV = \int_V \frac{\partial \varphi}{\partial t} dV + \oint_S \varphi \, \vec{u} \cdot \vec{n} \, dS$$

materielles Volumen $= fn(t)$

Kontrollvolumen $\neq fn(t)$

geschlossene Kontrollfläche



$\varphi\, \vec{u} \cdot \vec{n}\, dS$ konvektiver Transport der Größe φ über die Grenze des Kontrollvolumens.

$$\frac{D\Phi}{Dt} = \underbrace{\frac{\partial}{\partial t}\int_V \varphi\, dV}_{\text{lokale Änd.}} + \underbrace{\oint_S \varphi\, \vec{u}\cdot\vec{n}\, dS}_{\text{konvektive Änd.}}$$

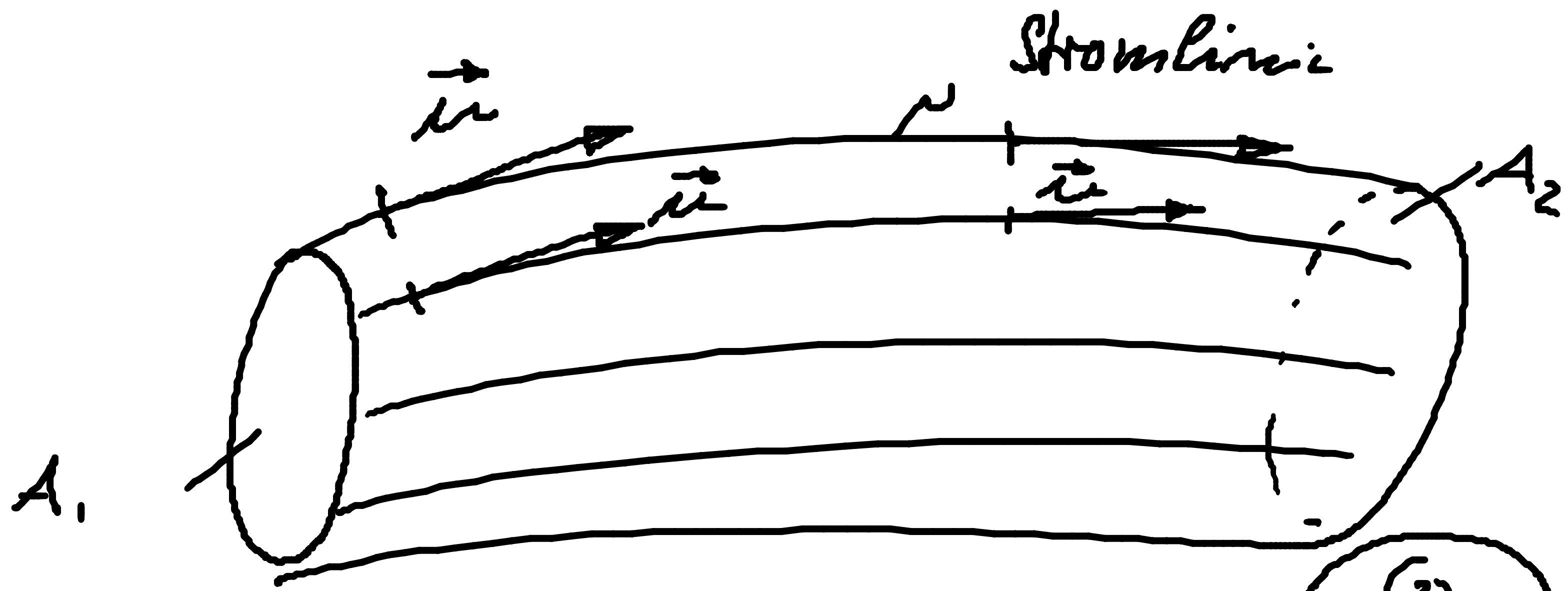
Reynoldsches Transporttheorem.

