

Nuclear Excited Studied by proton scattering  
With a High-Resolution Magnetic Spectrometer

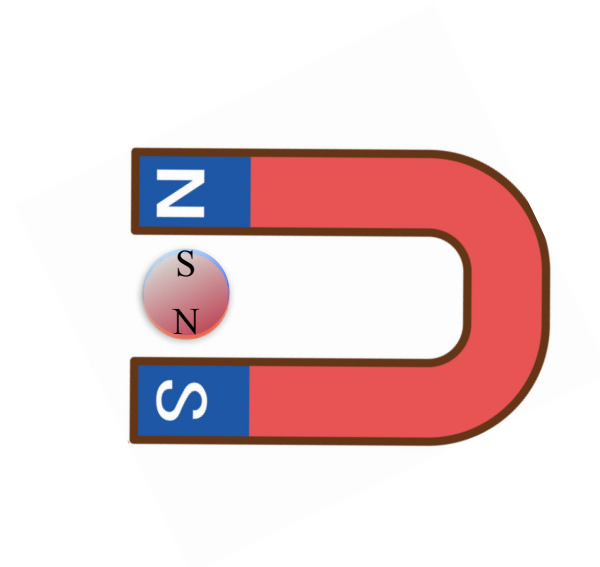
## Lecture VI

# Spin Magnetic Response of Nuclei *n-p* Correlation

43 34 13 1 at <https://menti.com>  
<https://www.menti.com/alwqsv7cirbx>



# Spin Magnetic Response of Nuclei and n-p Correlation



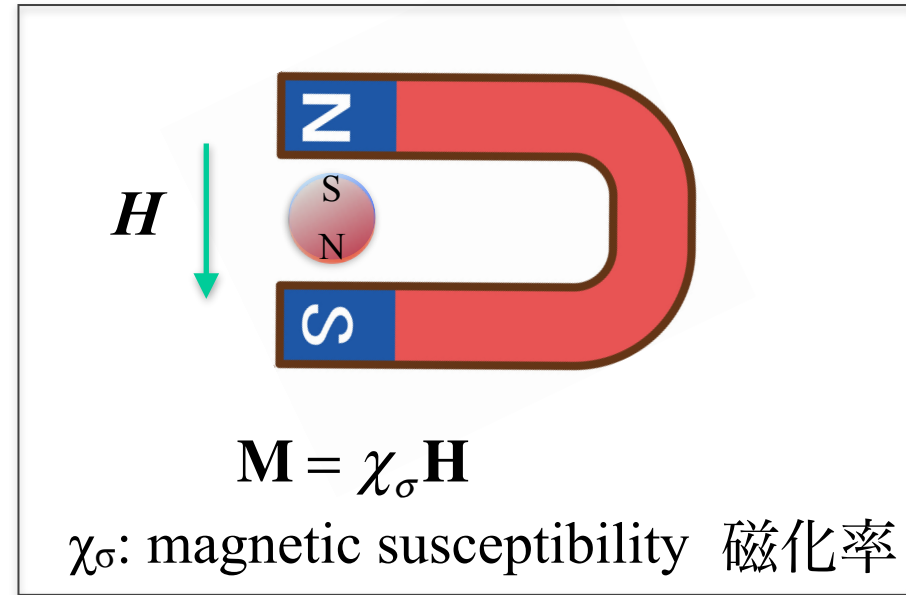
# Spin-Magnetic Susceptibility

Magnetic dipole ( $M1$ ) operator

$$O(M1) = g_\ell^{IS} \ell + \underline{g_s^{IS} \sigma} + g_\ell^{IV} \ell \cdot \tau + \underline{g_s^{IV} \sigma \cdot \tau}$$

IS(1) and IV( $\tau$ ) terms

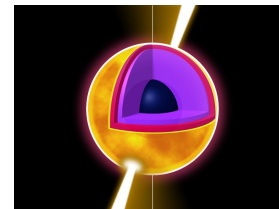
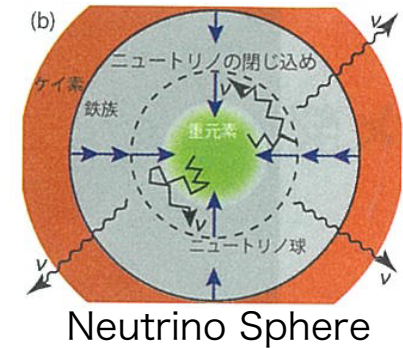
$$\chi_\sigma^{spin} = \frac{8}{3N} \sum_f \frac{1}{\omega} \left| \langle f | \sum_i \sigma_i | 0 \rangle \right|^2$$



- Spin part of magnetization of nuclear matter
- Magnetic response of nuclear matter (in e.g. magnetar)
- Neutrino trap in the core of supernova

Neutrino transparency

- Ferromagnetic state in a neutron star



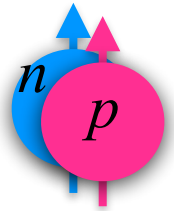
Magnetar  $10^{14-16}$  Gauss

# Collective Vibrational Excitations

		Magnetic $\Delta S = 1$ Operators			
		Isoscalar Electric	Isovector Electric	Isoscalar Magnetic	Isovector Magnetic
$(\Delta T, \Delta S)$		(0, 0)	(1, 0)	(0, 1)	(1, 1)
Monopole	$\Delta L = 0$				
Dipole	$\Delta L = 1$	—			
Quadrupole	$\Delta L = 2$				
...					
Multipole					

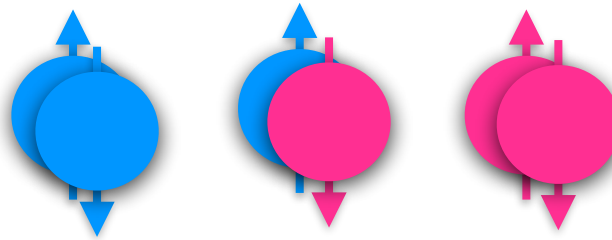
$\vec{\sigma}$   
 $\vec{\sigma} \vec{\tau}$

# IS and IV pairings



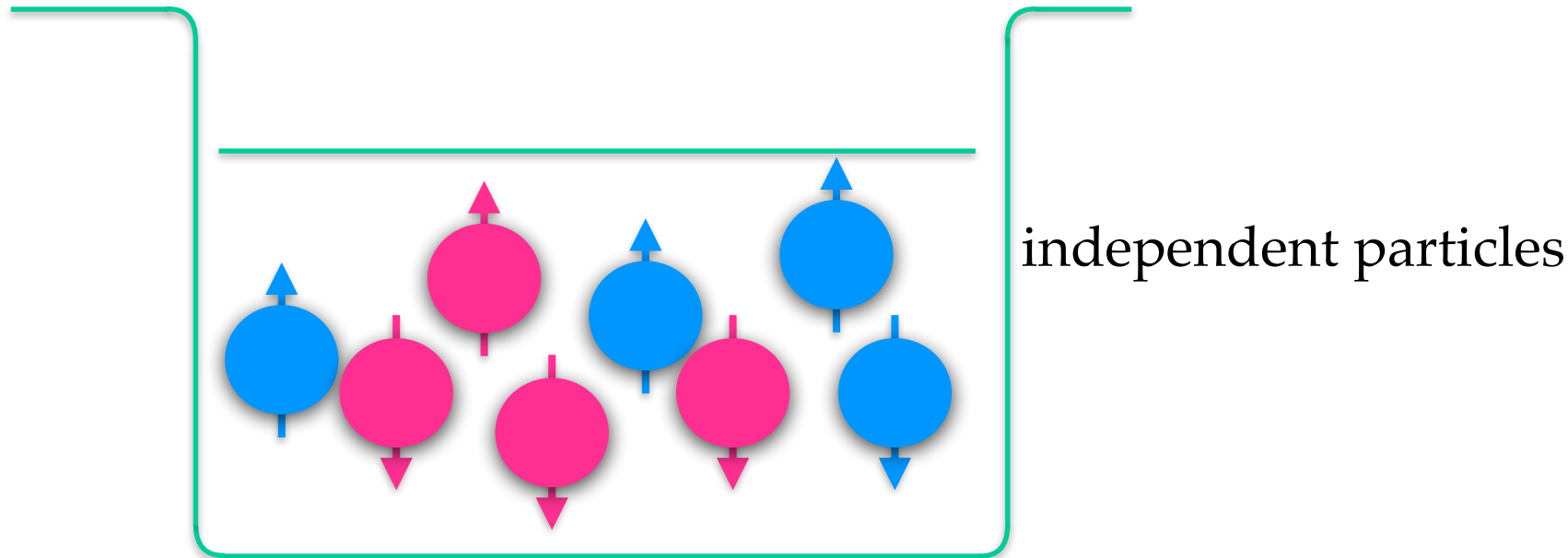
IS ( $T=0, S=1$ )

