Nuclear Excited Studied by proton scattering With a High-Resolution Magnetic Spectrometer

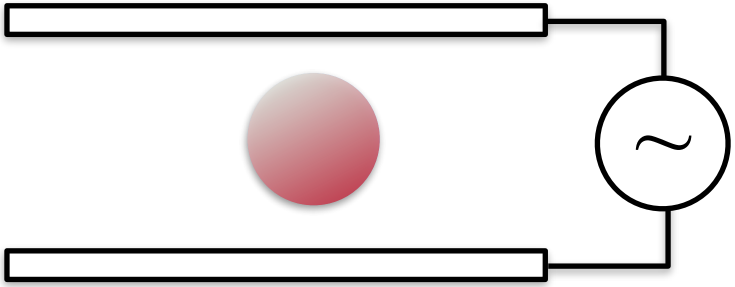
Lecture III

Electric Response of Nuclei
Sum Rules

5542 4365 at https://menti.com

https://www.menti.com/alqpregyewub

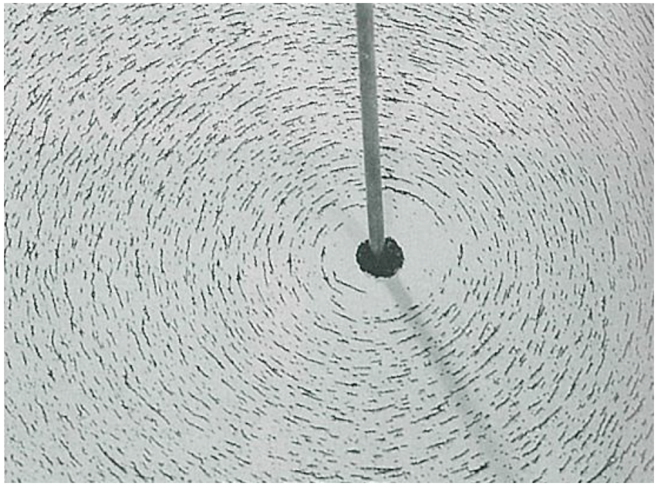
Electric Response of Nuclei



Static Electromagnetism

Static Electricity

Paper pieces are attracted by a comb



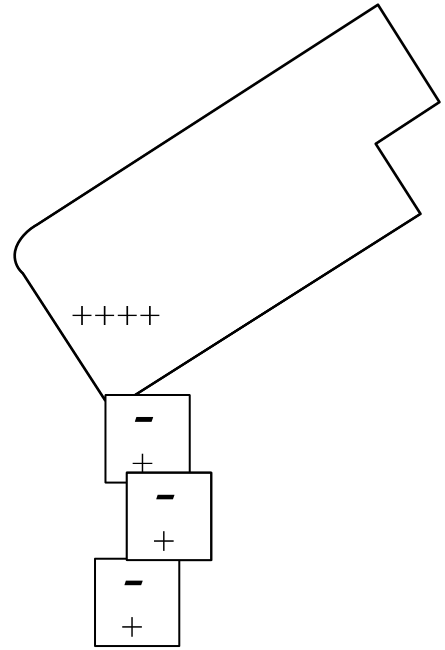
Static Magnetism

Iron powders align with the magnetic field

Halliday / Resnick / Walker, *Electricity and Magnetism*

Static Electricity

Induced Polarization



A come with + charge approaches

→ A neutral papers is polarized

→ The paper is attracted.

→ A next paper is polarized and is attracted.

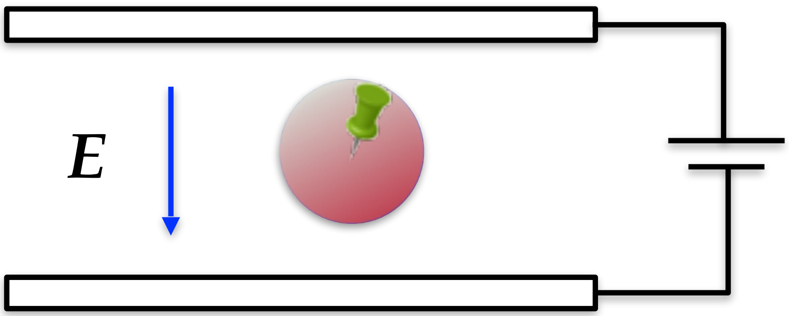
Photo from *Fundamentals of Physics, Electricity and Magnetism*, Halliday/Resnick/Walker

Static Electric Dipole Polarizability (α_D)

Electric dipole moment

$$p = \alpha_D \times E$$

α_D: electric dipole polarizability

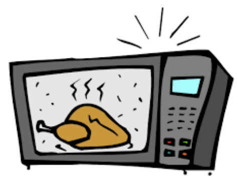


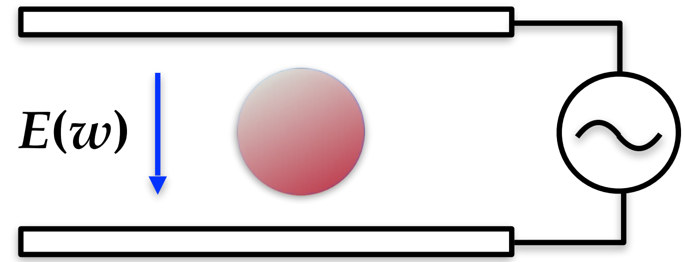
nucleus

in a static electric field

with fixing the c.m. position

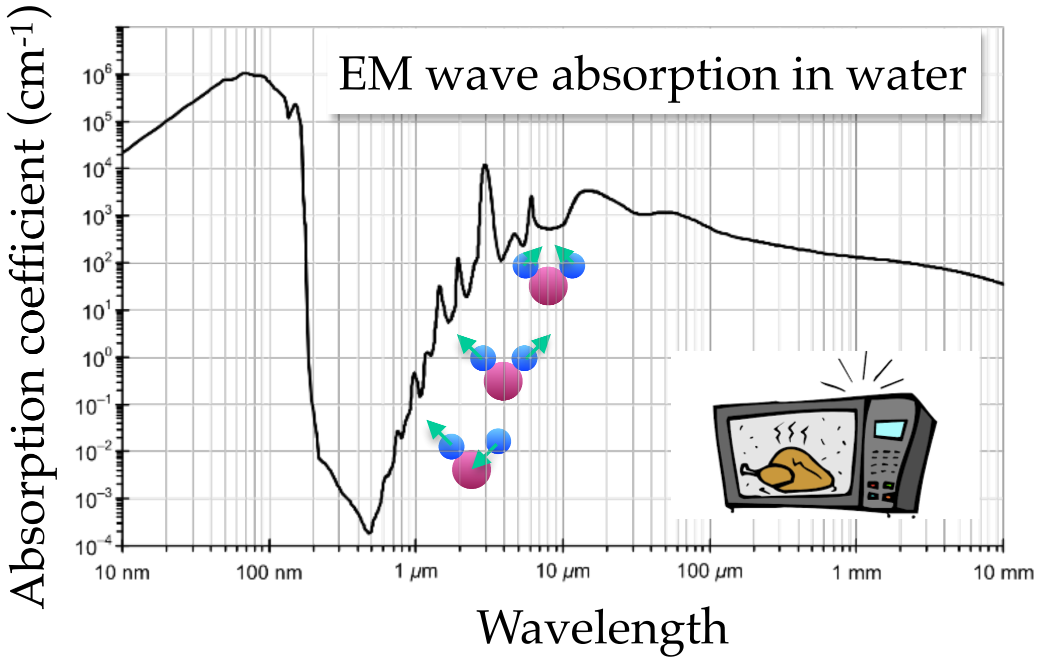
Electric Dipole Response of Nuclei

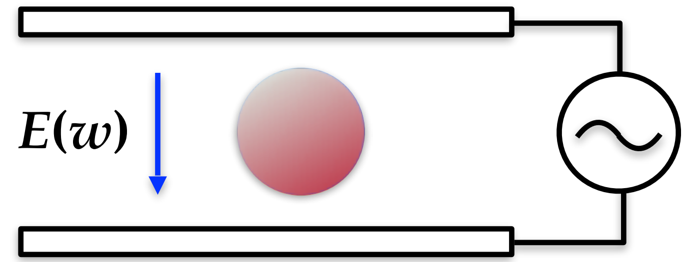




dielectric material
in an oscillating electric field

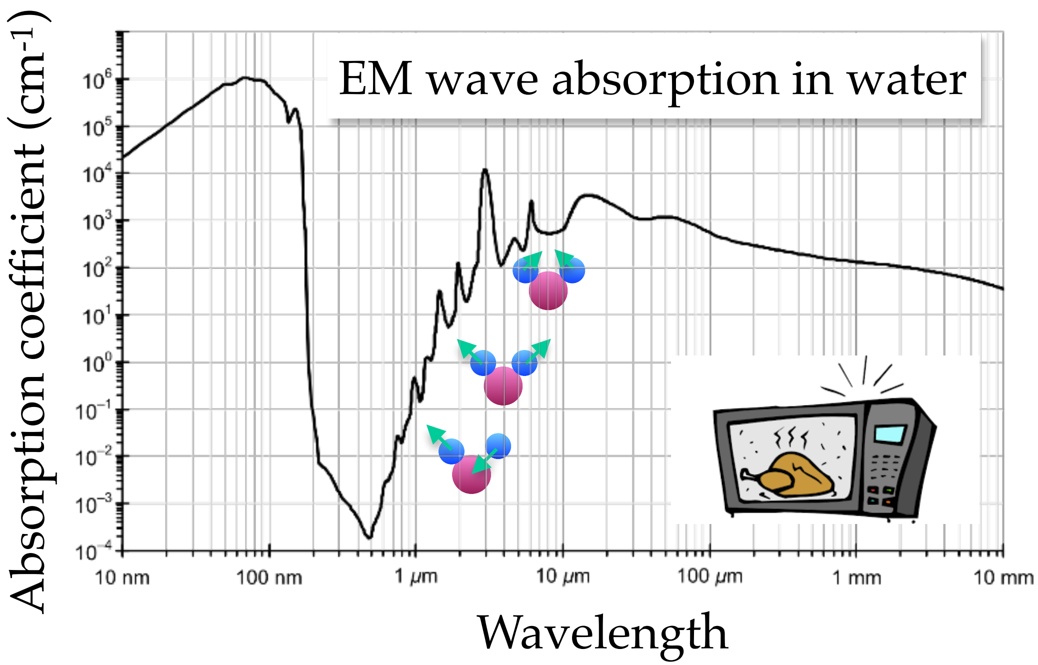
Electric Dipole Response of Nuclei

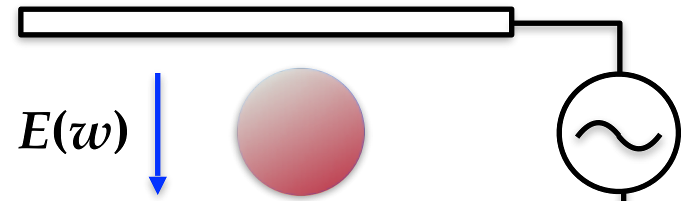




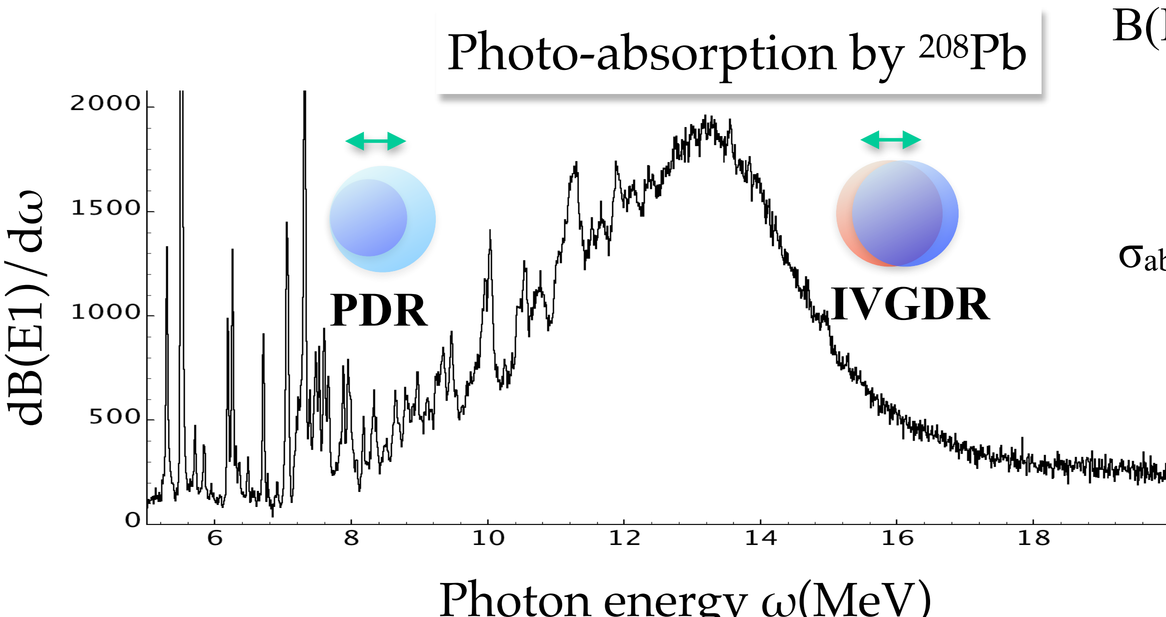
dielectric material
in an oscillating electric field

Electric Dipole Response of Nuclei





dielectric material
in an oscillating electric field



B(E1): reduced transition probability

$$B(E1) = \frac{1}{2J_i + 1} \left| < \Psi_f || O(E1) || \Psi_i > \right|^2$$

σ_{abs}: (E1) photo-absorption cross section

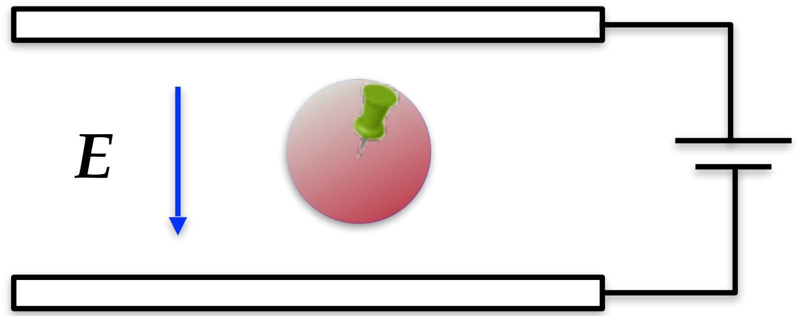
$$\sigma_{\rm abs} = \frac{16\pi^3 \times 10}{9} \alpha E_x \frac{dB(E1)}{dE_x}$$

Static Electric Dipole Polarizability (α_D)