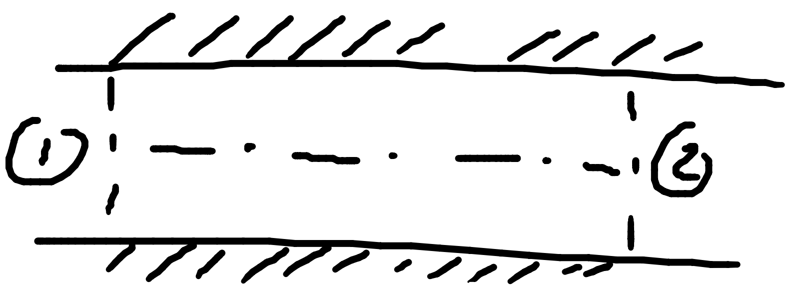
Spezielle Form

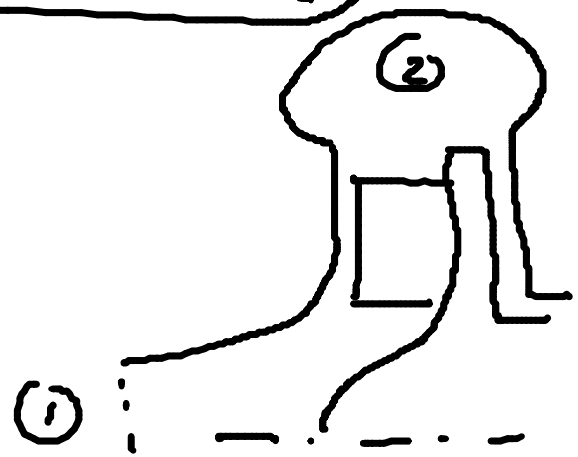
$$\frac{D}{Dt}\int_{V(t)} \varphi \underbrace{\varrho \, dV}_{dm} = \frac{D}{Dt}\int_m \varphi \, dm = \int_m \frac{D\varphi}{Dt} dm$$