Isoscalar  $np$ -pairing



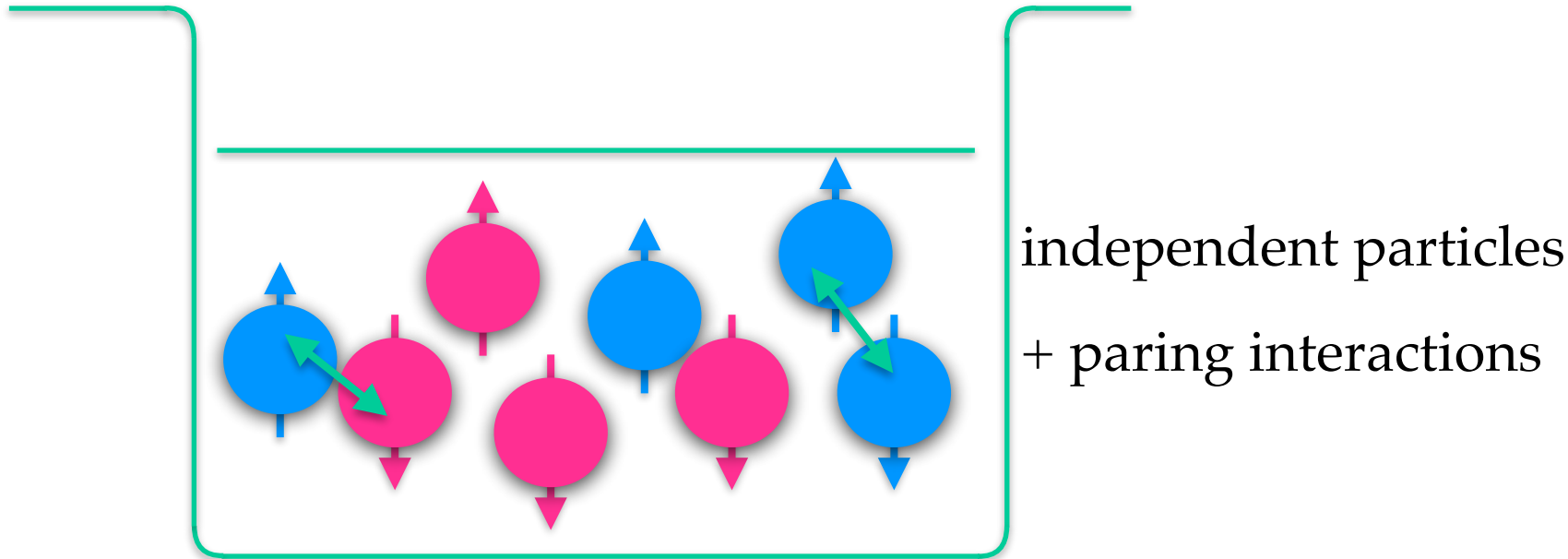
IV ( $T=1, S=0$ )

# pairing in the ground state W.F.

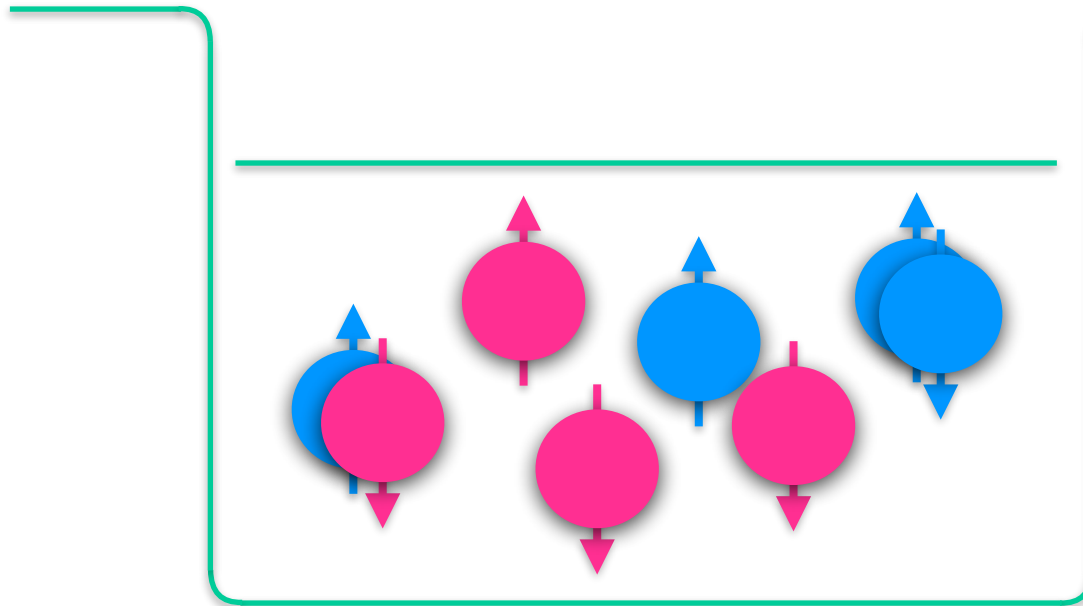


unperturbed ground state

# pairing in the ground state W.F.



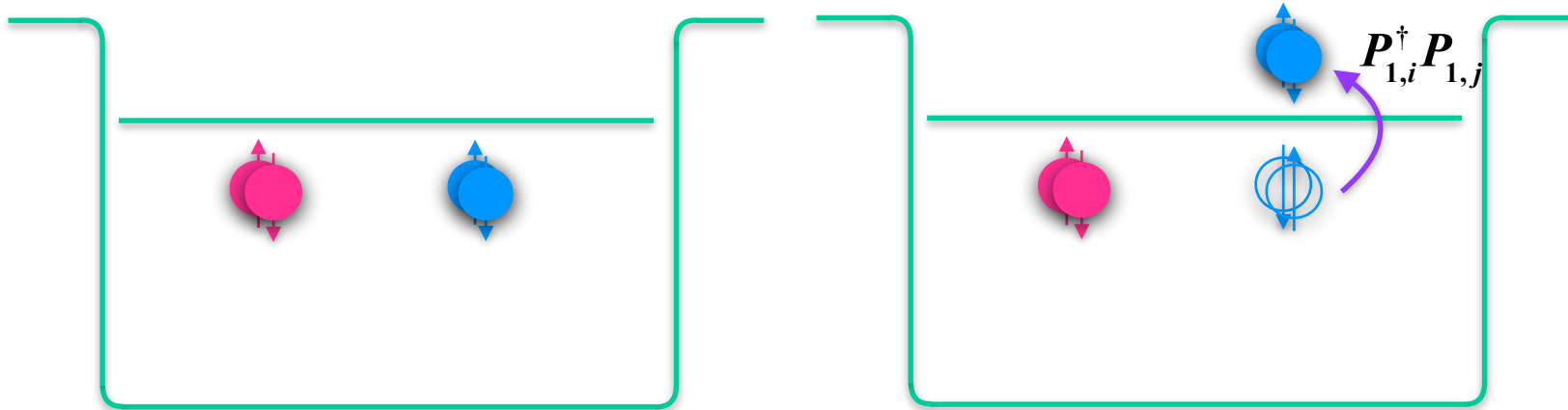
# pairing in the ground state W.F.



independent particles  
+ pairing interactions

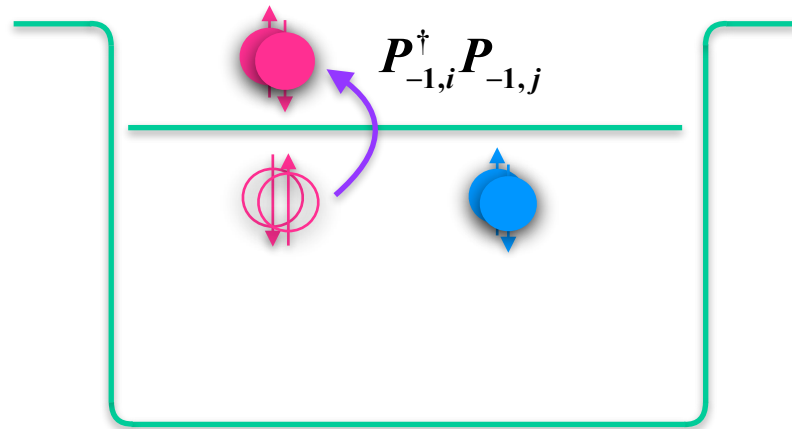


# pairing in the ground state W.F.



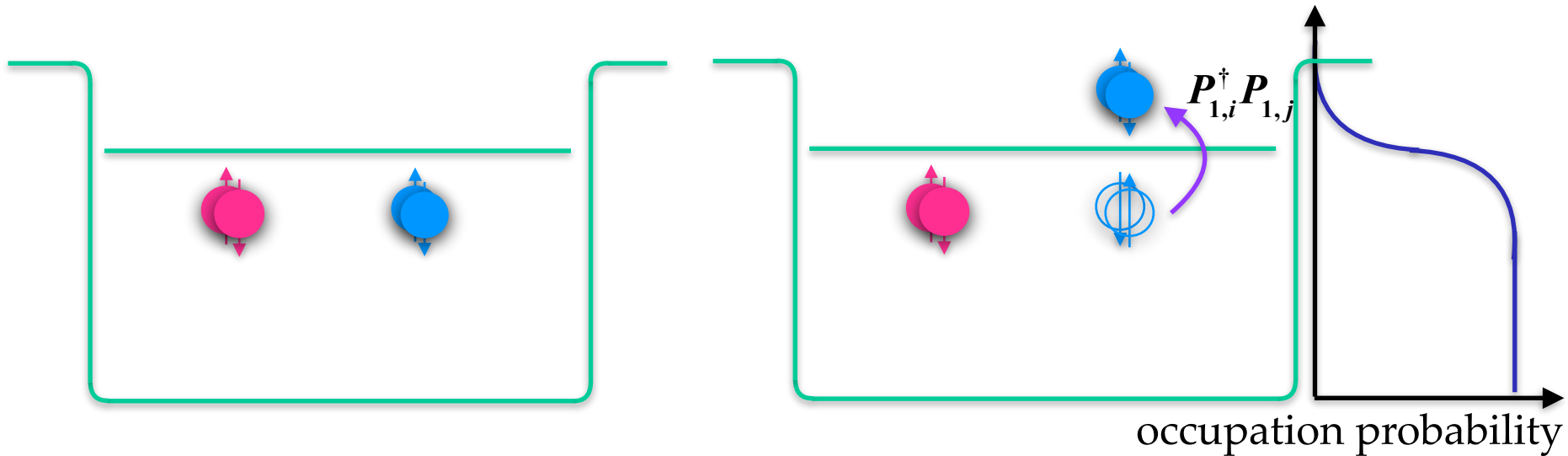
$$P_{1,i}^\dagger = v_i^\dagger v_i^\dagger$$