Electric dipole moment

$$p = \alpha_D \times E$$

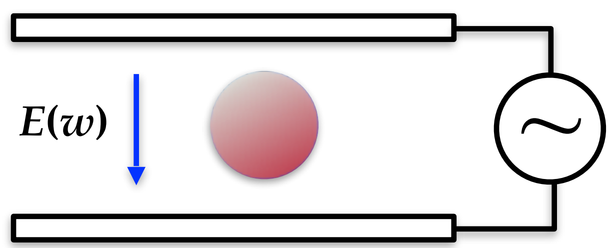
α_D: electric dipole polarizability



nucleus
in a static electric field
with fixing the c.m. position

Inversely energy-weighted sum-rule of B(E1)

$$\alpha_D = \frac{8\pi e^2}{9} \int \frac{dB(E1)}{E_x}$$



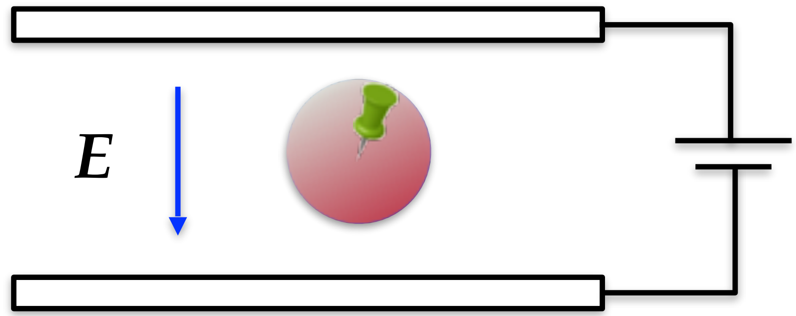
first order perturbation calc. A.B. Migdal: 1944

Static Electric Dipole Polarizability (α_D)

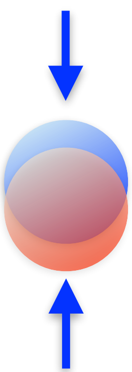
Electric dipole moment

$$p = \alpha_D \times E$$

***α_D*: electric dipole polarizability**



nucleus
in a static electric field
with fixing the c.m. position



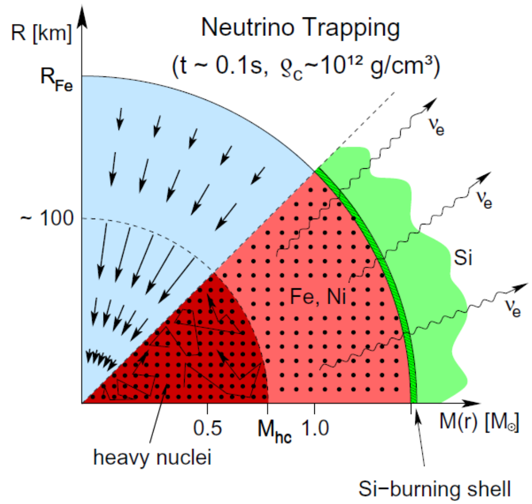
The **restoring force** originates from the **symmetry energy** of **the Nuclear Equation of State**

→ Lecture IV

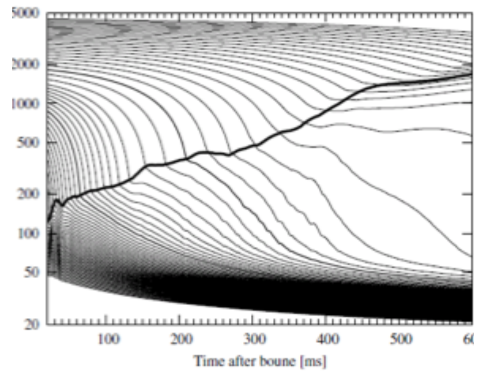
Nuclear EOS

is important for nuclear physics and nuclear-astrophysics

Core-collapse supernova

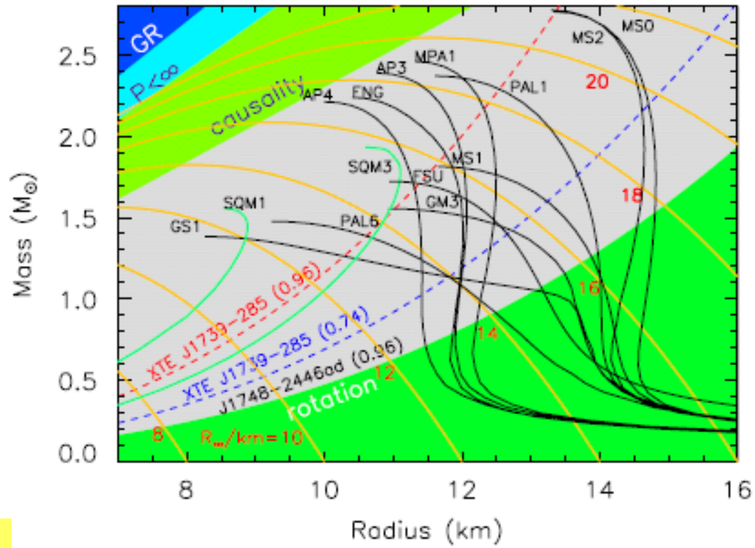


Langanke and Martinez-Pinedo



Y. Suwa et al., ApJ764, 99 (2013).

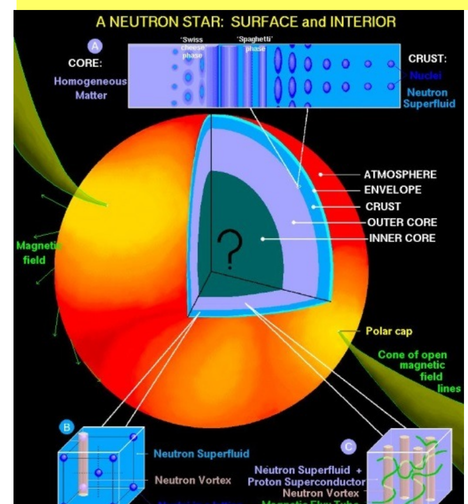
Neutron star mass vs radius



Lattimer et al., Phys. Rep. 442, 109(2007)

Nucleosynthesis

Neutron star structure

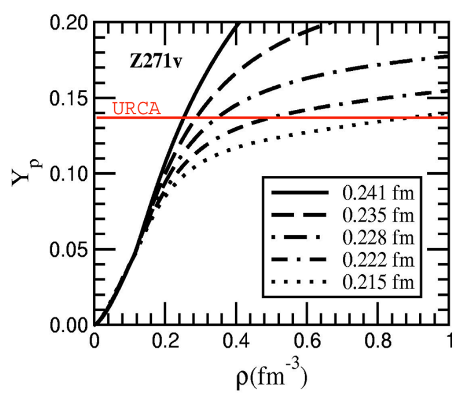


http://www.astro.umd.edu/~miller/nstar.html

Neutron Star Merger Gravitational Wave

https://www.youtube.com/watch?v=IZhNWh_lFuI

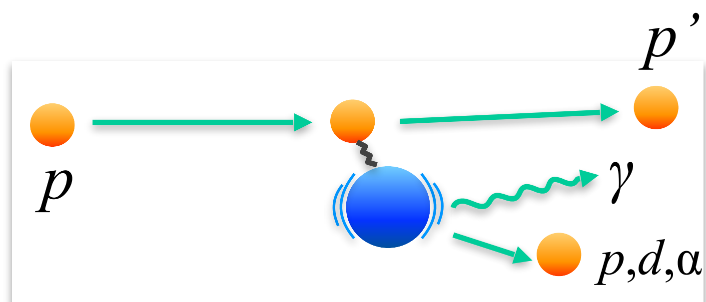
Neutron star cooling



Lattimer and Prakash, Science 304, 536 (2004).

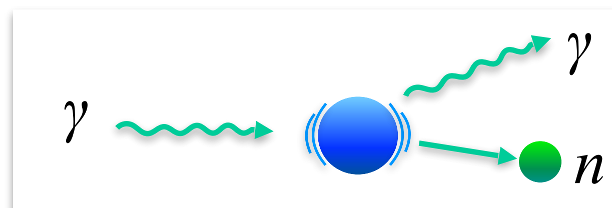
Probes for the Electric Dipole Response of Nuclei

1. Virtual photon excitation
   (Coulomb excitation)
   - proton inelastic scattering at 0 deg.



E_x distribution in one shot measurement
total photo-absorption c.s.
up to 32 MeV at RCNP

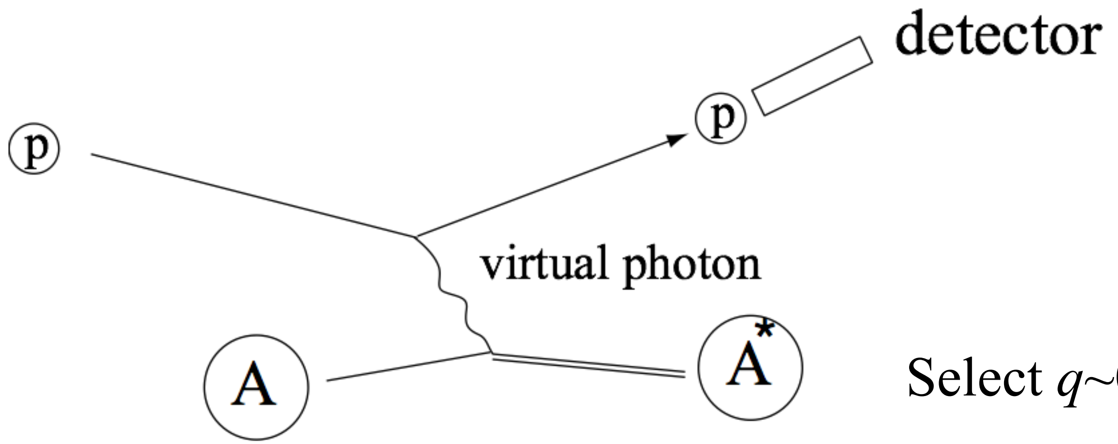
2. Real photon absorption
   - (γ,γ') Nuclear Resonance Fluorescence
   - (γ,n), (γ,2n), (γ,p), … photo-disintegration

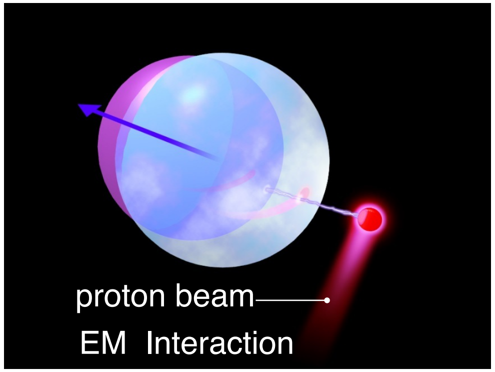


pure EM probe
precise absolute c.s.
model-independent separation of E1 and M1
partial strength including *n*
up to 19.5 MeV at ELI-NP

Probing the E1 Response by Proton Scattering

Missing Mass Spectroscopy by Virtual Photon Excitation



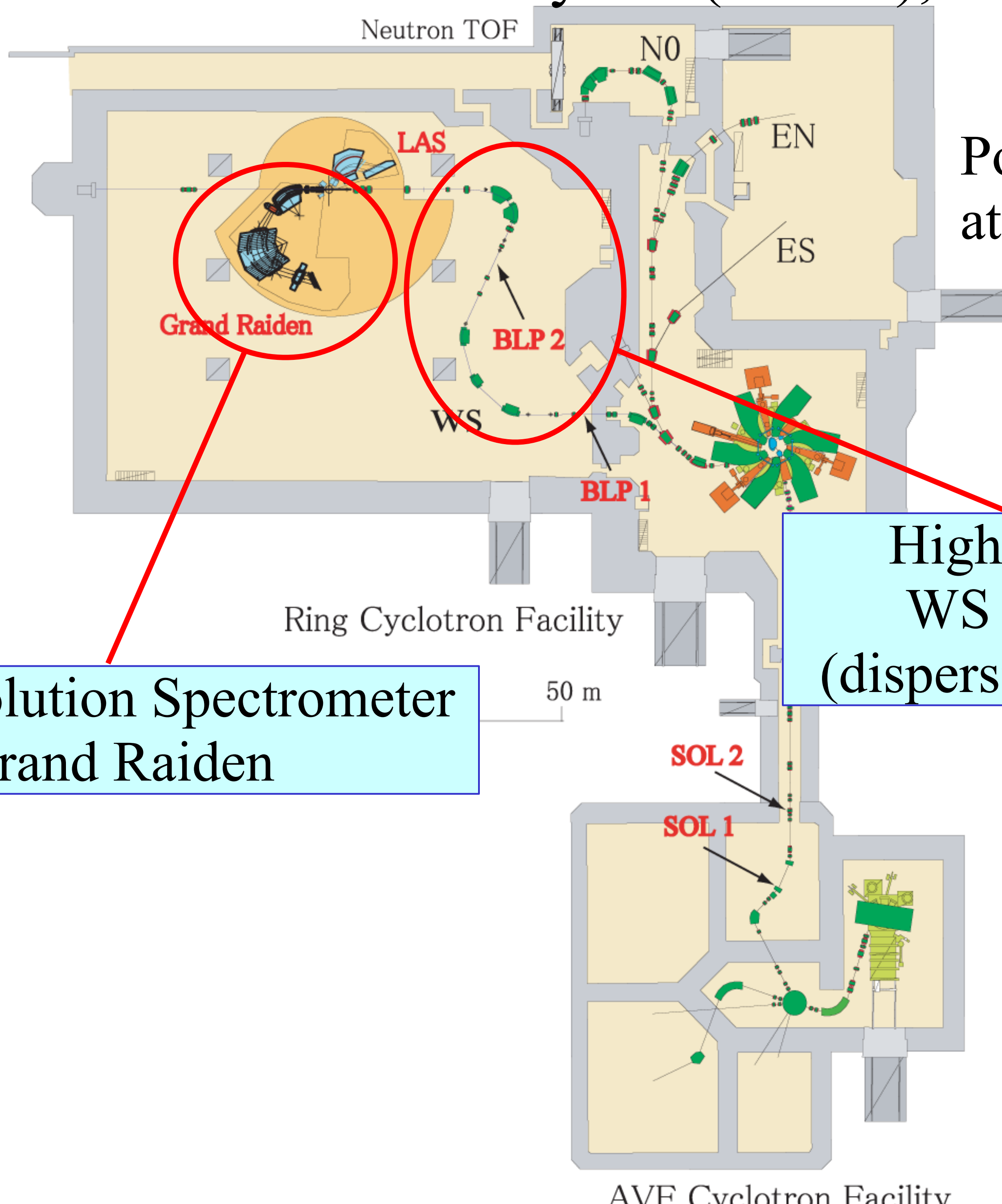


Select q~0 (~0 deg.)

- **Missing mass spectroscopy**:
  Total strength is measured (sum of all the decaying channel: inclusive)
- **Multipole decomposition** of the strength in the continuum:
  Includes the contribution of unresolved small states
- **Coulomb excitation**: EM Interaction
  Determination of the Absolute transition strength.