$$= \int_V \frac{D\varphi}{Dt} \varrho \, dV$$

Unser Ziel: Kontigleichung für eine Stromröhre



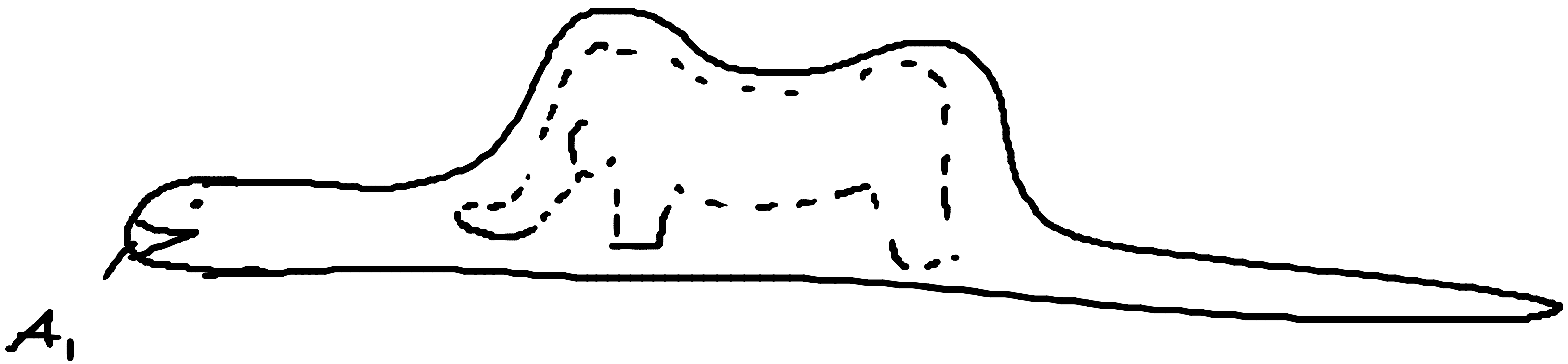


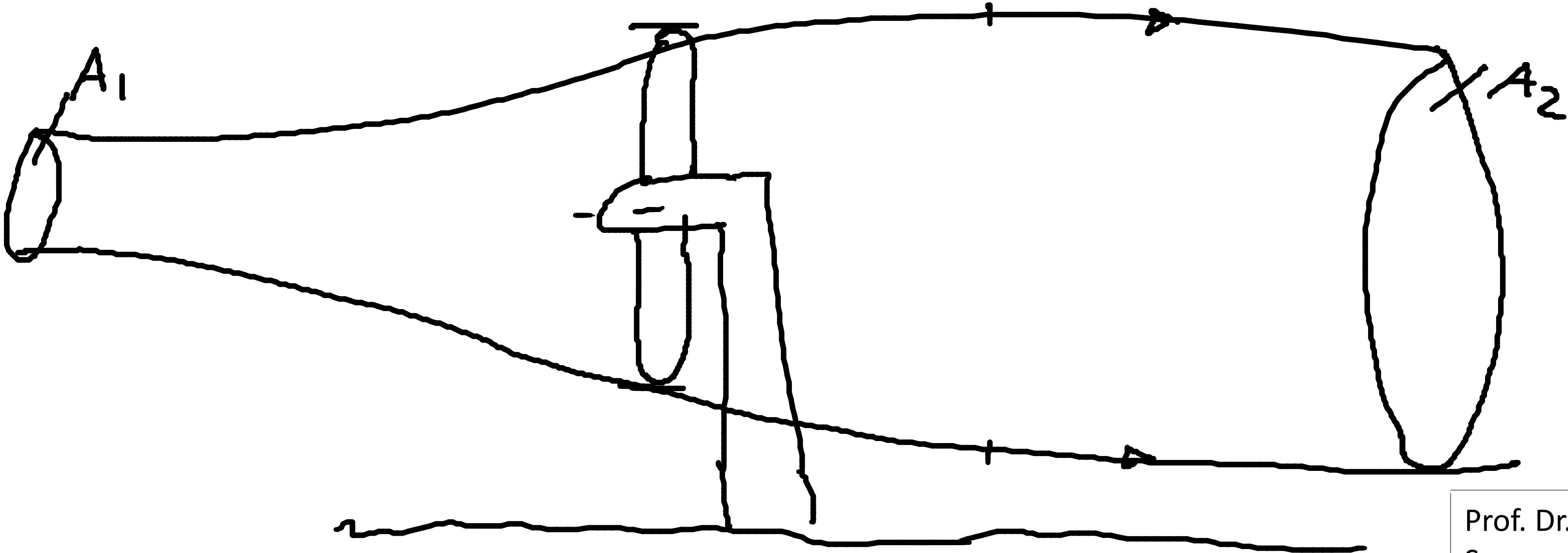


Technische Universität Darmstadt

Fluid System Technik

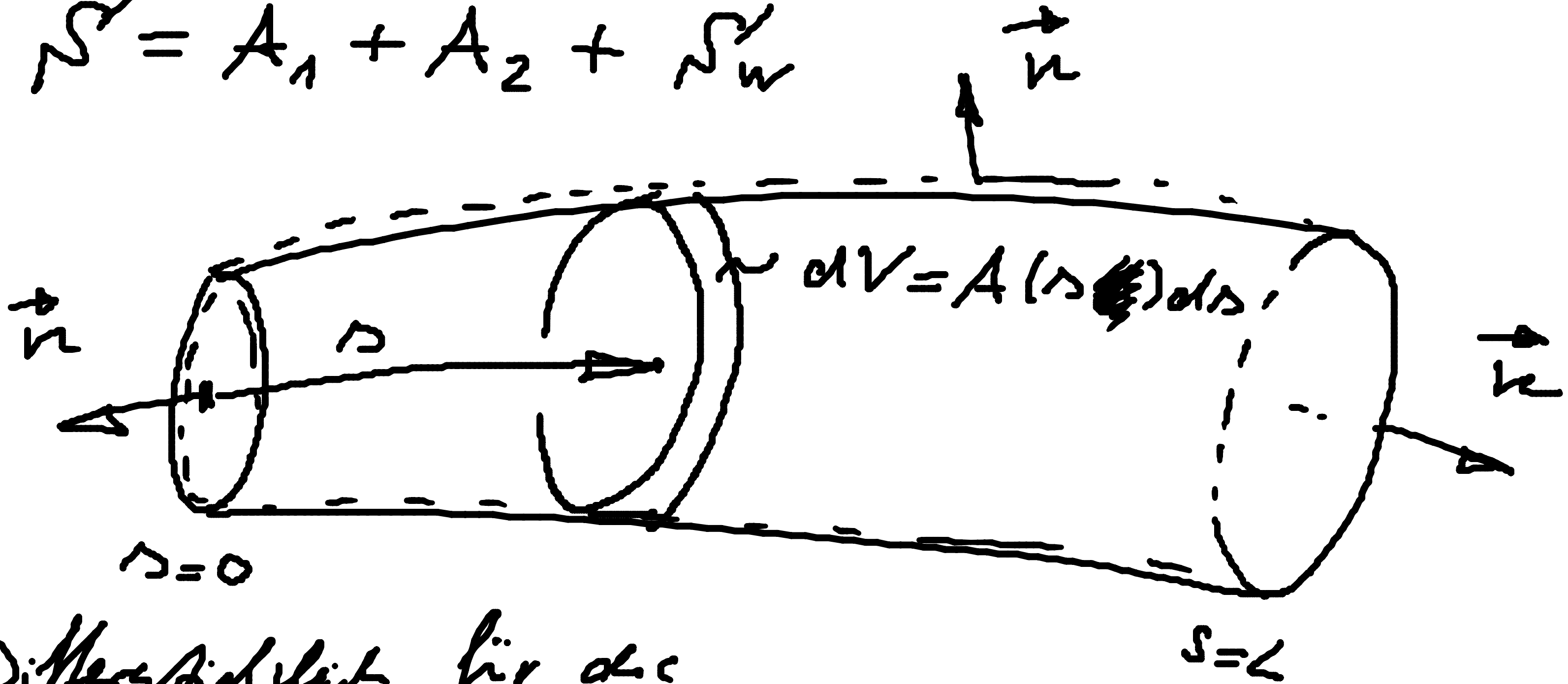
Grundlagen der Turbomaschinen und Fluidsysteme


A_1


A_1
A_2

Spezielle Form der Kontinuitätsgleichung für eine Stromröhre

$$S' = A_1 + A_2 + S'_W$$

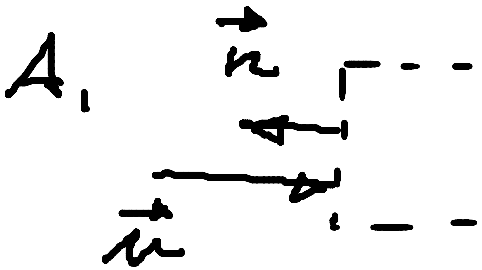


Differentialgleichung für die Stromlinie:

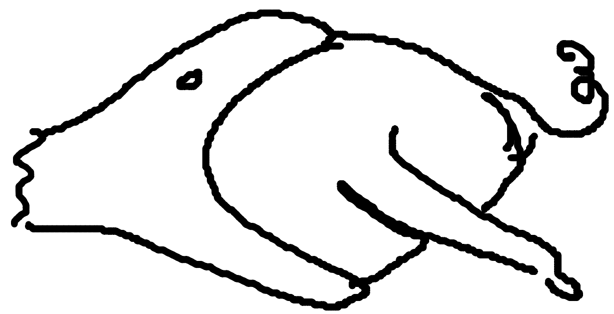
$$\frac{\vec{u}}{|\vec{u}|} = \frac{d\vec{x}}{ds}; \quad \vec{x}(s=0) = \vec{x}_0$$

$$\frac{Dm}{Dt} = 0 \Leftrightarrow \frac{\partial}{\partial t}\int_V \varrho \, dV + \oint_S \varrho \, \vec{u} \cdot \vec{n} \, dS$$

$$\int_0^L \frac{\partial}{\partial t} \varrho A \, ds + \underbrace{\int_{A_1} \varrho \, \vec{u} \cdot \vec{n} \, dA_1}_{-\dot{m}_1} + \underbrace{\int_{A_2} \varrho \, \vec{u} \cdot \vec{n} \, dA_2}_{\dot{m}_2} + \underbrace{\int_{S_w} \varrho \, \vec{u} \cdot \vec{n} \, dS_w}_{\neq 0 \text{ für atmende Wände}} = 0$$



Senckenberg Museum Frankf.