$$P_{-1,i}^\dagger = \pi_i^\dagger \pi_i^\dagger$$



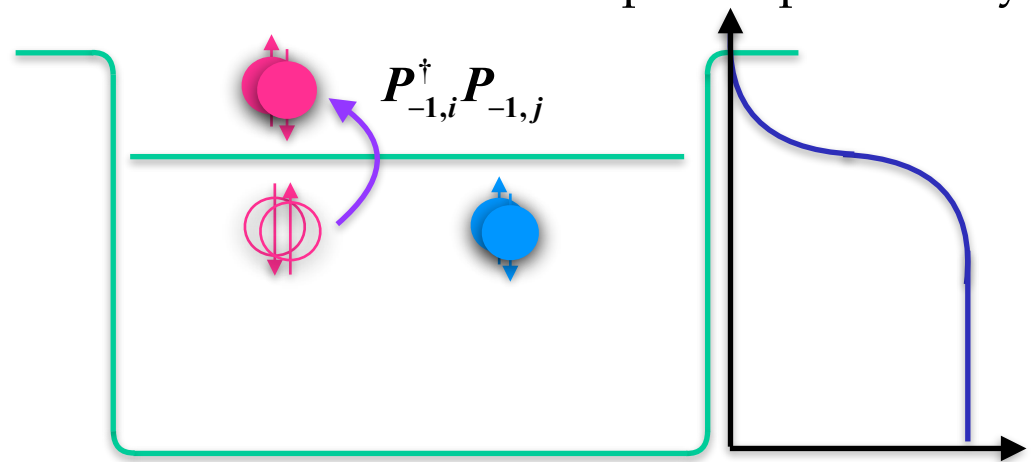
isovector “pairing” correlation  
= BCS type correlation

# pairing in the ground state W.F.



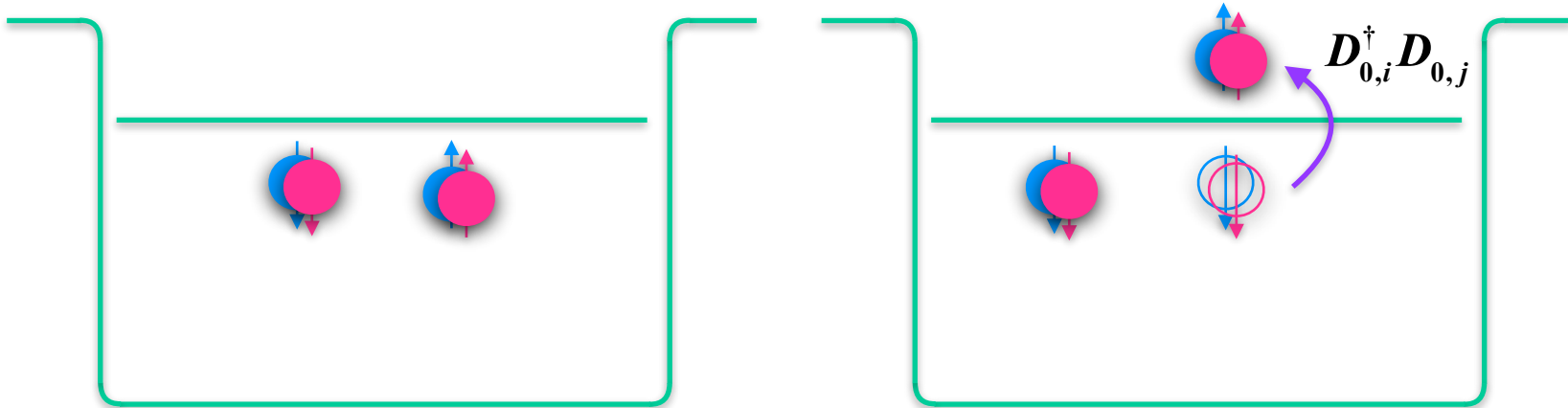
$$P_{1,i}^\dagger = v_i^\dagger v_i^\dagger$$

$$P_{-1,i}^\dagger = \pi_i^\dagger \pi_i^\dagger$$



isovector "pairing" correlation  
= BCS type correlation

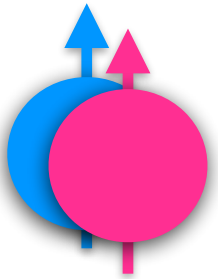
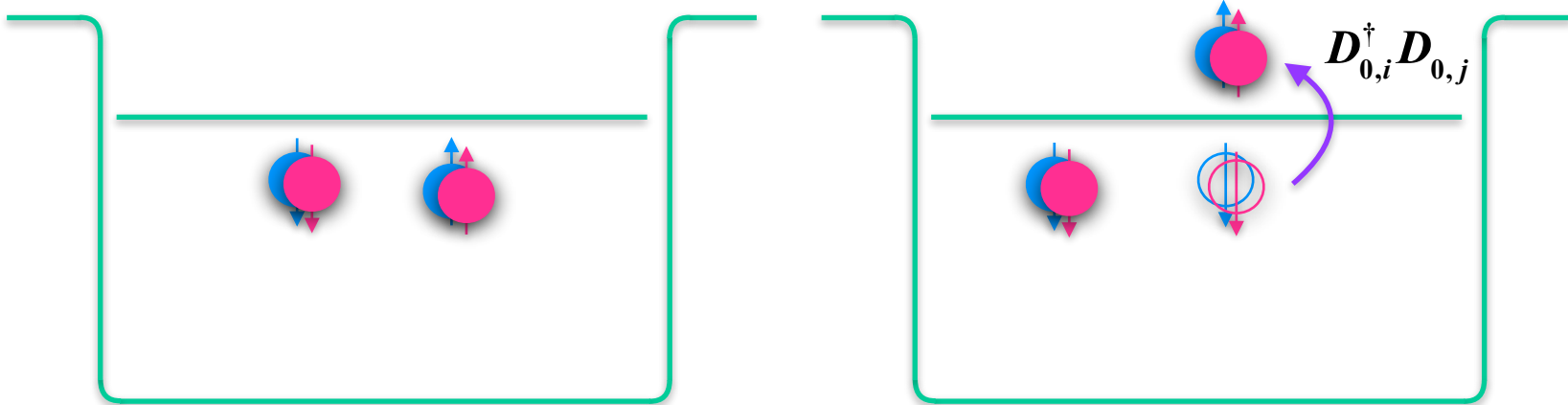
# $np$ pairing in the ground state W.F.



$$D_{0,i}^\dagger = \frac{1}{\sqrt{2}} \left( v_i^\dagger \pi_i^\dagger - \pi_i^\dagger v_i^\dagger \right)$$

isoscalar “pairing” correlation  
by *e.g.* tensor correlation

# $np$ pairing in the ground state W.F.

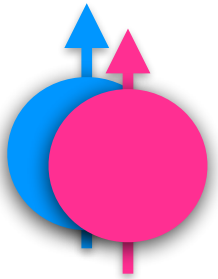
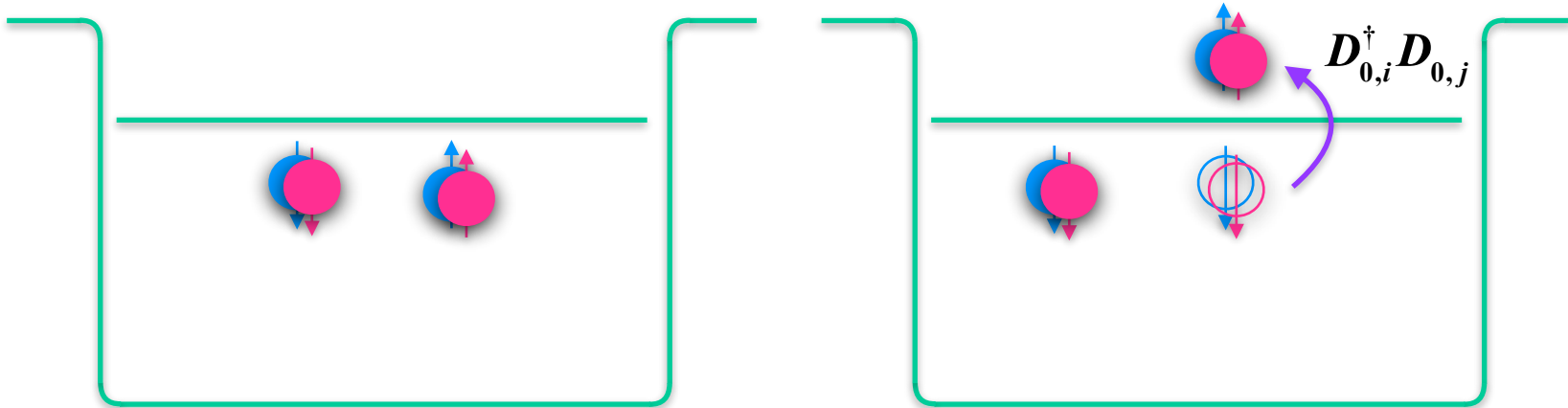