Research Center for Nuclear Physics (RCNP), Osaka University

Polarized *p* beam at 295 MeV



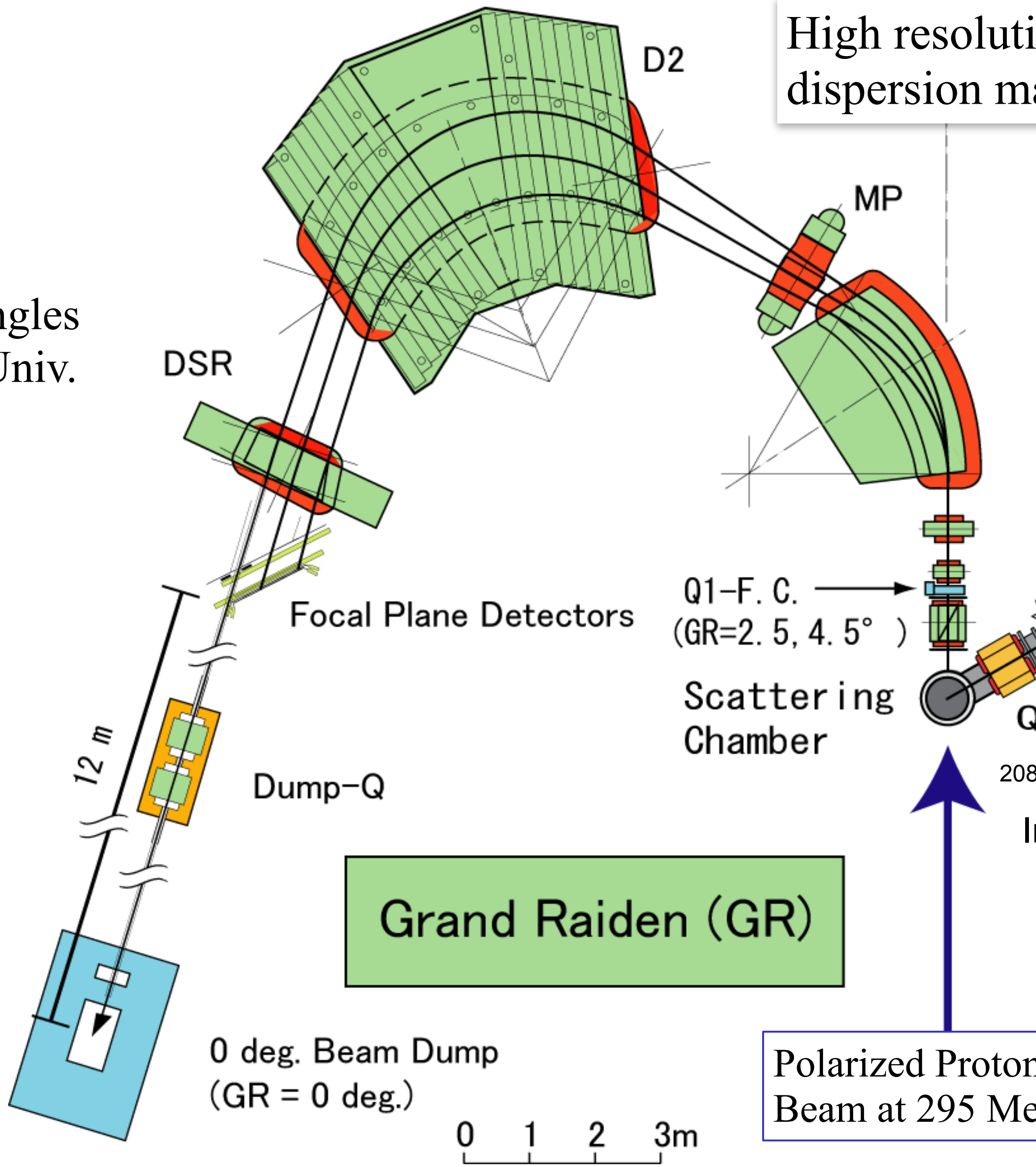
High-resolution Spectrometer
Grand Raiden

High-resolution
WS beam-line
(dispersion matching)

High-Resolution Spectrometer "Grand Raiden"

High resolution of 20-30 keV: dispersion matching.

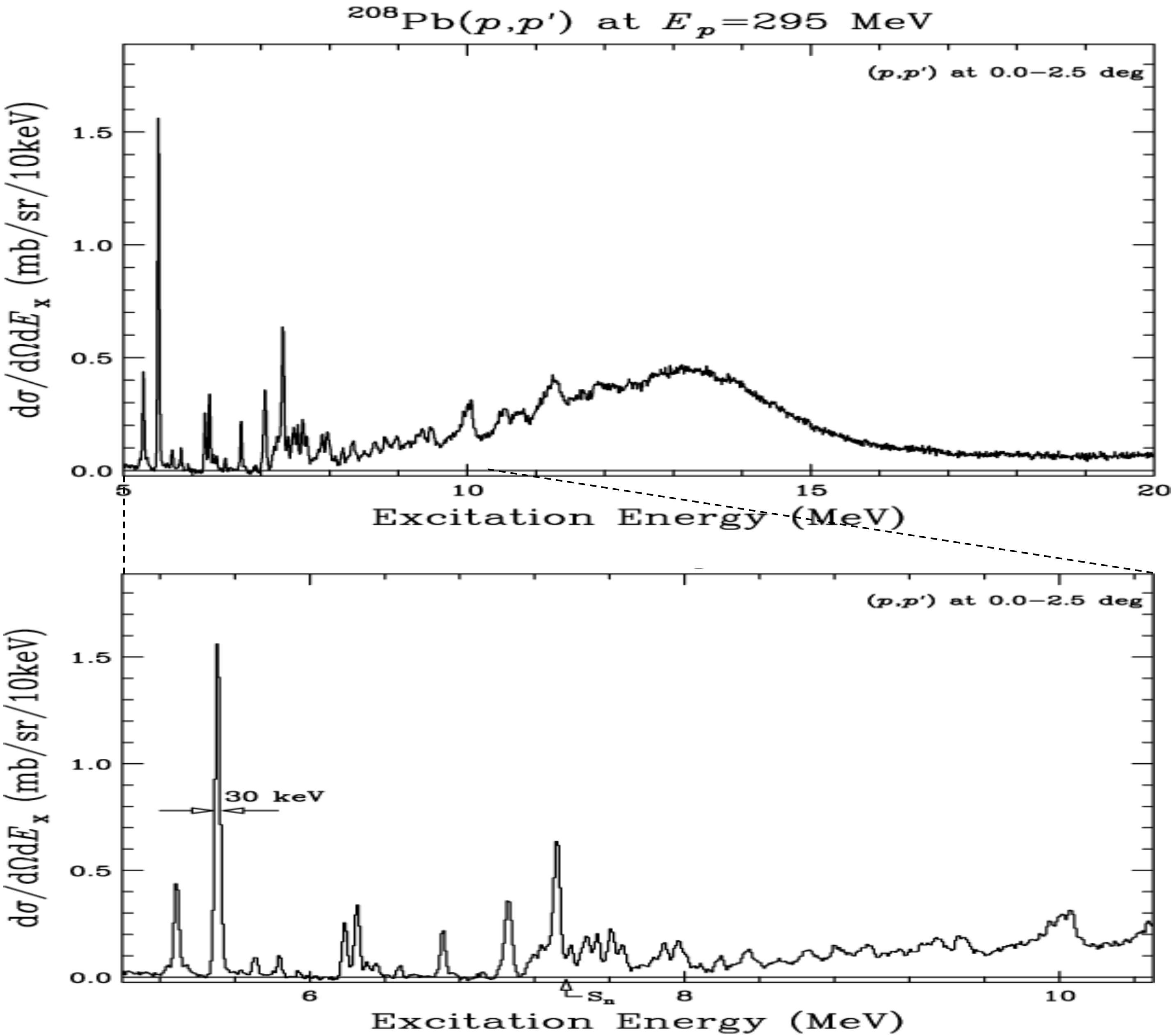
Proton scattering
at very forward angles
at RCNP, Osaka Univ.



^{208}Pb target: 5.2 mg/cm^2
Intensity : 1-8 nA

Polarized Proton Beam at 295 MeV

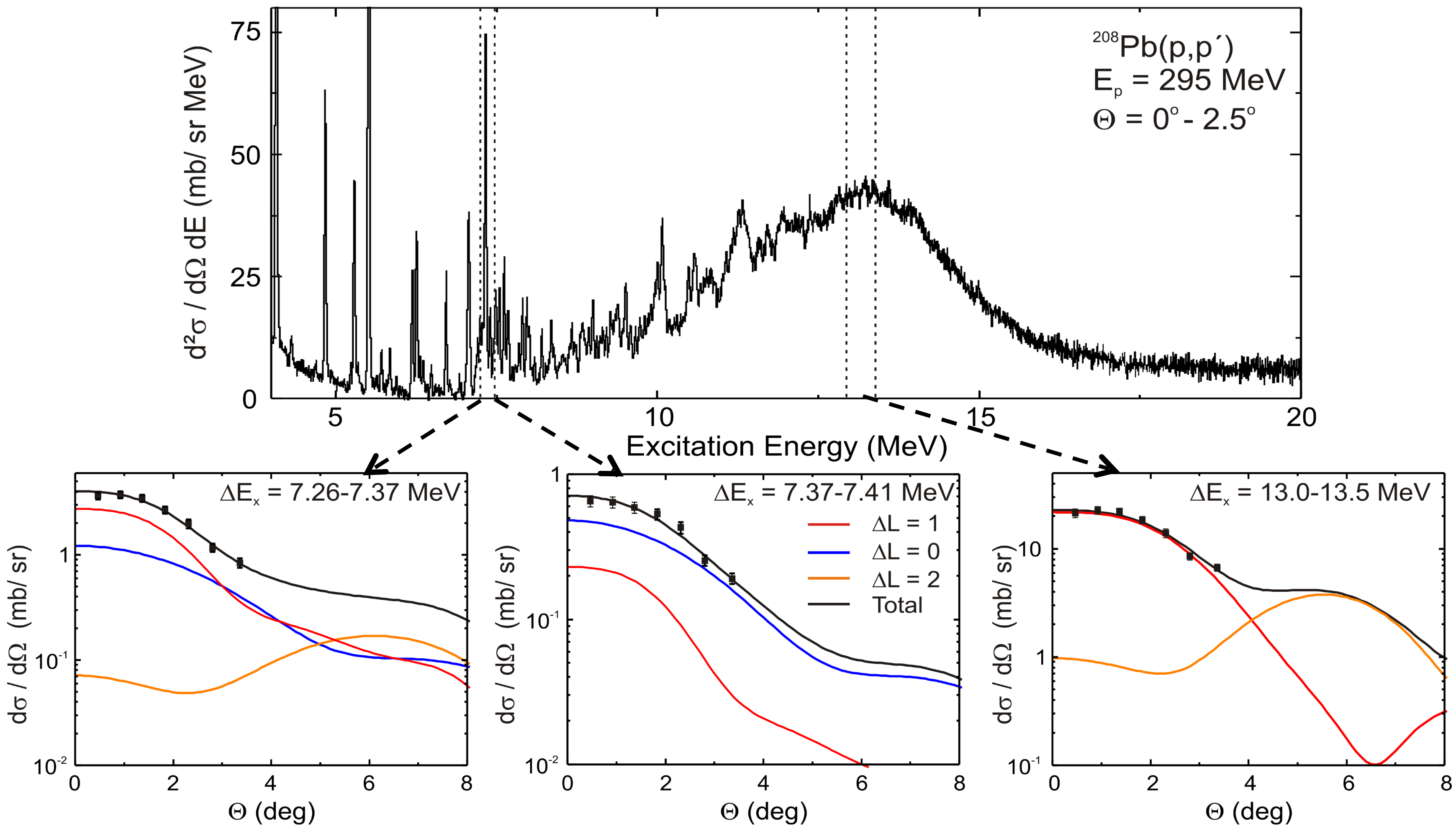
AT et al., NIMA605, 326 (2009)



²⁰⁸Pb(p,p′) at E_p=295 MeV
(p,p′) at 0.0–2.5 deg
dσ/dΩdE_x (mb/sr/10keV)
0.0
0.5
1.0
1.5
5
10
15
20
Excitation Energy (MeV)
(p,p′) at 0.0–2.5 deg
dσ/dΩdE_x (mb/sr/10keV)
0.0
0.5
1.0
1.5
30 keV
S_n
6
8
10
Excitation Energy (MeV)

B(E1): continuum and GDR region

Method 1: Multipole Decomposition



- Neglect of data for Θ>4: (p,p´) response too complex
- Included E1/M1/E2 or E1/M1/E3 (little difference)

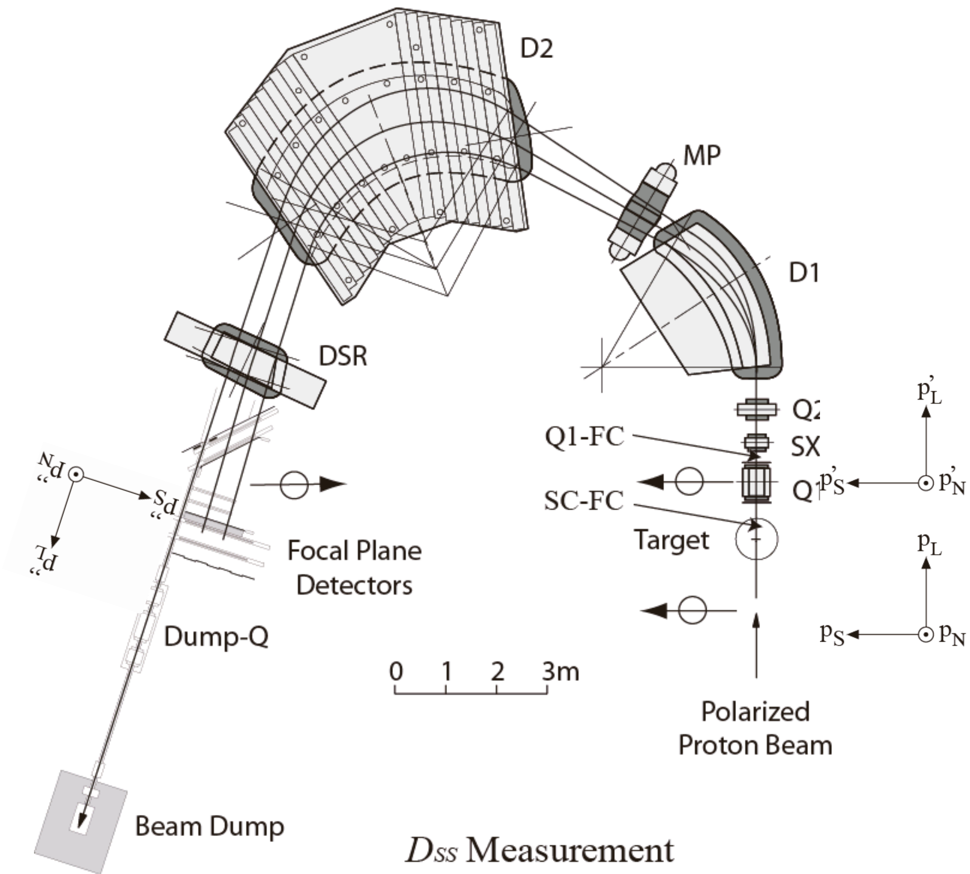
Grazing Angle = 3.0 deg

Polarization Transfer Measurement

spin precession

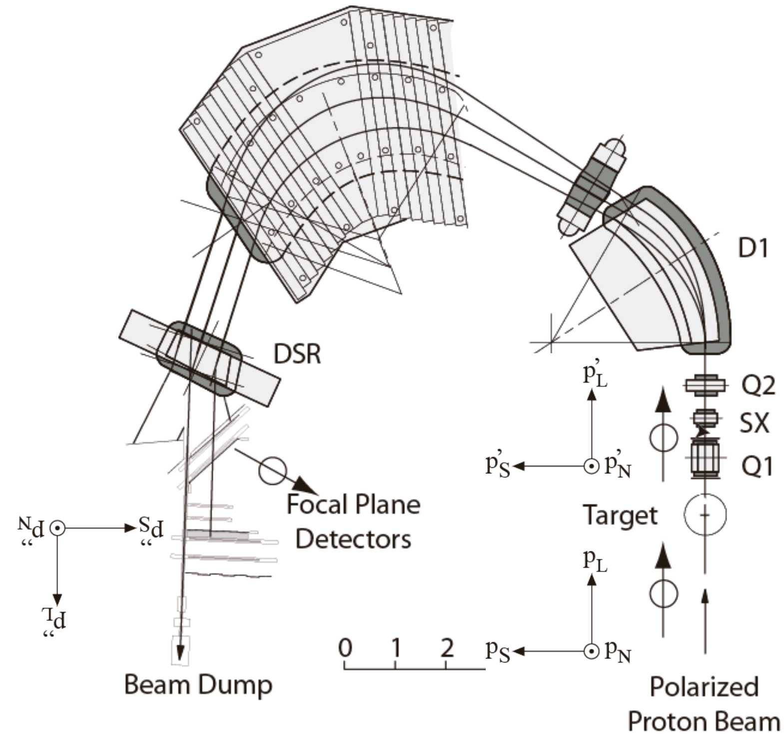
$$\theta_p = \gamma\left(\frac{g}{2} - 1\right)\theta_b$$