TECHNISCHE UNIVERSITÄT DARMSTADT
FLUID SYSTEM TECHNIK

Grundlagen der Turbomaschinen und Fluidsysteme

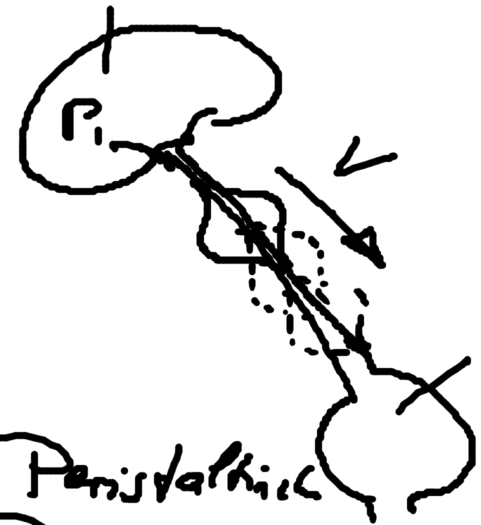
Prof. Dr. Ing. Peter Pelz

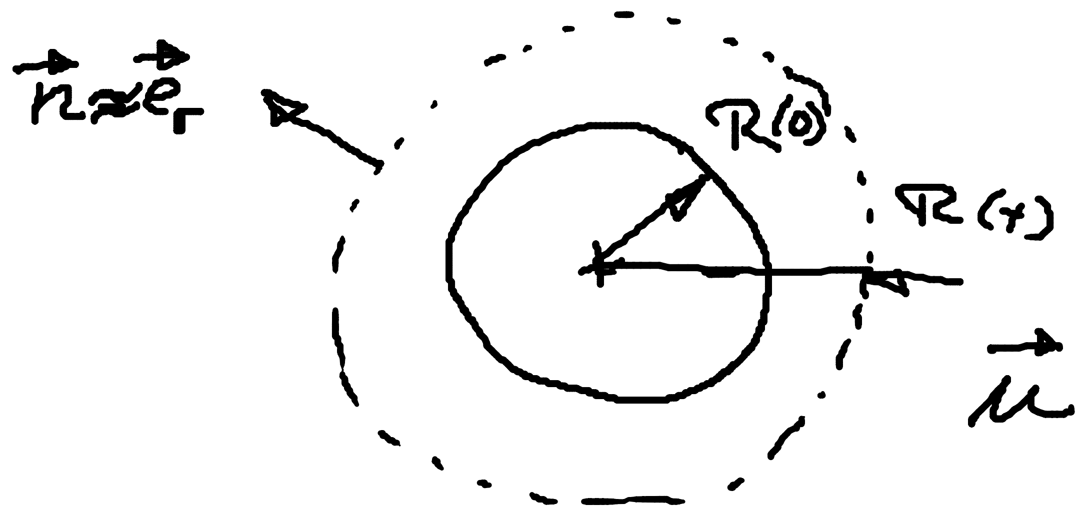
Zum Term

$$\int_{S_W} \rho\, \vec{u}\cdot\vec{n}\, dS$$

Spezialfall: kreisförmige Quelle in der Strömung.