$n$ -spin and  $p$ -spin:  
aligned

$$\langle \vec{s}_n \cdot \vec{s}_p \rangle > 0$$

isoscalar “pairing” correlation  
by *e.g.* tensor correlation

# $np$ pairing in the ground state W.F.

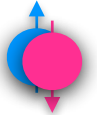


$n$ -spin and  $p$ -spin:  
aligned

$$\langle \vec{s}_n \cdot \vec{s}_p \rangle > 0$$

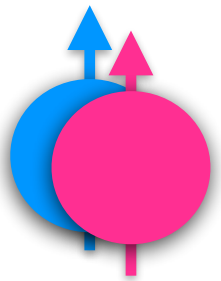
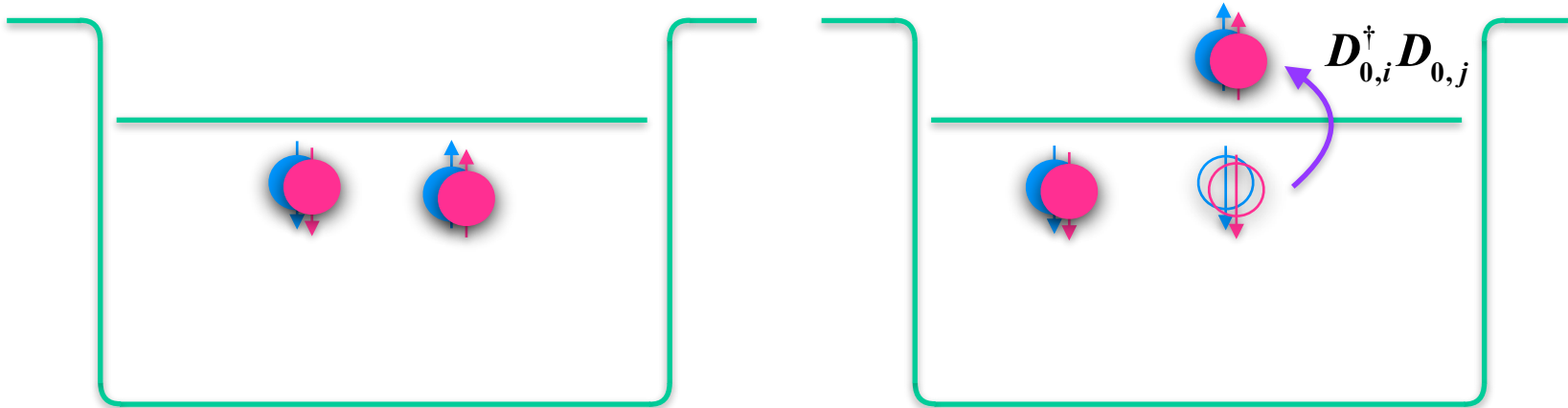
$$\langle \vec{s}_n \cdot \vec{s}_p \rangle =$$

$$\vec{S}^2 = (\vec{s}_n + \vec{s}_p)^2 = \vec{s}_n^2 + \vec{s}_p^2 + 2\vec{s}_n \cdot \vec{s}_p \quad \left\{ \begin{array}{l} +\frac{1}{4} \quad \text{for IS } np \text{ pair (deuteron)} \\ -\frac{3}{4} \quad \text{for IV } np \text{ pair} \end{array} \right.$$



isoscalar “pairing” correlation  
by *e.g.* tensor correlation

# $np$ pairing in the ground state W.F.



$n$ -spin and  $p$ -spin:  
aligned

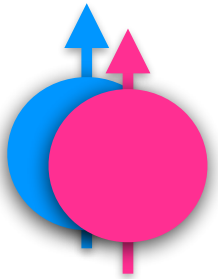
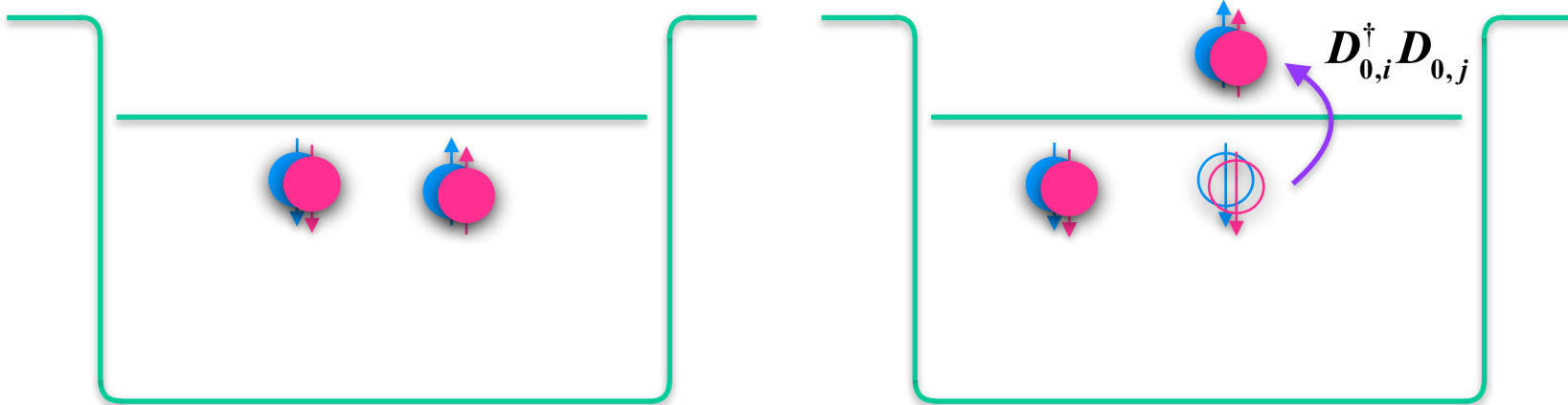
$$\langle \vec{s}_n \cdot \vec{s}_p \rangle > 0$$

$$\langle \vec{s}_n \cdot \vec{s}_p \rangle =$$

isoscalar “pairing” correlation  
by *e.g.* tensor correlation

$$\left\{ \begin{array}{ll} +\frac{1}{4} & \text{for IS } np \text{ pair (deuteron) } \begin{array}{c} \uparrow \\ \uparrow \end{array} \\ \text{statistical weight} = 3 \\ -\frac{3}{4} & \text{for IV } np \text{ pair} \\ \text{statistical weight} = 1 \end{array} \right.$$

# $np$ pairing in the ground state W.F.



$n$ -spin and  $p$ -spin:  
aligned

$$\langle \vec{s}_n \cdot \vec{s}_p \rangle > 0$$

isoscalar “pairing” correlation  
by *e.g.* tensor correlation



induces correlation between the  
directions of the  $n$ -spin and  $p$ -spin

# *np* spin correlation function

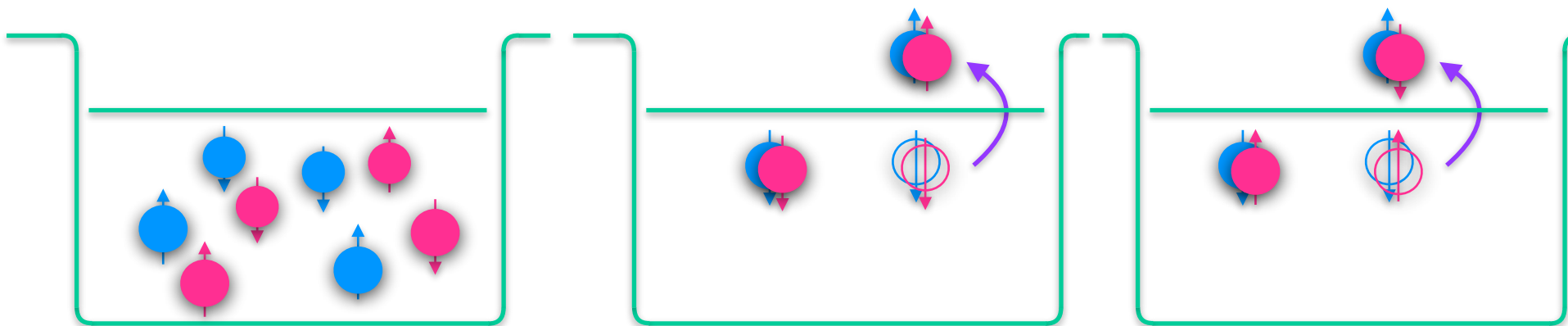
$$\vec{S}_n \equiv \sum_i^N \vec{S}_{n,i} \quad \vec{S}_p \equiv \sum_i^Z \vec{S}_{p,i}$$

$\langle \vec{S}_n \cdot \vec{S}_p \rangle$  : *np* spin correlation function  
of the nuclear ground state



$$\vec{S}_n \equiv \sum_i^N \vec{S}_{n,i} \quad \vec{S}_p \equiv \sum_i^Z \vec{S}_{p,i}$$

$\langle \vec{S}_n \cdot \vec{S}_p \rangle$  :  $np$  spin correlation function  
of the nuclear ground state



unperturbed ground state

IS  $np$  pairing

IV  $np$  pairing

$$\langle \vec{S}_n \cdot \vec{S}_p \rangle = 0$$

$$\langle \vec{S}_n \cdot \vec{S}_p \rangle > 0$$

$$\langle \vec{S}_n \cdot \vec{S}_p \rangle < 0$$

also for IV  $pp/nn$  pairings

# How to Study the $np$ Spin Correlation Function?

→ IS/IV spin-M1 excitations and Sum-Rule

$$\vec{S}_n + \vec{S}_p = \sum_i^A \frac{1}{2} \vec{\sigma}_i$$

$$\vec{S}_n - \vec{S}_p = \sum_i^A \frac{1}{2} \vec{\sigma}_i \tau_z$$

$$\langle (\vec{S}_n - \vec{S}_p)^2 \rangle = \frac{1}{4} \langle (\vec{\sigma} \tau_z)^2 \rangle$$

$$= \frac{1}{4} \sum_f \langle 0 | \vec{\sigma} \tau_z | f \rangle \langle f | \vec{\sigma} \tau_z | 0 \rangle$$

$$= \frac{1}{4} \sum_f |\langle f | \vec{\sigma} \tau_z | 0 \rangle|^2$$

$$= \frac{1}{4} \sum |M(\vec{\sigma} \tau_z)|^2$$

IV spin-M1 squared nuclear matrix elements (SNME)

$$\langle (\vec{S}_n + \vec{S}_p)^2 \rangle = \frac{1}{4} \sum |M(\vec{\sigma})|^2$$