θ_p: precession angle with respect to the beam direction
θ_b: bending angle of the beam
g: Lande's g-factor
γ: gamma in special relativity



D_{SS} Measurement

$\theta_b \cong 162°$



D_{LL} Measurement

$\theta_b \cong 180°$

B(E1): continuum and GDR region

Method 2: Decomposition by Spin Observables

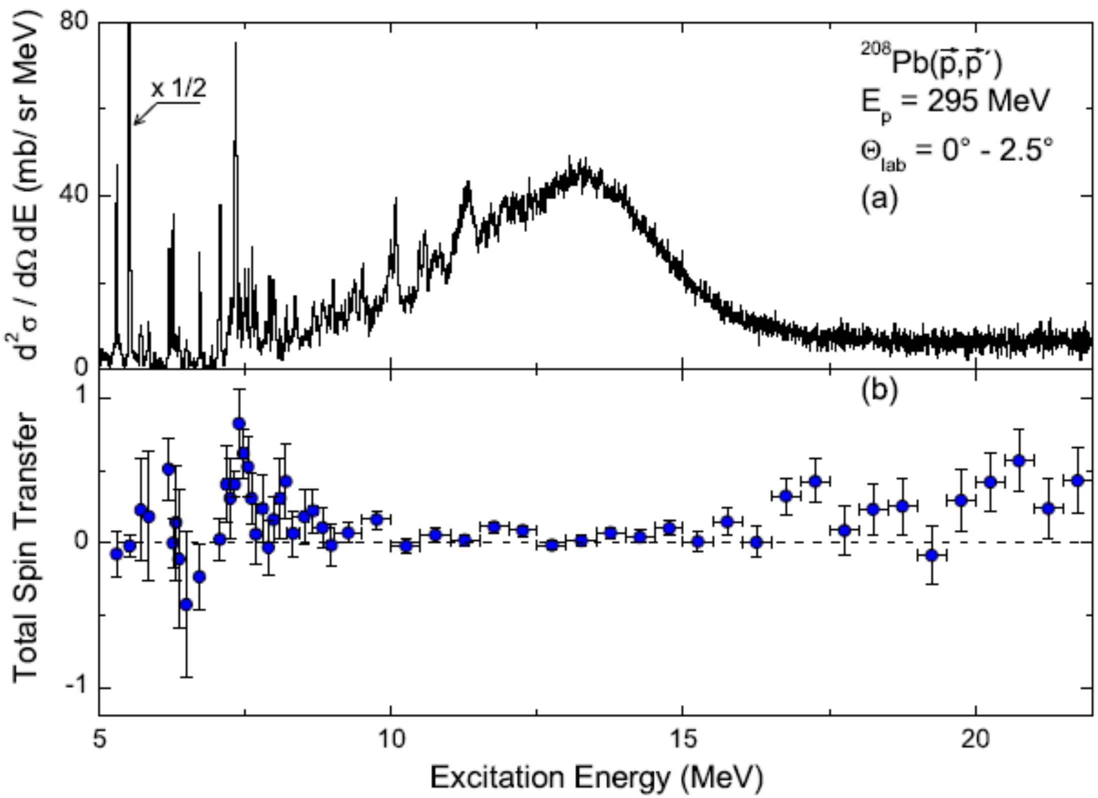
- Polarization observables at 0° ➡ spinflip / non-spinflip separation

model-independent

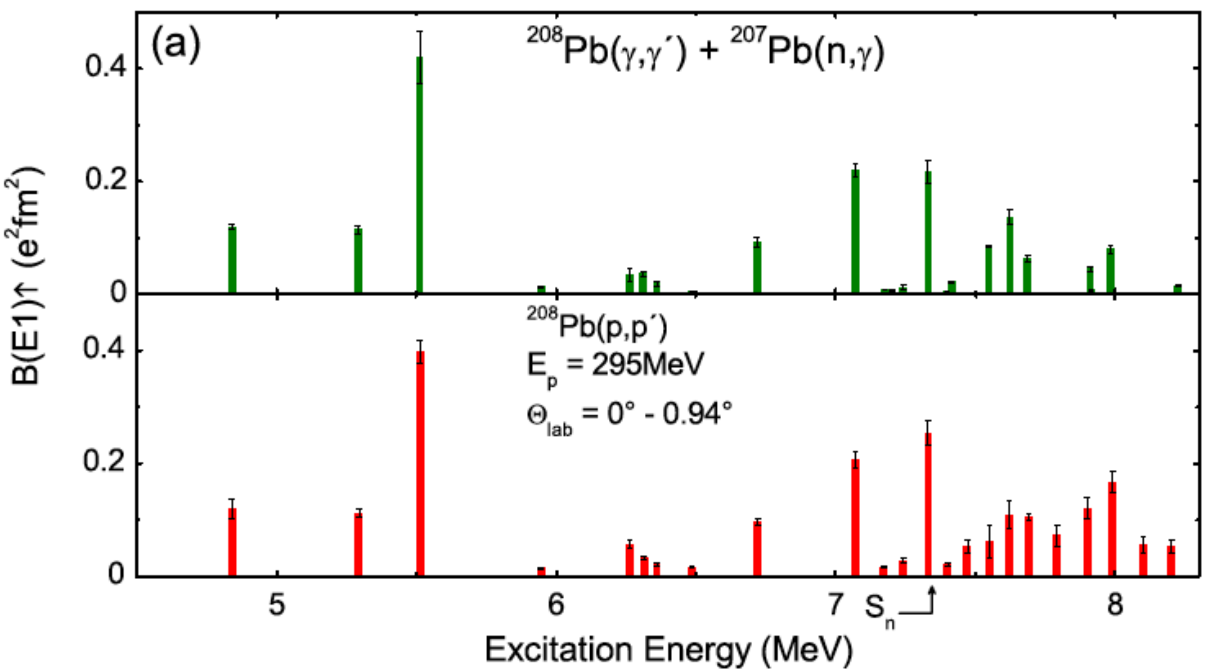
E1 / spin-M1 decomposition

T. Suzuki, PTP 103 (2000) 859

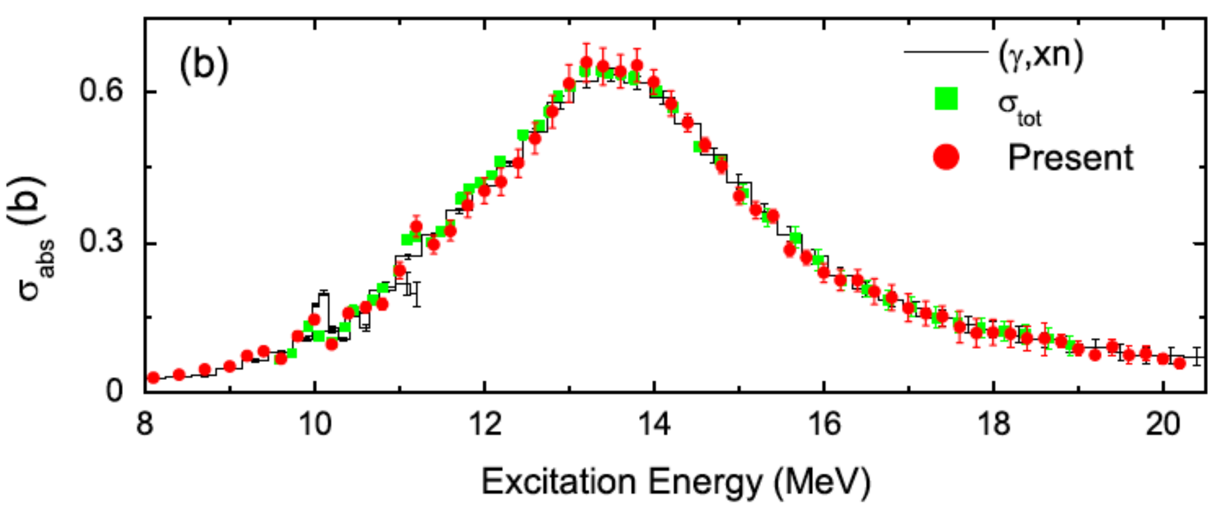
$$\text{Total Spin Transfer } \Sigma \equiv \frac{3-(2D_{SS}+D_{LL})}{4} = \begin{cases} 1 \text{ for } \Delta S = 1 & \text{spin-M1} \\ 0 \text{ for } \Delta S = 0 & \text{E1} \end{cases}$$



Comparison with (γ,γ') and (γ,xn)