Niere





$$\vec{u}\cdot\vec{n} = u_r = \dot{R}$$

$$\int_{S_W} \rho\, \vec{u}\cdot\underbrace{\vec{n}\, dS}_{2\pi R\, ds} = \int_0^L \rho\, \underbrace{\dot{R}\, 2\pi R}_{\frac{\partial A}{\partial t}}\, ds = \int_0^L \rho\, \frac{\partial A}{\partial t}\, ds$$

$$A = \pi R^2$$

$$\frac{\partial A}{\partial t} = 2\pi R \dot{R}$$

vgl. Kap 9.1
Spurk.

$$\int_0^L \frac{\partial \rho}{\partial t} A\, ds - \dot{m}_1 + \dot{m}_2 + \underbrace{\int_0^L \rho\, \frac{\partial A}{\partial t}\, ds} = 0$$

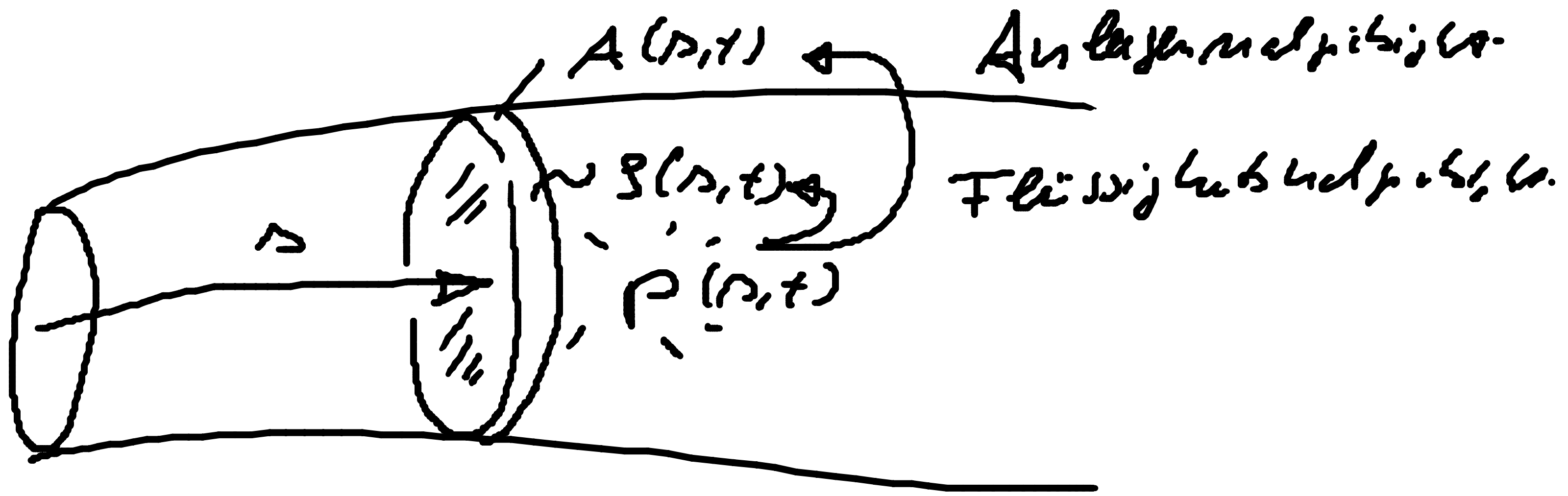
$$\boxed{\int_0^L \frac{\partial}{\partial t}(\varrho A)\,ds - \dot m_1 + \dot m_2 = 0}$$

Kontigleichung für eine Stromröhre

<u>Dichte</u> ist für Meßtechnik unzugänglich.

<u>Druck</u> ist einfach zu messen.

Zeitliche Änderung von ϱA $\rightarrow$ Druckänderung.



$$\underset{\substack{\uparrow\\ \text{Trick}}}{\frac{\partial}{\partial t}(A\varrho) =} \frac{\partial p}{\partial t}\underbrace{\frac{d}{dp}(A\varrho)}_{\substack{\text{quasi-statische}\\ \text{Zustandsänderung od.}\\ \text{Gleichgewichtsbeding.}}}\Bigg|_{\text{Entropie = konst.}}$$

$$\int_0^L \frac{\partial p}{\partial t}\,\cancel{H_{eff}}\,ds - \dot m_1 + \dot m_2 = 0$$