IS spin-M1 SNME

$$\begin{aligned} \langle \vec{S}_n \cdot \vec{S}_p \rangle &= \frac{1}{4} \langle (\vec{S}_n + \vec{S}_p)^2 - (\vec{S}_n - \vec{S}_p)^2 \rangle \\ &= \frac{1}{16} \left( \sum |M(\vec{\sigma})|^2 - \sum |M(\vec{\sigma} \tau_z)|^2 \right) \end{aligned}$$

$$\begin{aligned} \langle \vec{S}_n^2 + \vec{S}_p^2 \rangle &= \frac{1}{4} \langle (\vec{S}_n + \vec{S}_p)^2 + (\vec{S}_n - \vec{S}_p)^2 \rangle \\ &= \frac{1}{16} \left( \sum |M(\vec{\sigma})|^2 + \sum |M(\vec{\sigma} \tau_z)|^2 \right) \end{aligned}$$

closure

# Spin- $M1$ Reduced Transition Strength

$M1$  Operator

$$\hat{O}(M1) = \left[ \sum_{k=1}^Z (g_l^p \vec{l}_k + g_s^p \vec{s}_k) + \sum_{k=Z+1}^A (g_l^n \vec{l}_k + g_s^n \vec{s}_k) \right] \mu_N$$

$M1$  Reduced Transition Strength

$$B(M1) = \frac{3}{4\pi} \frac{1}{2J_i + 1} \left| \left\langle f \left\| g_l^{IS} \vec{l} + \frac{g_s^{IS}}{2} \vec{\sigma} - \left( g_l^{IV} \vec{l} + \frac{g_s^{IV}}{2} \vec{\sigma} \right) \tau_z \right\| i \right\rangle \right|^2$$

$T=0$  Isoscalar (IS) Spin- $M1$  Reduced Transition Strength

$$B(M1)_\sigma = \frac{3}{4\pi} \frac{1}{2J_i + 1} \left( \frac{g_s^{IS}}{2} \right)^2 \left| \langle f \| \vec{\sigma} \| i \rangle \right|^2 \mu_N^2 \quad M(\sigma) = \langle f \| \vec{\sigma} \| i \rangle$$

IS Reduced Matrix Element

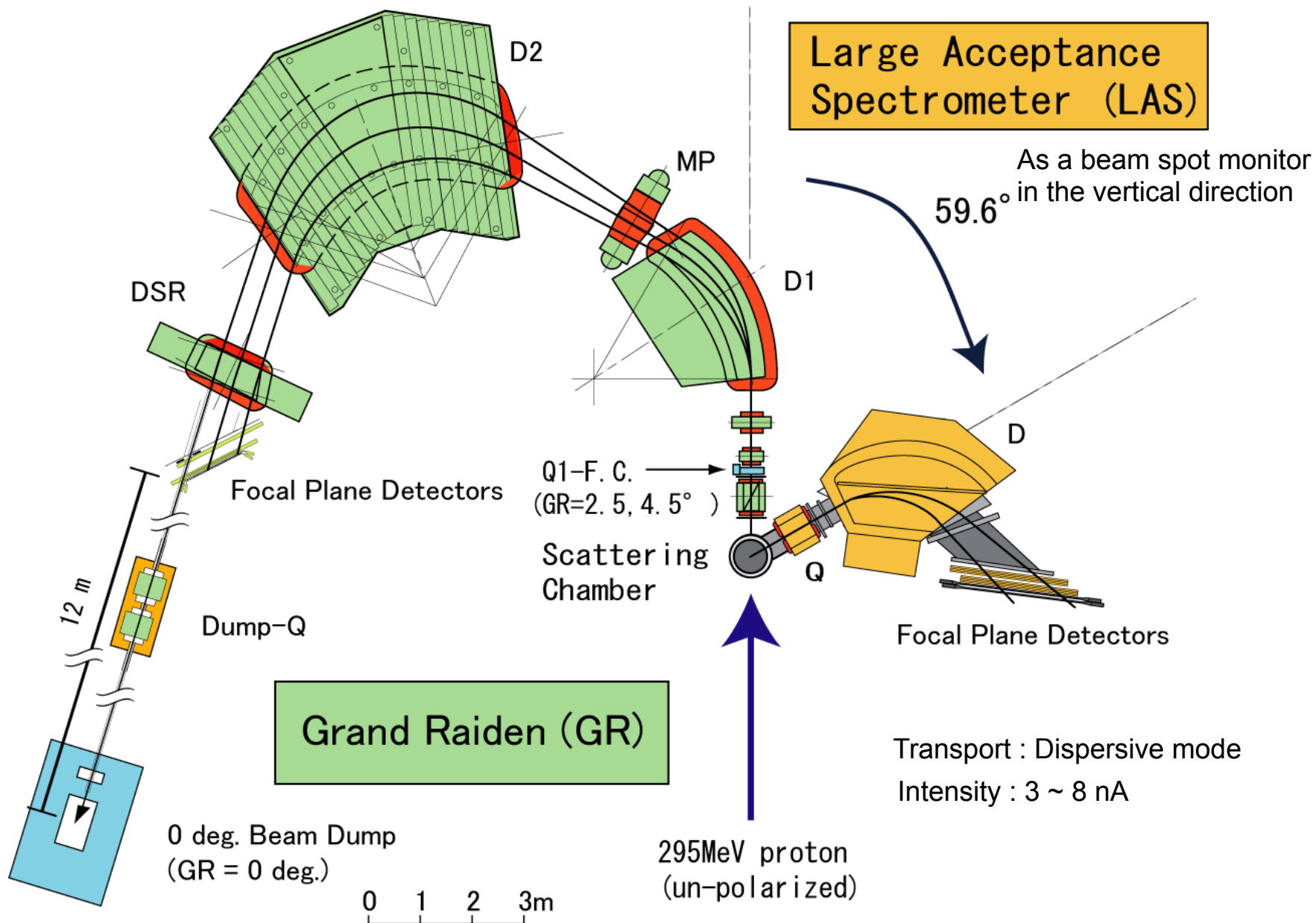
$T=1$  Isovector (IV) Spin- $M1$  Reduced Transition Strength

$$B(M1)_{\sigma\tau} = \frac{3}{4\pi} \frac{1}{2J_i + 1} \left( \frac{g_s^{IV}}{2} \right)^2 \left| \langle f \| \vec{\sigma} \tau_z \| i \rangle \right|^2 \mu_N^2 \quad M(\sigma\tau) = \langle f \| \vec{\sigma} \tau_z \| i \rangle$$

IV Reduced Matrix Element

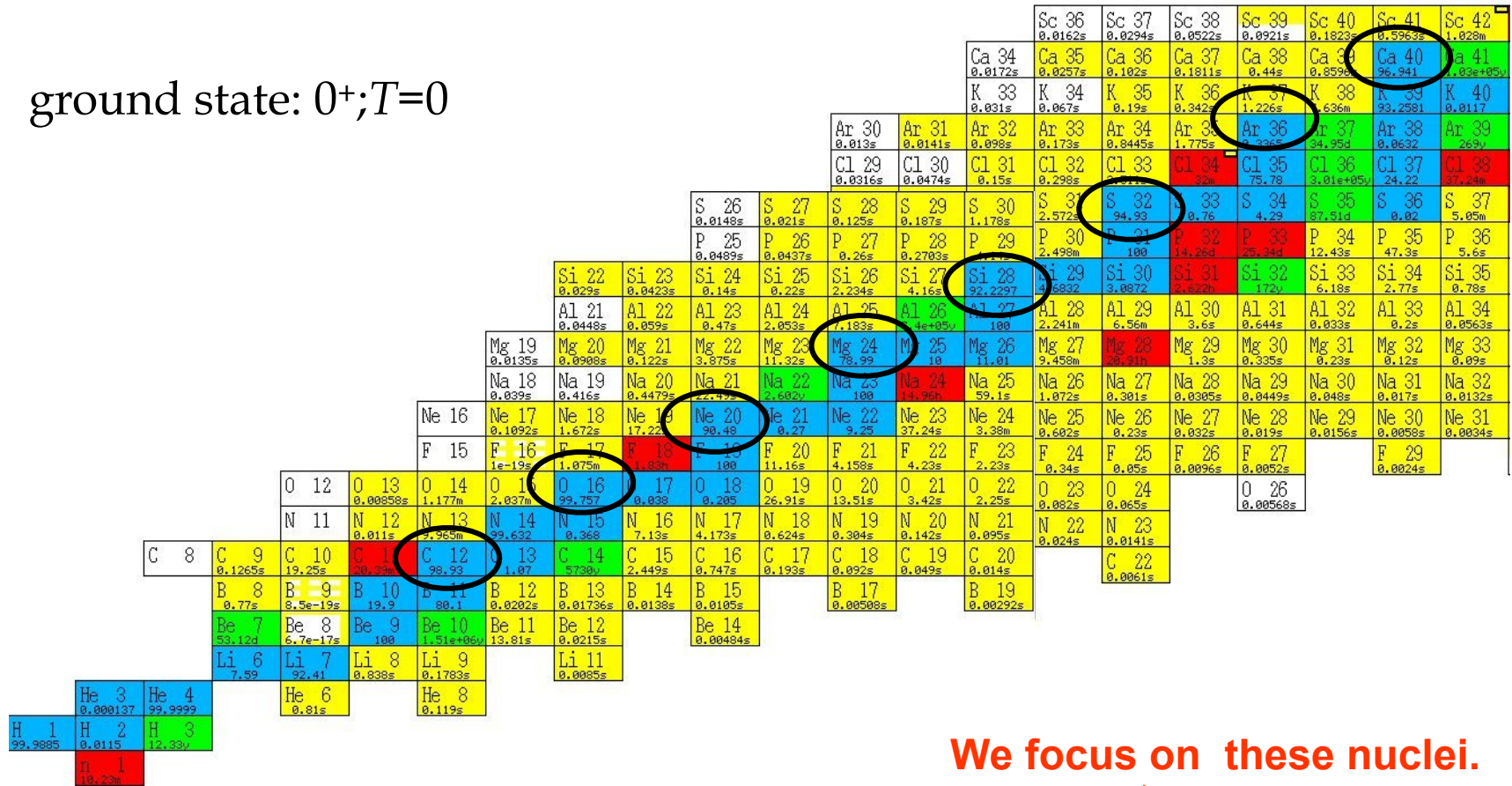
# Experimental Methods

# Spectrometer Setup for 0-deg (p,p') at RCNP



# Self-Conjugate ( $N=Z$ ) even-even Nuclei

ground state:  $0^+; T=0$



We focus on these nuclei.