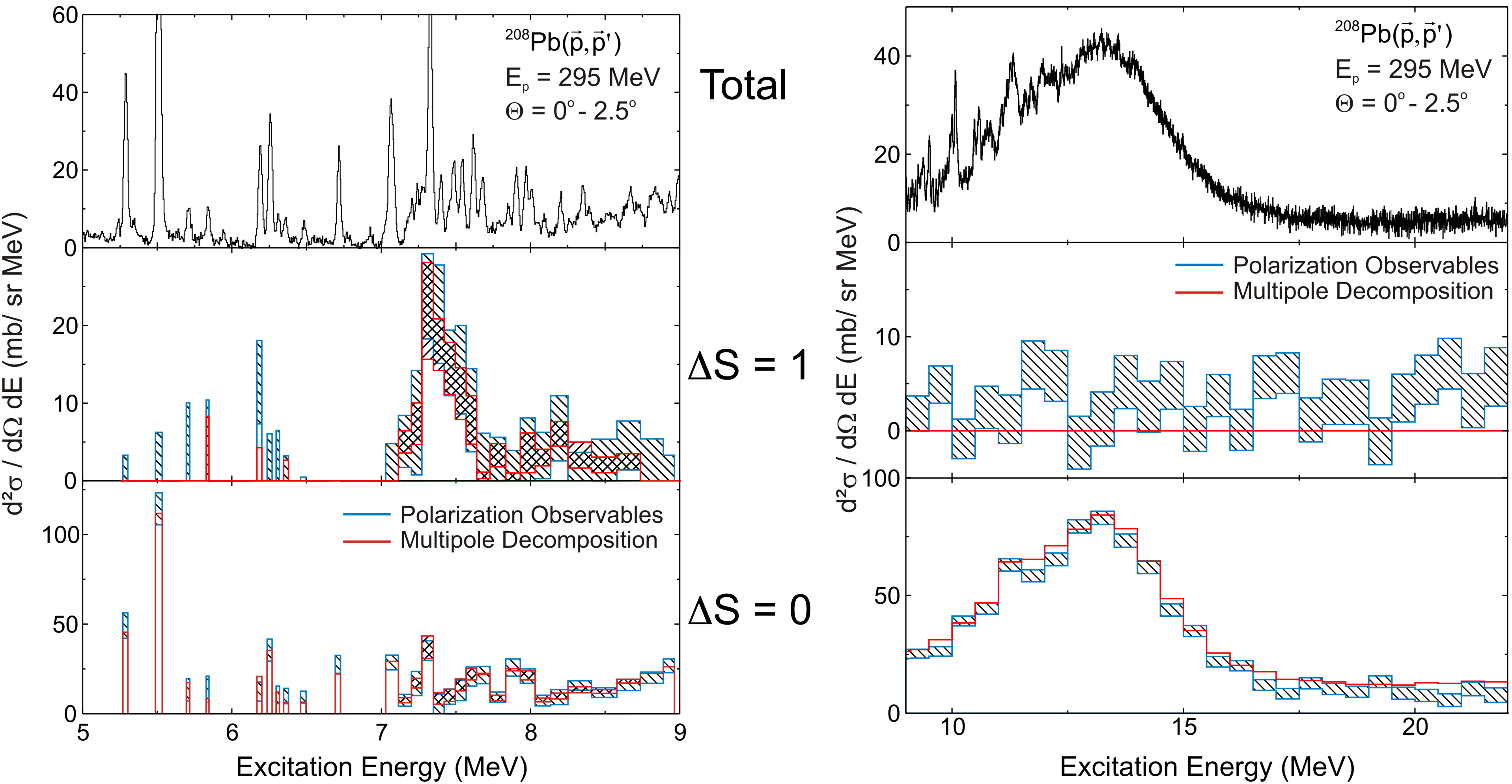


low-lying discrete states

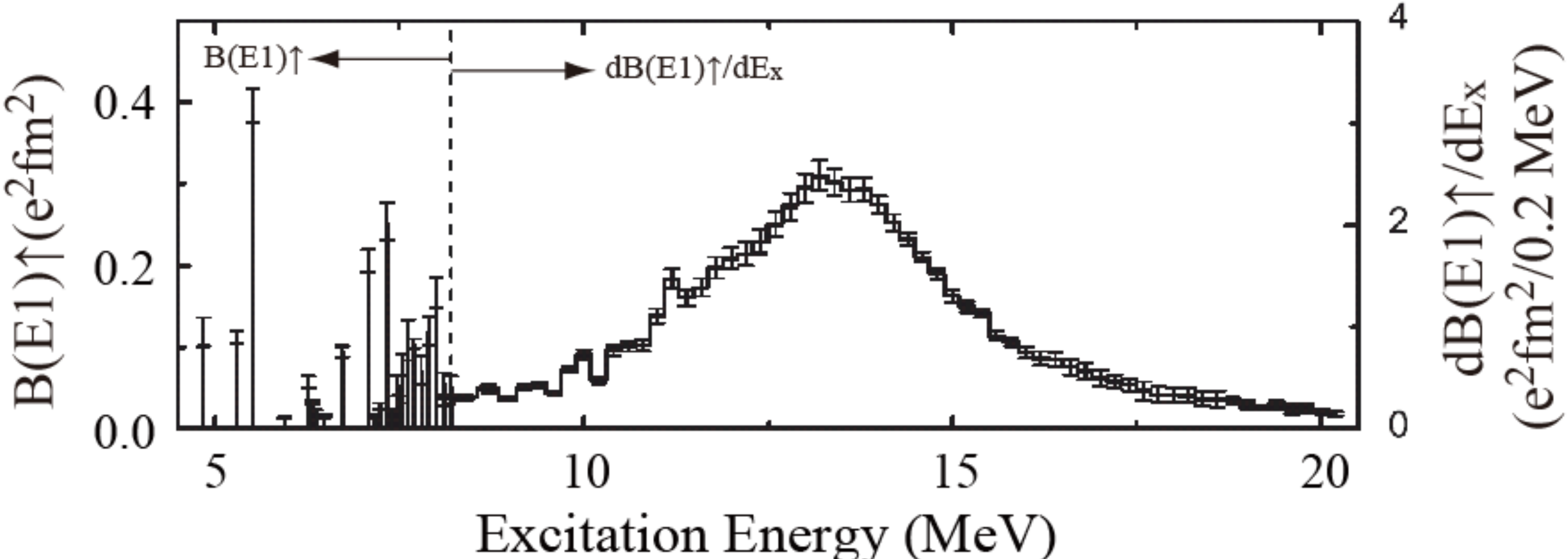


GDR region

Comparison between the two methods



E1 Response of ^{208}Pb and α_D



E_x	0	10	20	130 MeV
α_D	2.7	16.2		1.2 fm³

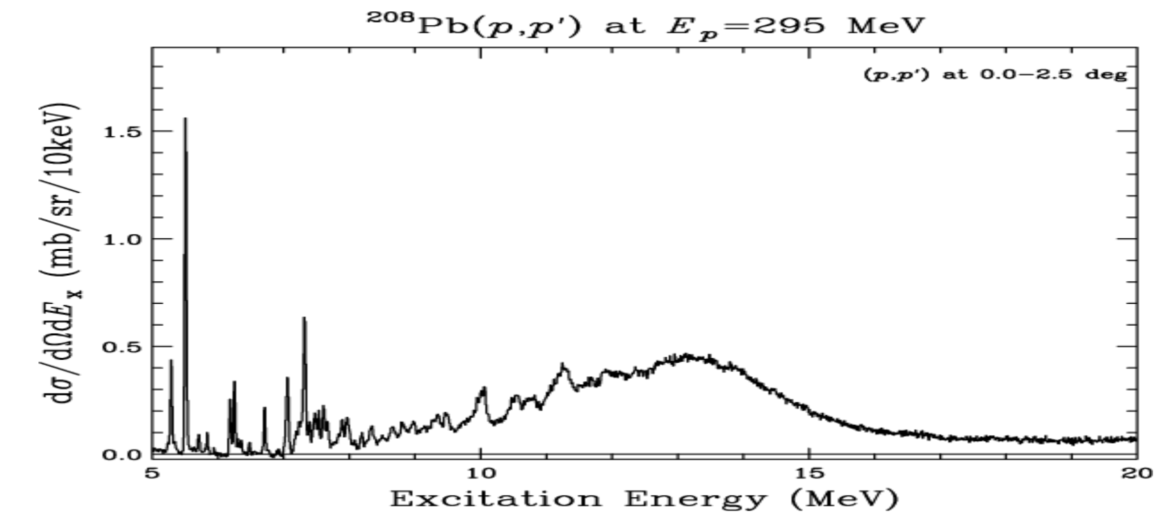
total 20.1±0.6 fm³

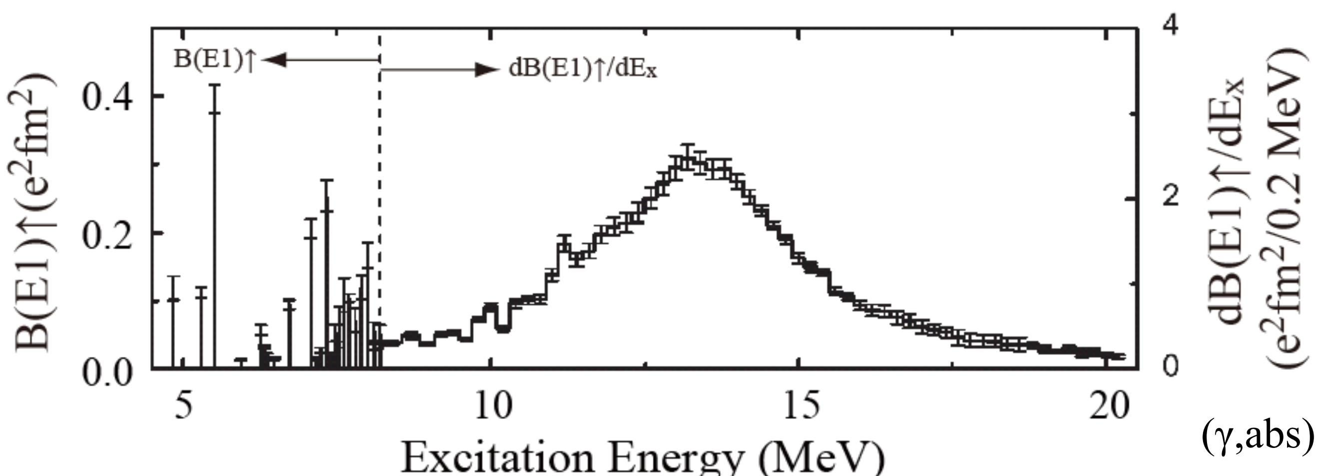
combined data

The dipole polarizability of ^{208}Pb has been precisely determined.

AT et al., PRL107, 062502(2011)

Electric Dipole Polarizability: ^{208}Pb, ^{120}Sn

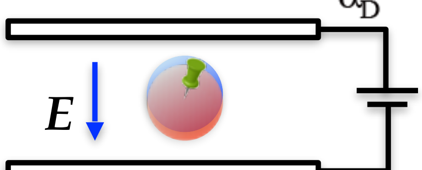




(γ,abs) at Mainz

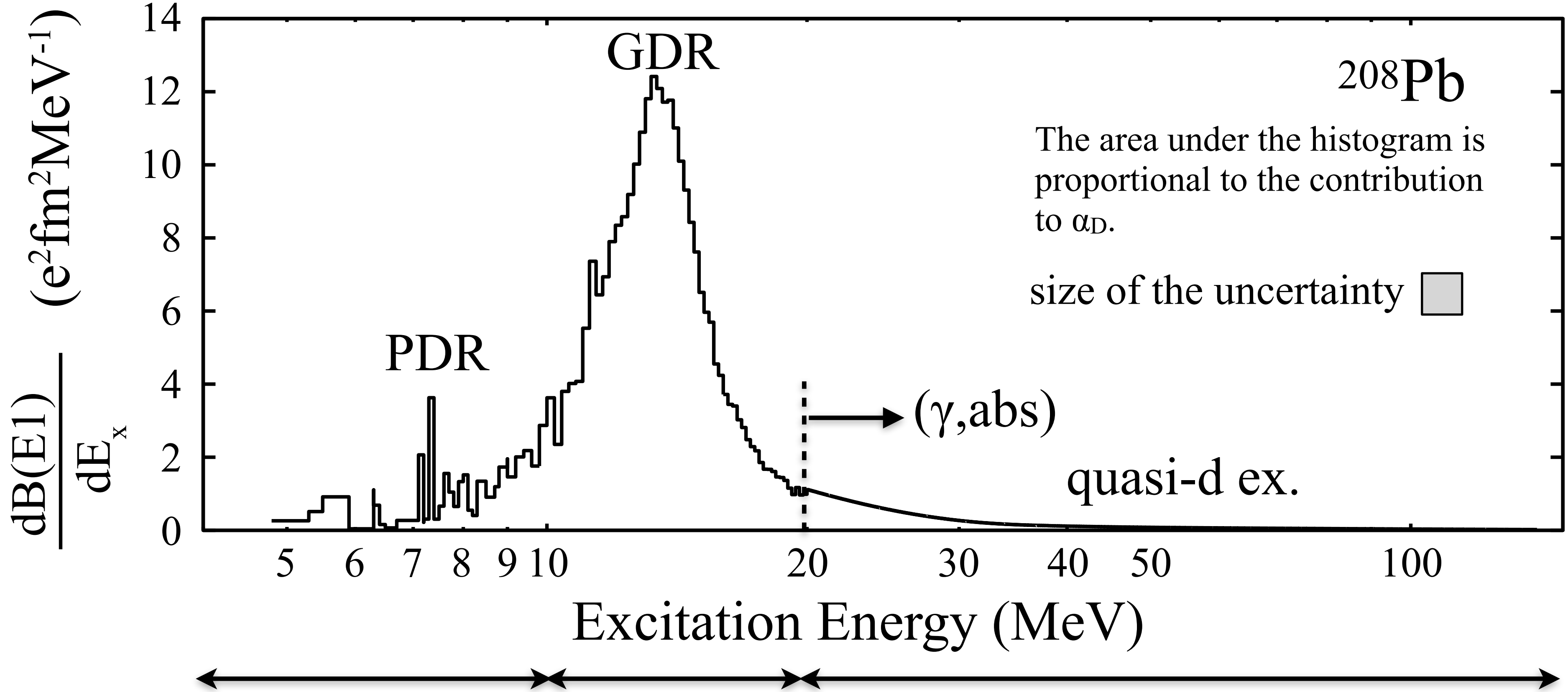
E_x	0 — 10	10 — 20	20 — 130 MeV
α_D	2.7	16.2	1.2 fm^3

total 20.1±0.6 fm^3



AT et al., PRL107, 062502(2011)

Relative Contribution of the Electric Dipole Responses to α_D

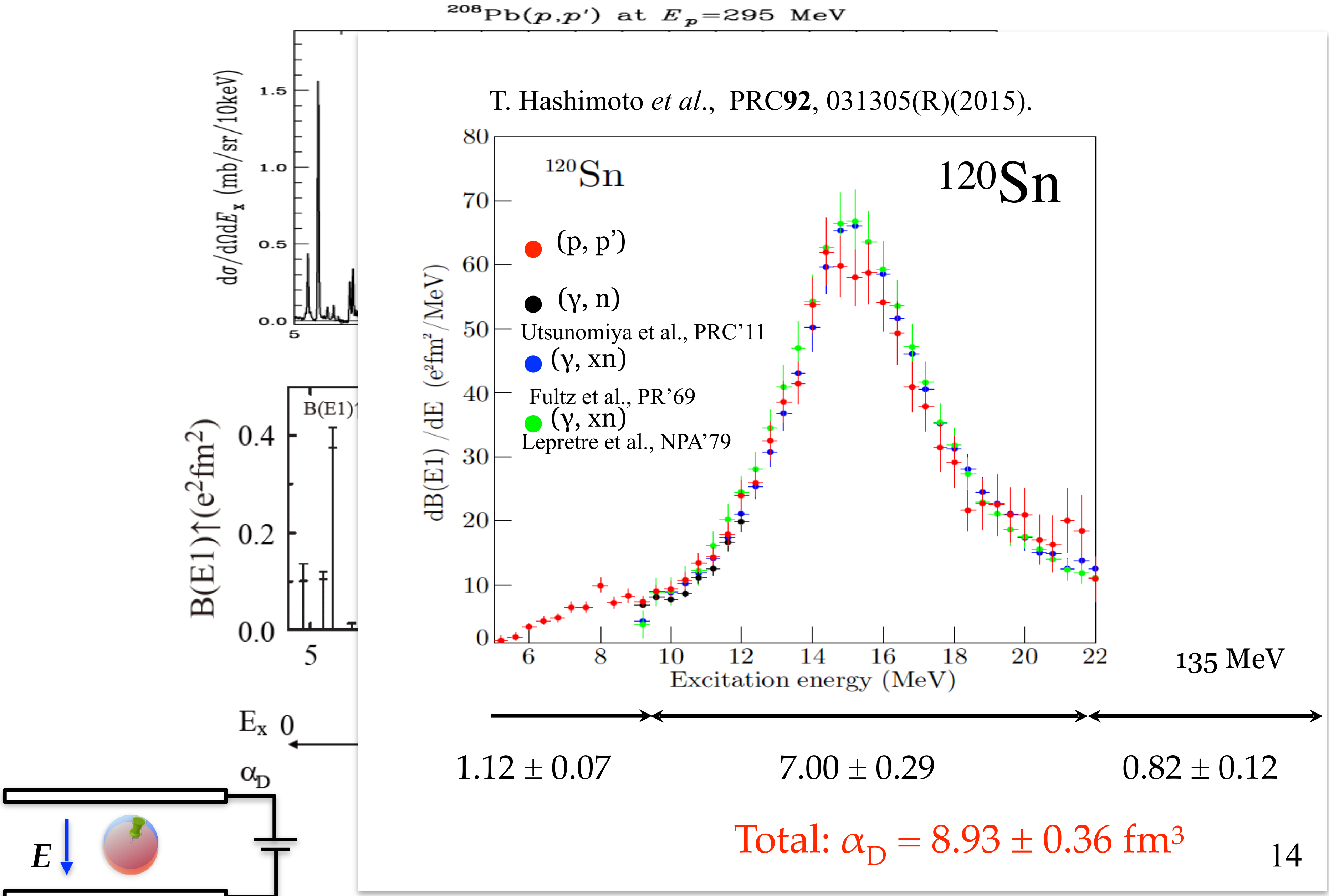


α_D	2.7	16.2	1.2 fm³

total 20.1±0.6 fm³

$$\alpha_D = \frac{8\pi e^2}{9} \int \frac{dB(E1)}{E_x}$$

Electric Dipole Polarizability: ^{208}Pb, ^{120}Sn



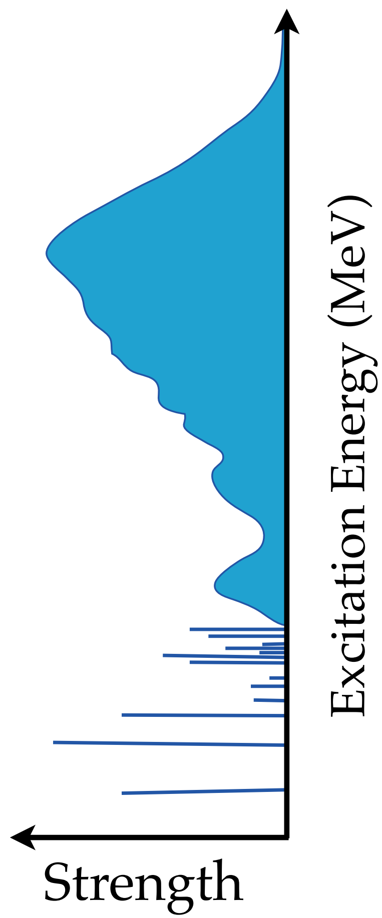
Sum Rules

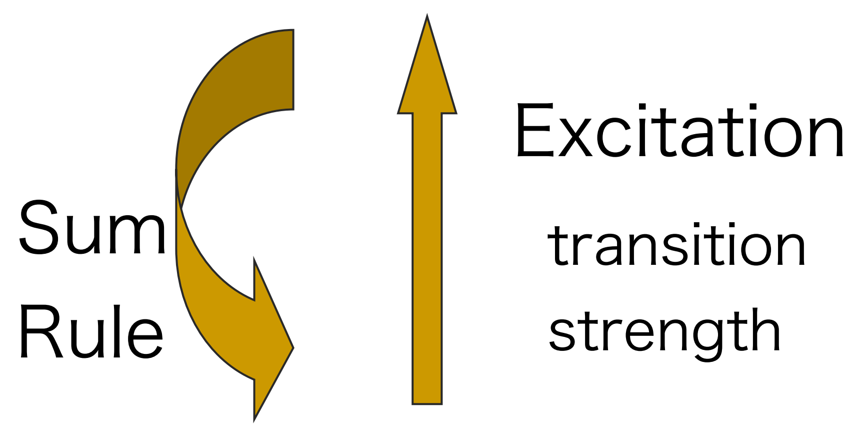
Sum-Rule

The integrated value of the transition strength from the ground state to all the excited states is a property of the ground state.