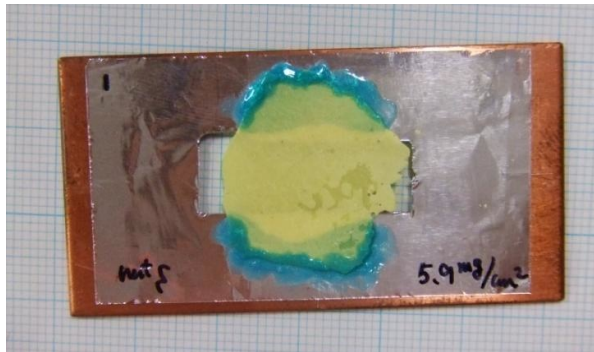
Stable self-conjugate even-even nuclei:

$({}^4\text{He}), {}^{12}\text{C}, {}^{16}\text{O}, {}^{20}\text{Ne}, {}^{24}\text{Mg}, {}^{28}\text{Si}, {}^{32}\text{S}, {}^{36}\text{Ar}, {}^{40}\text{Ca}$

# Targets

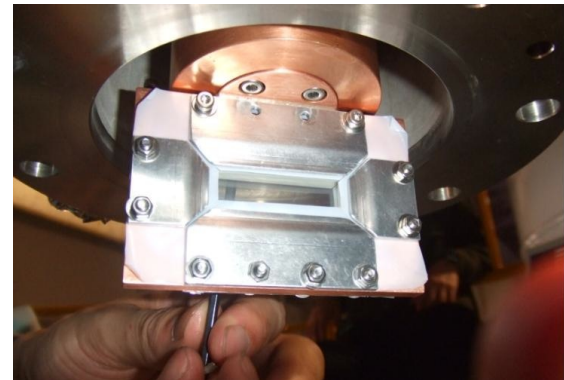
$^{12}\text{C}$ ,  $^{24}\text{Mg}$ ,  $^{28}\text{Si}$ : self-supporting target

Cooled  $^{32}\text{S}$  self-supporting target



H. Matsubara *et al.*, NIMB 267, 3682 (2009)

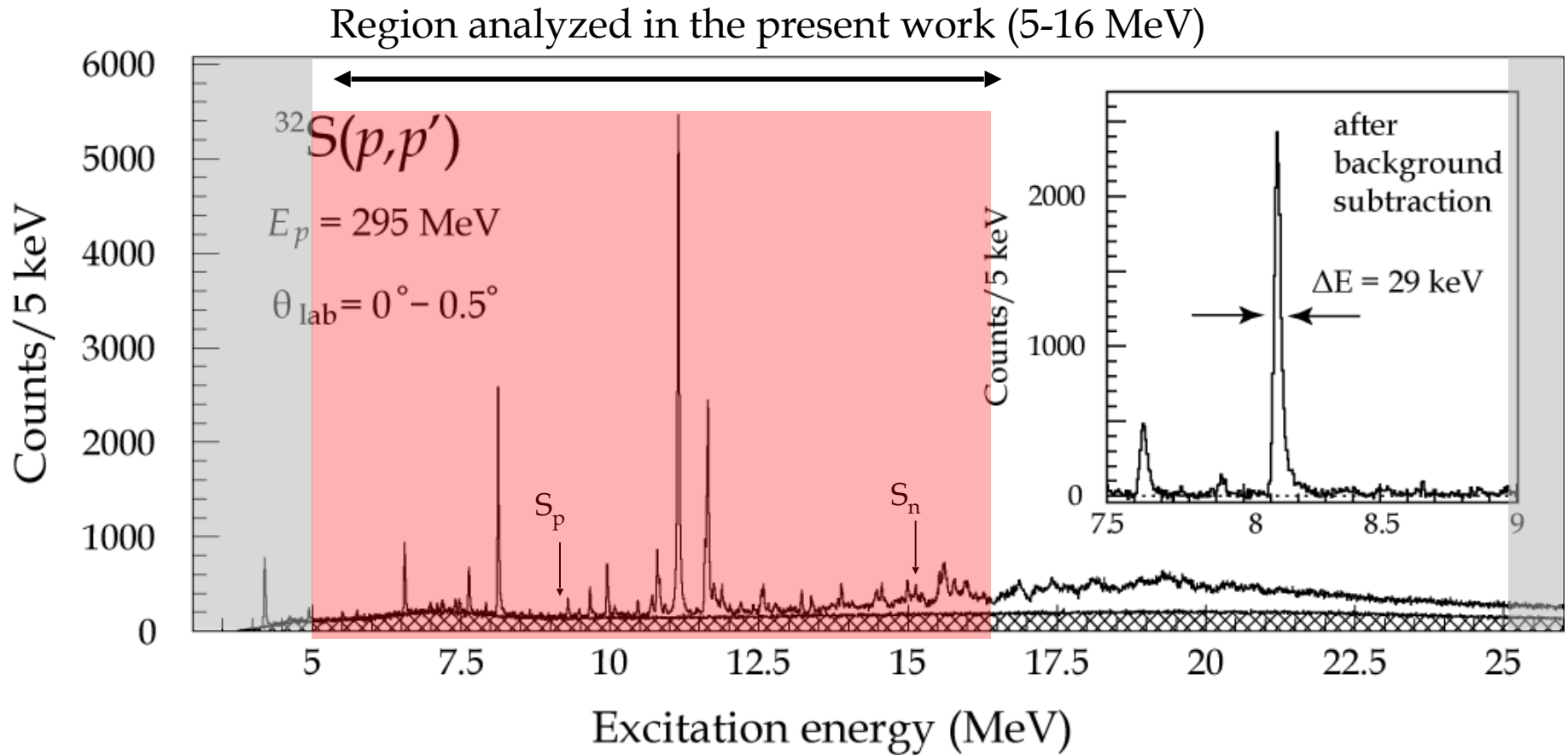
Gas Cell Target ( $^{36}\text{Ar}$ )



H. Matsubara *et al.*, NIMA 678, 122 (2012)

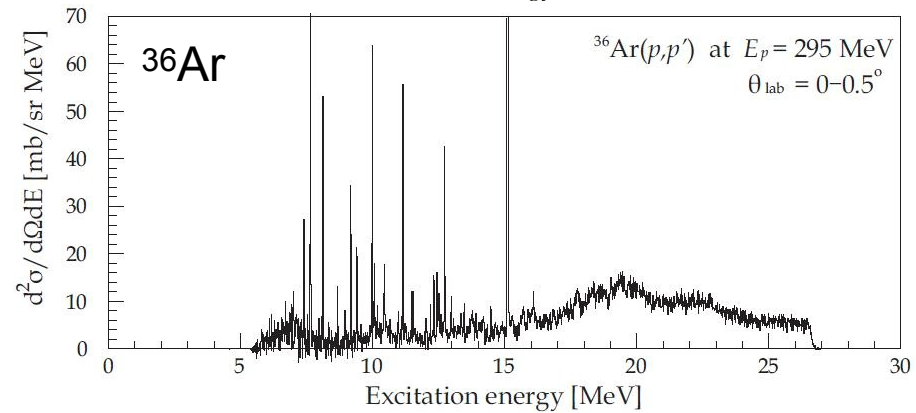
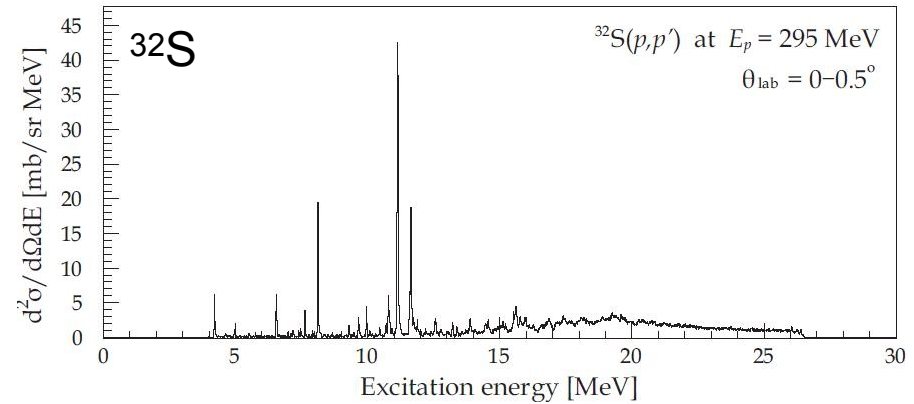
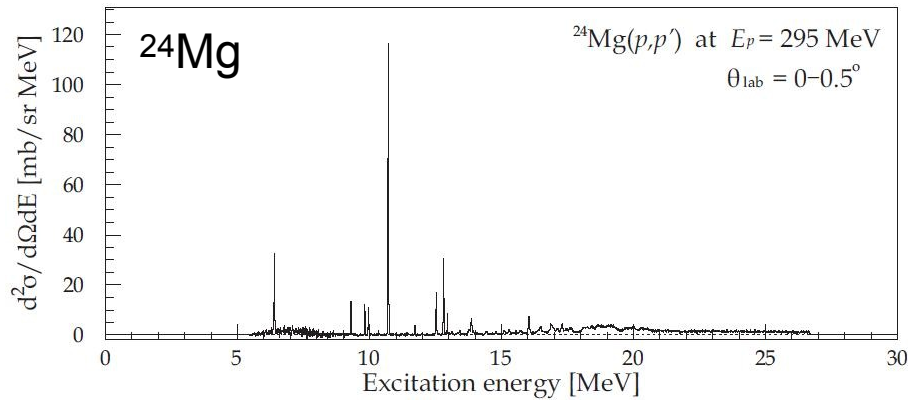
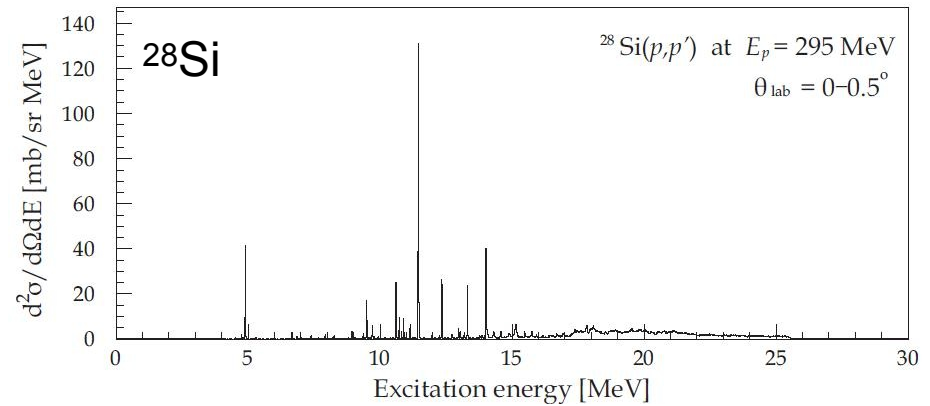
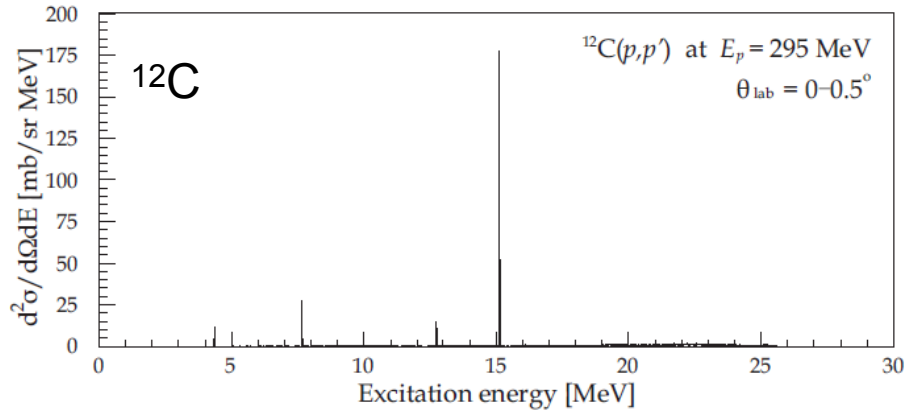
Aramide window of  $6\ \mu\text{m}^t$

# High energy-resolution spectrum





# Energy spectra at 0-degrees



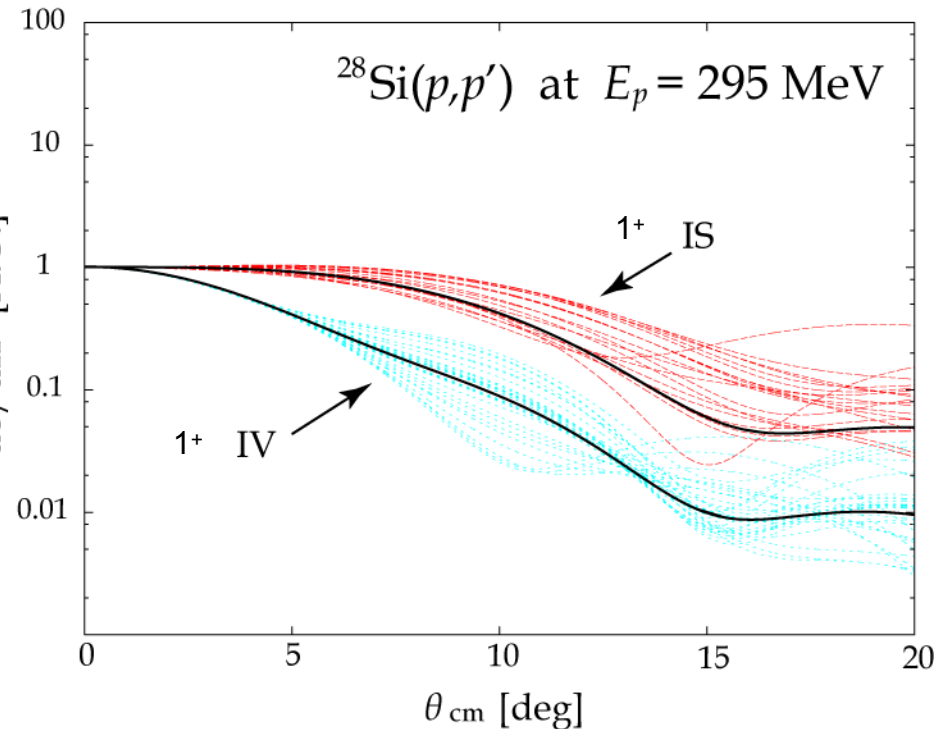
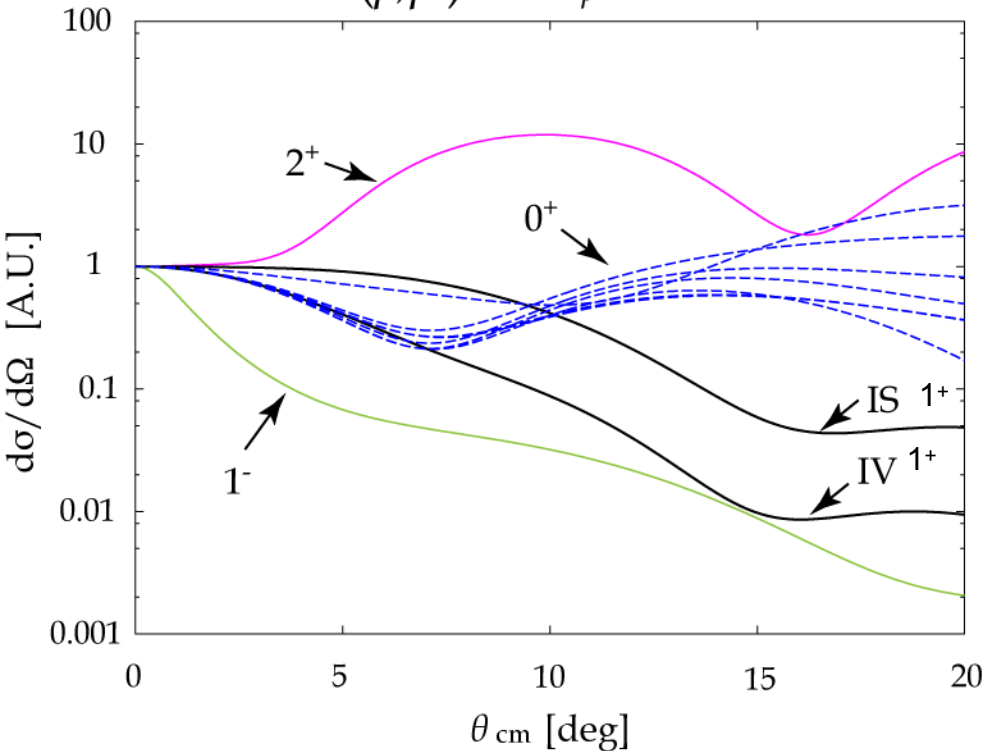
# Angular distribution for $J^\pi$ assignment

- Distorted wave Born approximation by DWBA07

Trans. density : USD, USDA, USDB (from shell model calculation)

NN interaction. : Franey and Love, PRC31(1985)488. (325 MeV data)