Excited State

Ground State

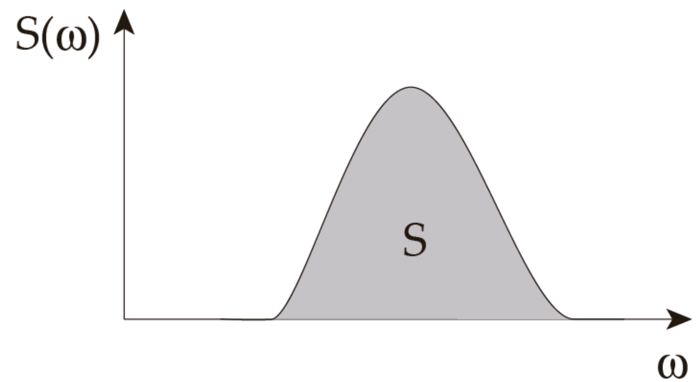




Strength Function and Moments

Strength function $S(\omega)$ of the excitations mediated by the operator O.

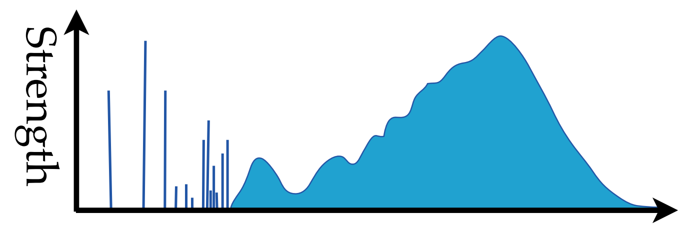
$$S(\omega) \equiv \sum_k \left| < k|O|0 > \right|^2 \delta(\omega - \omega_k)$$



ω: excitation energy

$|0\rangle$ ground state ($\omega = \omega_0 = 0$)

$|k\rangle$ excited state k ($\omega = \omega_k$)



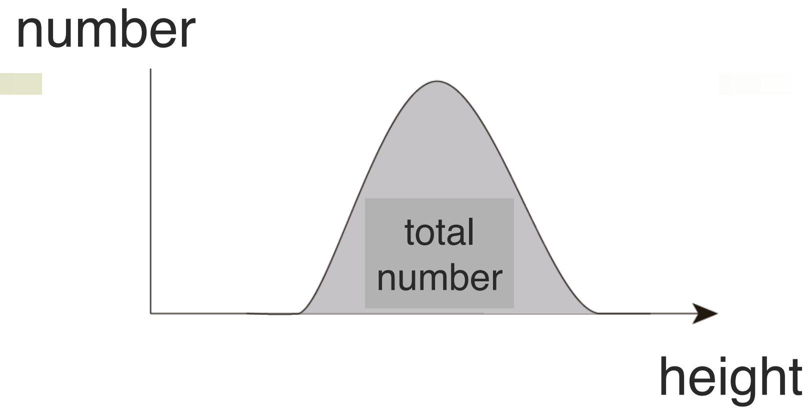
The pth moment of $S(\omega)$

$$m_p \equiv \int_0^\infty S(\omega)\omega^p d\omega = \sum_k \left|\langle k|O|0\rangle\right|^2 \omega_k^p$$

moments

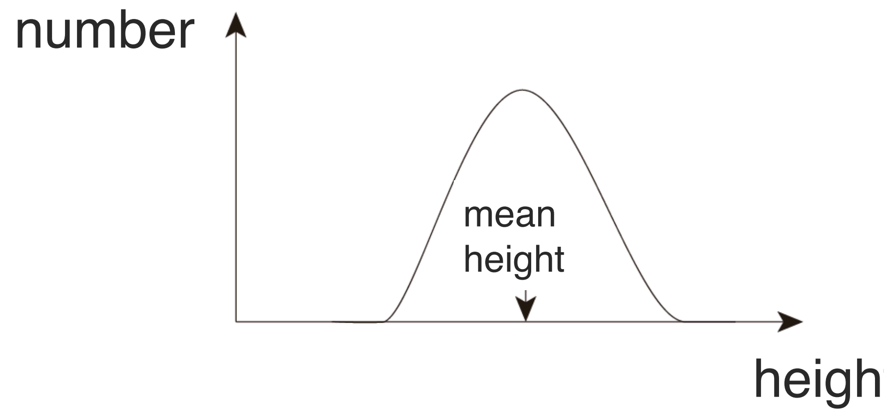
$$S = m_0 = \int_0^\infty S(\omega)d\omega = \sum_k \left|\langle k|O|0\rangle\right|^2$$

total number



$$\overline{\omega} = \frac{m_1}{m_0} = \frac{\int_0^\infty S(\omega)\omega \, d\omega}{\int_0^\infty S(\omega)d\omega} = \frac{\sum_k \left|\langle k|O|0\rangle\right|^2 \omega_k}{\sum_k \left|\langle k|O|0\rangle\right|^2}$$

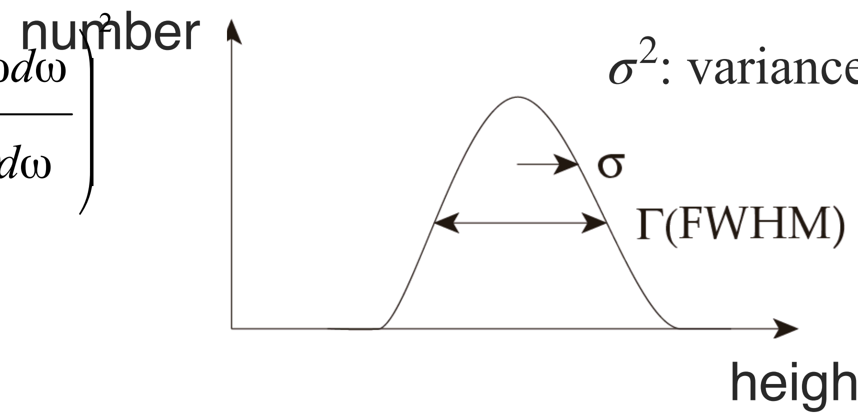
mean height



$$\sigma^2 = \frac{m_2}{m_0} - \left(\frac{m_1}{m_0}\right)^2 = \frac{\int_0^\infty S(\omega)\omega^2 d\omega}{\int_0^\infty S(\omega)d\omega} - \left(\frac{\int_0^\infty S(\omega)\omega d\omega}{\int_0^\infty S(\omega)d\omega}\right)^2$$

variance

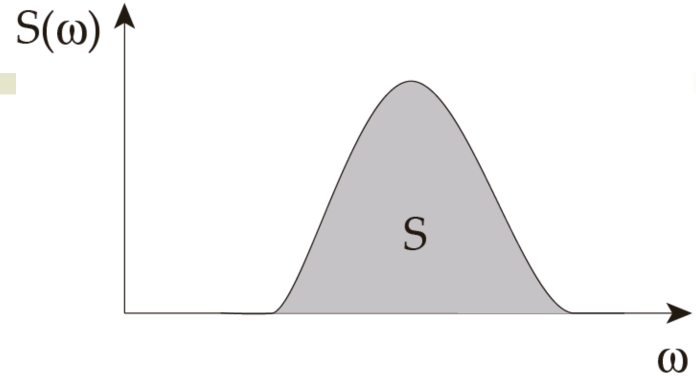
$$= \frac{\int_0^\infty S(\omega)(\omega - \overline{\omega})^2 d\omega}{\int_0^\infty S(\omega)d\omega}$$



moment

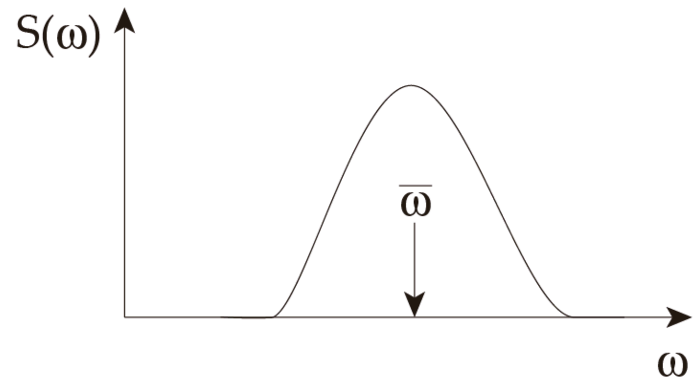
$$S = m_0 = \int_0^\infty S(\omega)d\omega = \sum_k \left|\langle k|O|0\rangle\right|^2$$

total strength



$$\overline{\omega} = \frac{m_1}{m_0} = \frac{\int_0^\infty S(\omega)\omega\ d\omega}{\int_0^\infty S(\omega)d\omega} = \frac{\sum_k \left|\langle k|O|0\rangle\right|^2 \omega_k}{\sum_k \left|\langle k|O|0\rangle\right|^2}$$

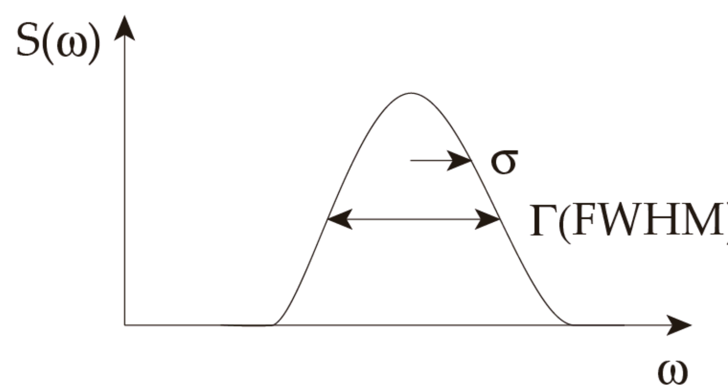
mean energy alternatively $\sqrt{\frac{m_1}{m_{-1}}}$



$$\sigma^2 = \frac{m_2}{m_0} - \left(\frac{m_1}{m_0}\right)^2 = \frac{\int_0^\infty S(\omega)\omega^2 d\omega}{\int_0^\infty S(\omega)d\omega} - \left(\frac{\int_0^\infty S(\omega)\omega d\omega}{\int_0^\infty S(\omega)d\omega}\right)^2$$

variance

$$= \frac{\int_0^\infty S(\omega)(\omega - \overline{\omega})^2 d\omega}{\int_0^\infty S(\omega)d\omega}^2$$



Moments of a strength Function

0th moment

$$m_0 = \int_0^\infty S(\omega)d\omega = \sum_k \left| \langle k | O | 0 \rangle \right|^2$$

Non Energy-Weighted Sum-Rule

1st moment

$$m_1 = \int_0^\infty S(\omega)\omega d\omega = \sum_k \left| \langle k | O | 0 \rangle \right|^2 \omega_k$$

Energy-Weighted Sum-Rule

-1st moment

$$m_{-1} = \int_0^\infty \frac{S(\omega)}{\omega} d\omega = \sum_k \left| \langle k | O | 0 \rangle \right|^2 \frac{1}{\omega_k}$$

Inversely Energy-Weighted Sum-Rule

A moment of a strength function an the exception value of the ground state for the corresponding operator.

O : Hermitian $O = O^\dagger$

$$m_p \equiv \int_0^\infty S(\omega)\omega^p d\omega = \sum_k |<k|O|0>|^2 \omega^p$$

$$= \sum_k <0|O|k><k|O|0> \omega^p$$

$$= \sum_k <0|O\omega^p|k><k|O|0>$$

completeness

$$= \sum_k <0|OH^p|k><k|O|0>$$

$$= <0|OH^pO|0>$$

expectation value of the ground state wave function!

Expressions using commutation/anti-commutation relations

$$m_0 = \langle 0 | O^2 | 0 \rangle$$

$$m_1 = \frac{1}{2} \langle 0 | [O,[H,O]] | 0 \rangle$$

$$m_2 = \frac{1}{2} \langle 0 | \{[O,H],[H,O]\} | 0 \rangle$$

$$m_3 = \frac{1}{2} \langle 0 | [[O,H],[H,[H,O]]] | 0 \rangle$$

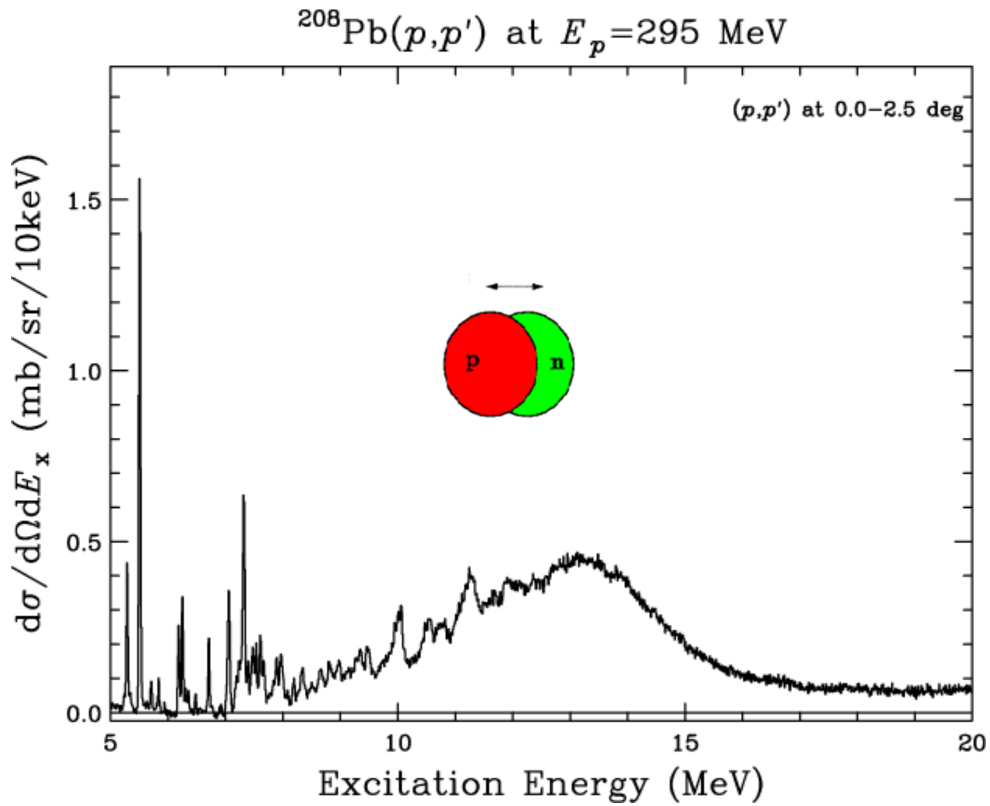
$$[A,B] = AB - BA$$

$$\{A,B\} = AB + BA$$

Giant Dipole Resonance (GDR)

Isovector (Electric) Giant Dipole Resonance

$$O^{\Delta L=1,\Delta S=0,\Delta T=1} = \sum_{i=1}^{A} r_i Y_1(\hat{r}_i)\vec{\tau}_i$$



E1 Energy Weighted Sum Rule: TRK Sum Rule

$$D = O(IVE1) = \sum_i e_i r_i Y_1(\hat{r})$$

$$\rightarrow e\left[\frac{N}{A}\sum_i^Z r_i Y^1(r_i) - \frac{Z}{A}\sum_i^N r_i Y^1(r_i)\right]$$

Correction of the operator for having no c.m. motion

$$H = \sum_i \frac{p_i^2}{2m_N} + V(r_1, r_2, \ldots, r_A)$$

Assumption: Potential V can be written only by the nucleon positions

$$[D, V] = 0$$

1st moment of D (Energy Weighted Sum Rule)

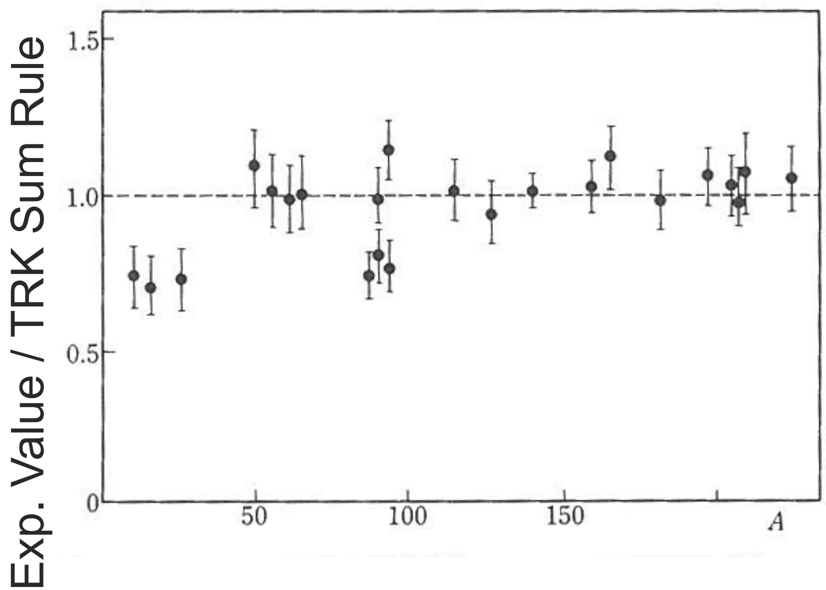
$$m_1^{IVM1} = \frac{1}{2}\langle 0|\left[D, [H, D]\right]|0\rangle$$

$$= \frac{8e^2}{9\pi m_N}\frac{ZN}{A}$$

Thomas-Reich-Kuhn (TRK) Sum-Rule

prove the equation! a good practice for nuclear theory students

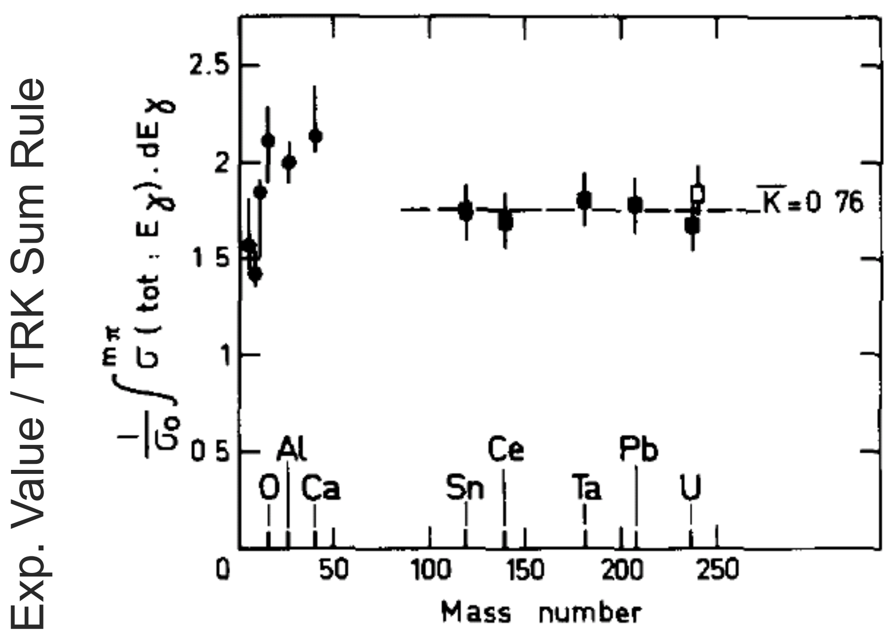
GDR Energy Weighted Sum Rule: TRK Sum Rule



up to 30 MeV

GDR almost exhausts the TRK sum-rule value

＝Giant Resonance



If integrated up to ~140 MeV, the exp. value becomes ~1.8 times the TRK sum-rule value.

enhancement factor:

$\kappa \simeq 0.76$

due to the momentum dependence of the NN interaction, by e.g. exchange force.

Gamow-Teller Strength and Ikeda Sum Rule

The difference between (p,n) and (n,p) of the 0th moment (non-energy weighted sum-rule) of the GT transition strength

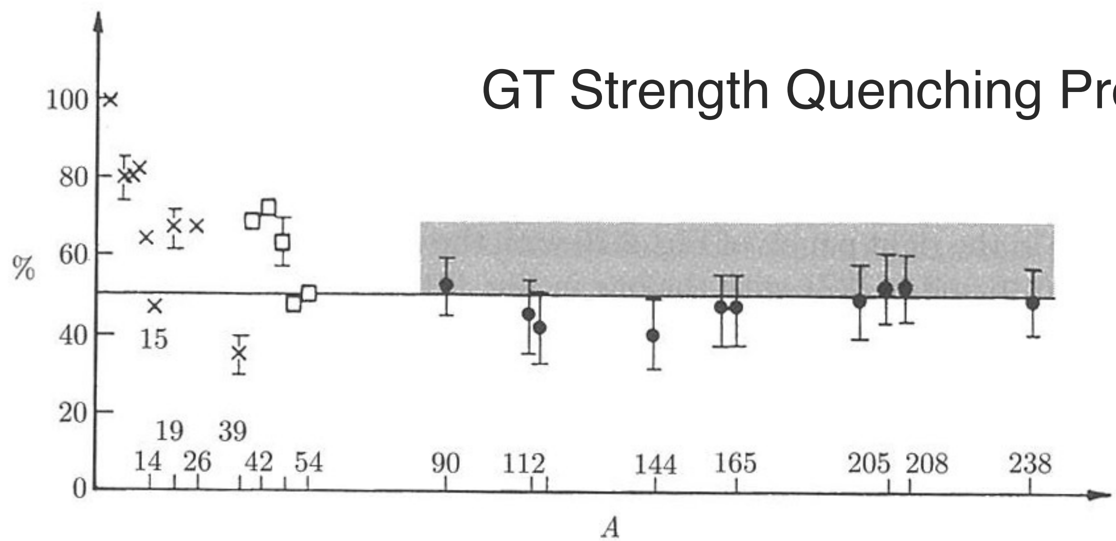
$$D = O^{0,\sigma,\tau} = \sum_{i=1}^{A} \boldsymbol{\sigma}_i \boldsymbol{\tau}_i$$

$$S_{-}^{(GT)} - S_{+}^{GT} = 3(N - Z)$$

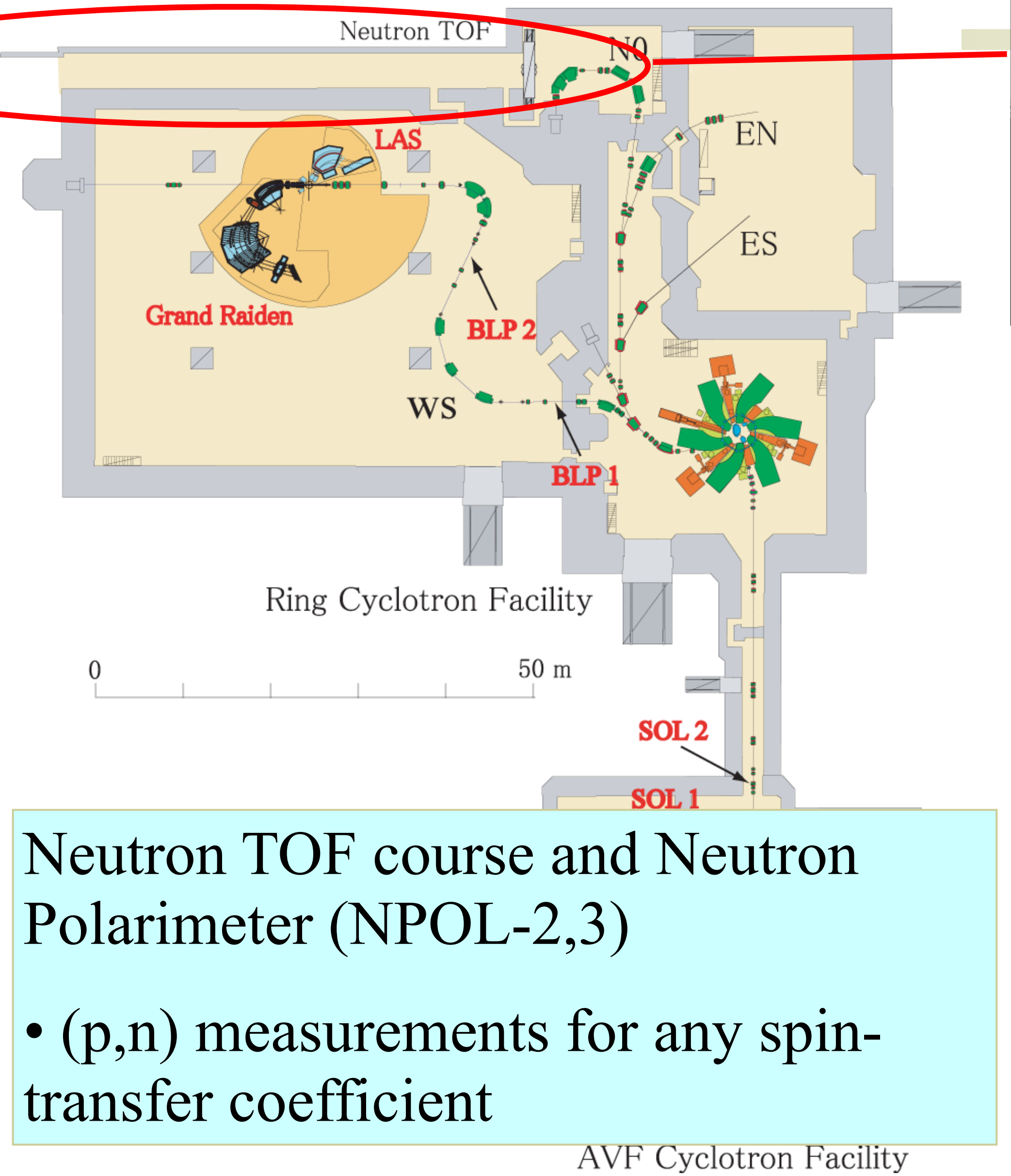
Ikeda-Fujii-Fujita Sum-Rule

GT Strength Quenching Problem

Exp. Value / Sum Rule

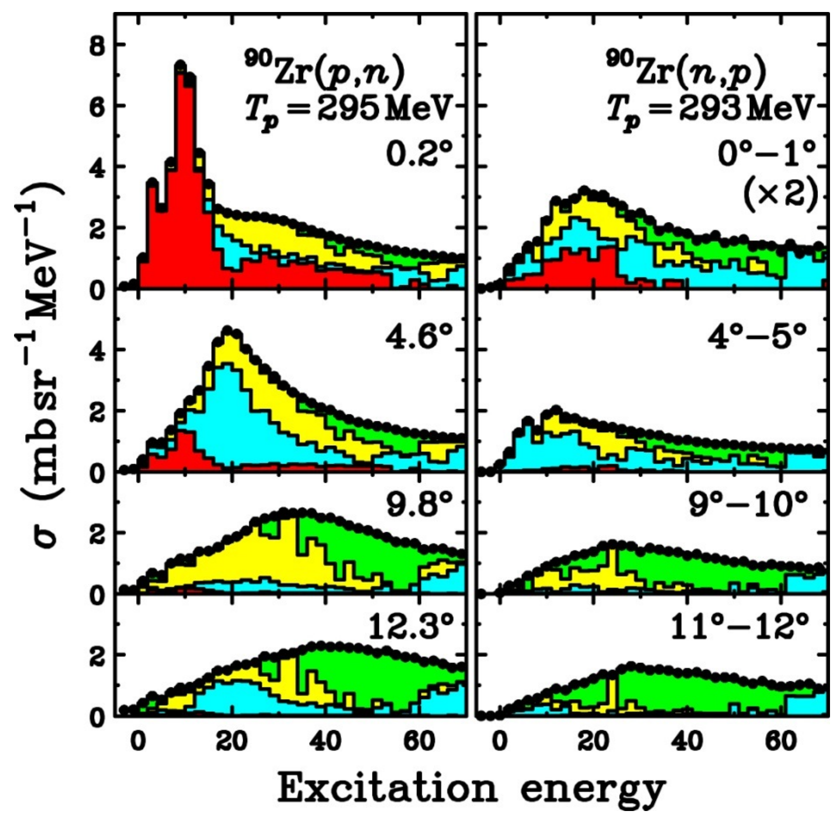


100m n-TOF course



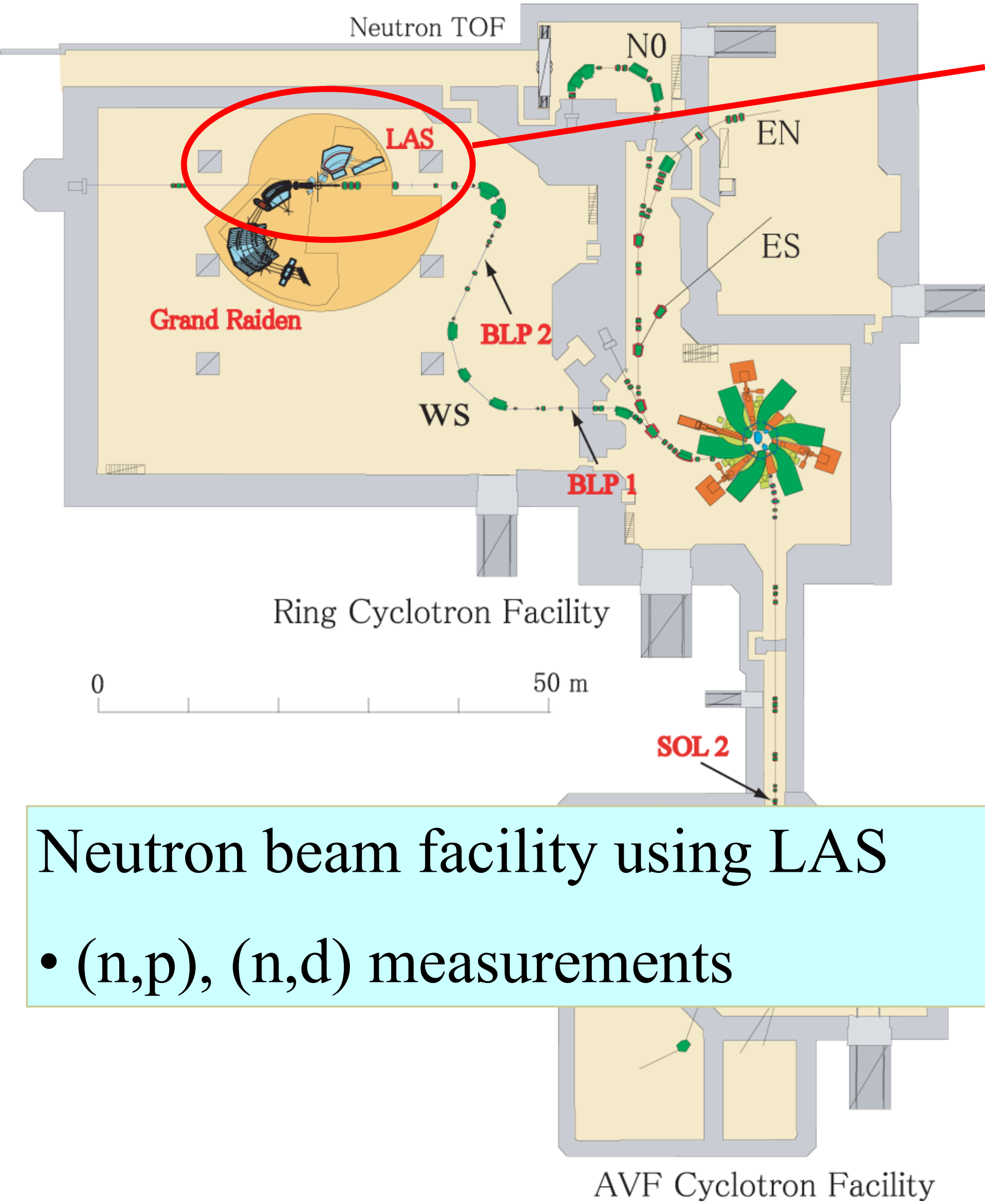
Neutron Polarimeter (NPOL-2)

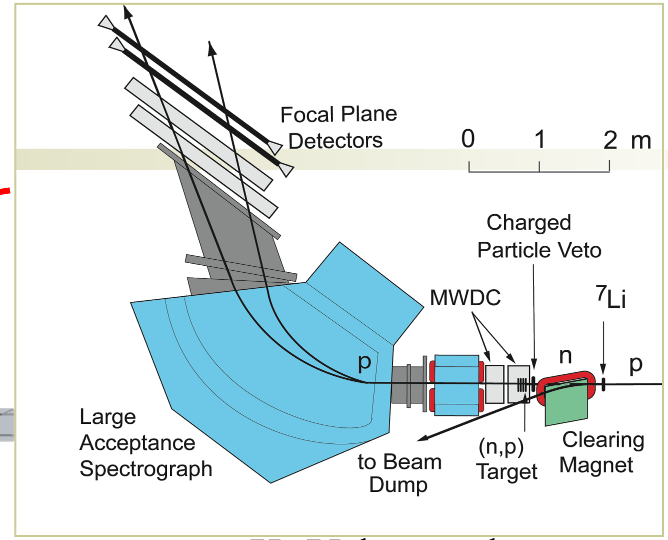
T. Wakasa et al.



Neutron TOF course and Neutron Polarimeter (NPOL-2,3)

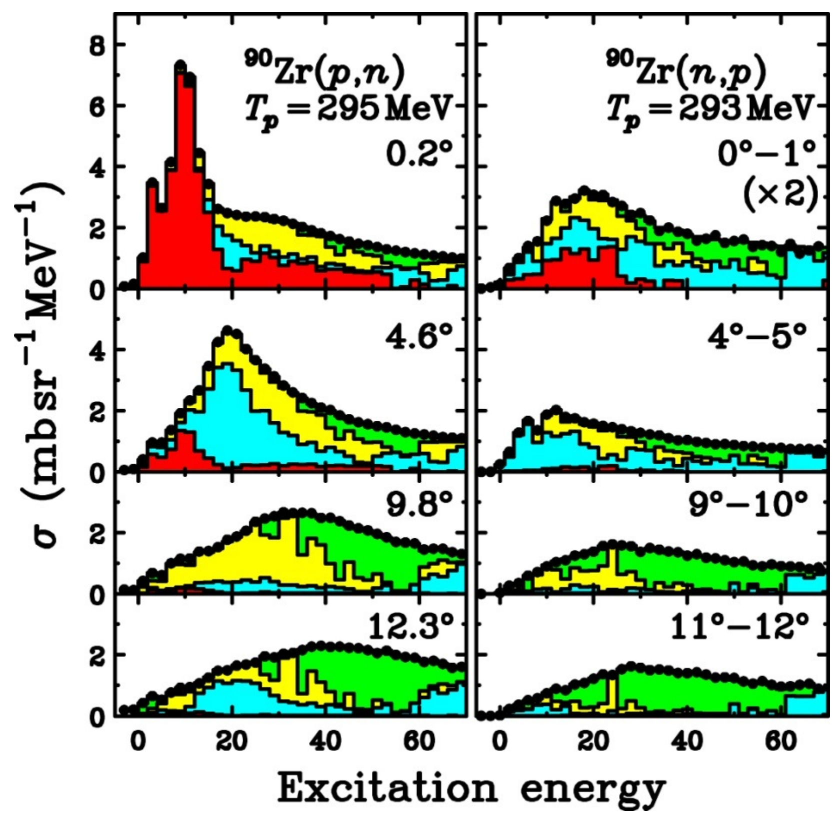
- (p,n) measurements for any spin-transfer coefficient





Neutron beam facility

K. Yako et al.

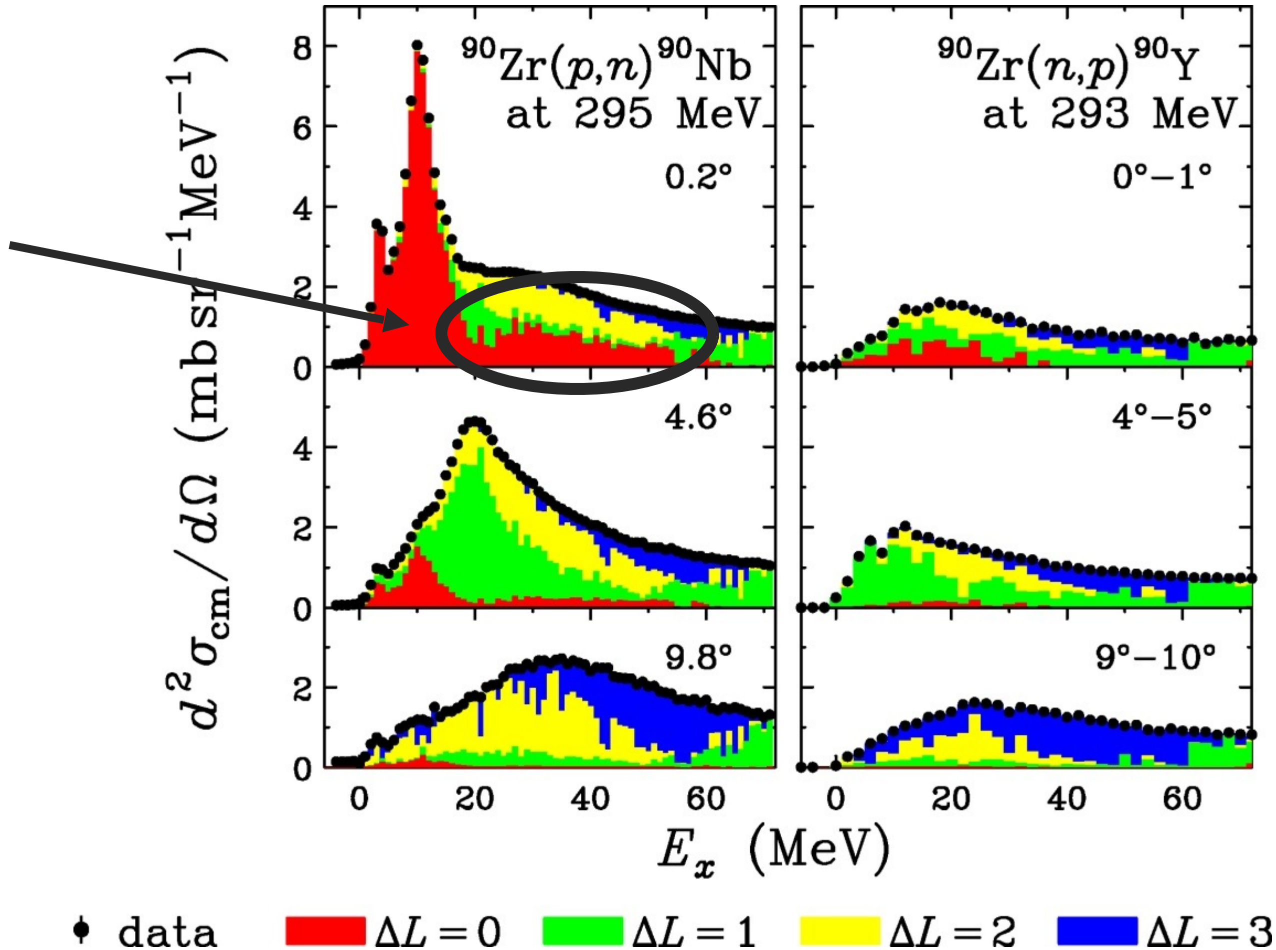


Neutron beam facility using LAS

- (n,p), (n,d) measurements

Gamow-Teller Strength and Ikeda Sum Rule

M. Ichimura, H. Sakai and T. Wakasa, PPNP. 56, 446 (2006).



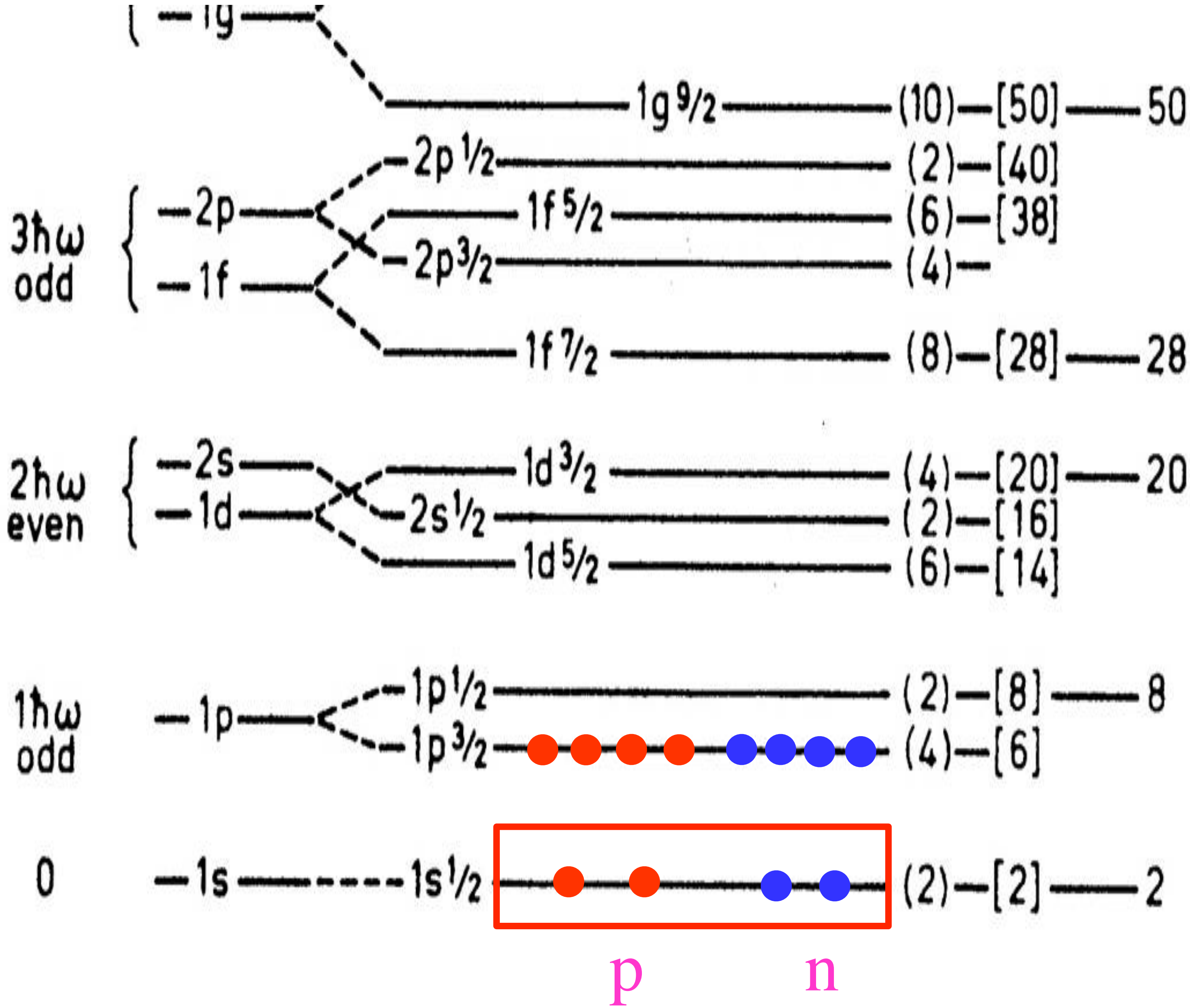
~90% of the sum rule was found up to 50 MeV

due to the admixture of the 2p-2h and more complex wave functions

the rests are attributed to the admixture of Δ-hole?

”naive” nuclear ground state

1+ state



admixture of 2-particle 2-hole states

1^+ state