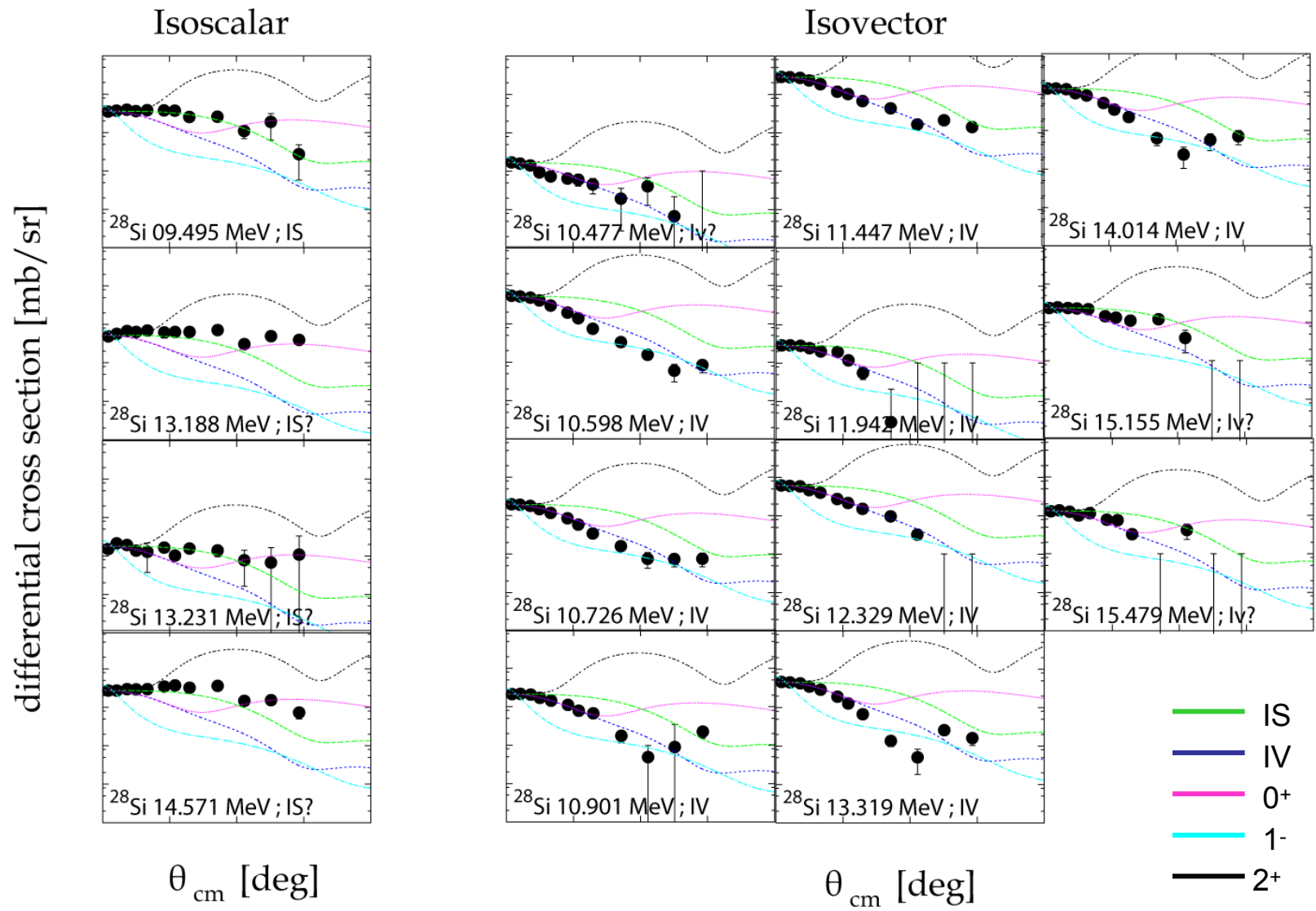
$^{28}\text{Si}(p,p')$  at  $E_p = 295$  MeV



- Forward peaking for  $L=0$  transition.
- $M1$  has the maximum at 0 degree.
- $0^+$ , IS- $1^+$ , IV- $1^+$  and others

- Distributions at 0-5 degree are similar.
- Difference between IS and IV is due to exchange tensor term.

# IS, IV spin- $M1$ angular dist. ( $^{28}\text{Si}$ )



# Unit cross section (UCS)

- Conversion factor from cross-section to Squared Nuclear Matrix Elements (SNME)
- Calibration from  $\beta$  and  $\gamma$ -decay measurements  
(on the assumption of the isospin symmetry).

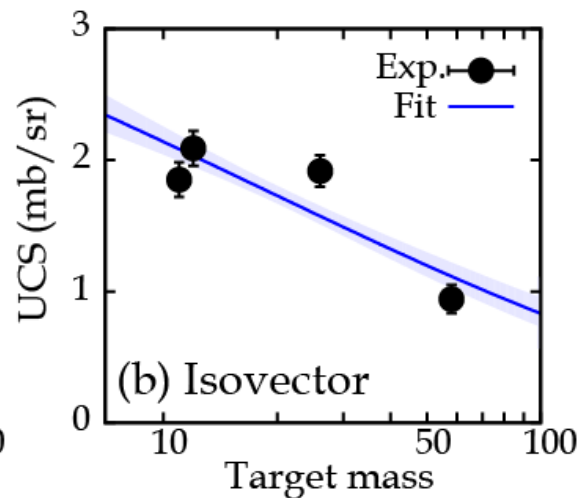
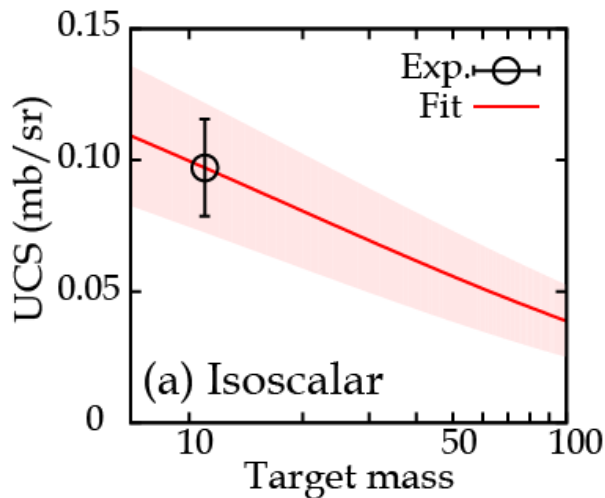
$$\frac{d\sigma}{d\Omega}(0^\circ) = \hat{\sigma}_T F(q, E_x) |M_f(O)|^2 \quad (T= IS \text{ or } IV)$$

UCS      Kinematical factor      SNME

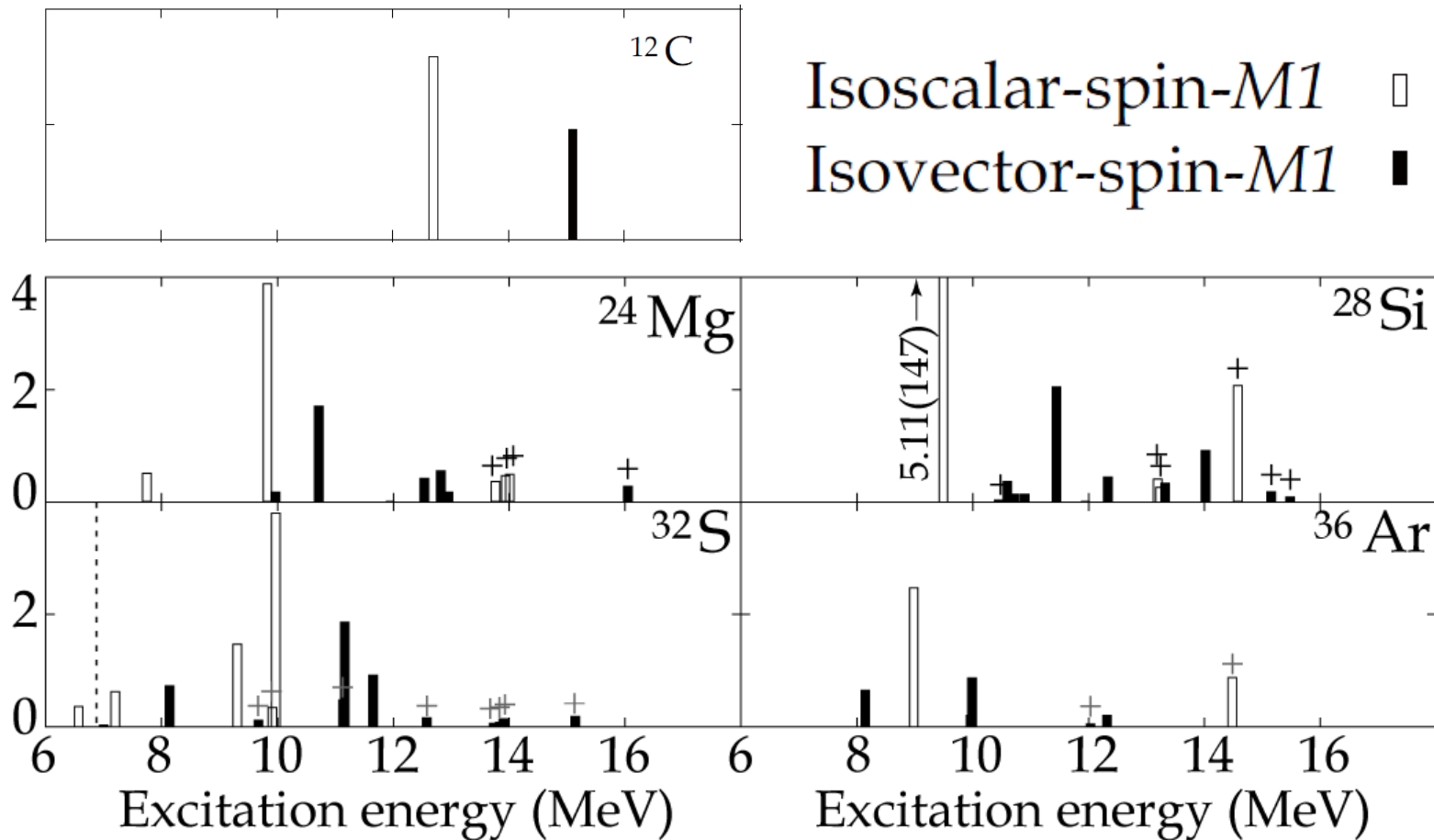
$$\hat{\sigma}_T(A) = N \exp(-x A^{1/3})$$

T.N. Taddeucci, NPA469 (1987).

- Function taken from the mass dependence of GT UCS

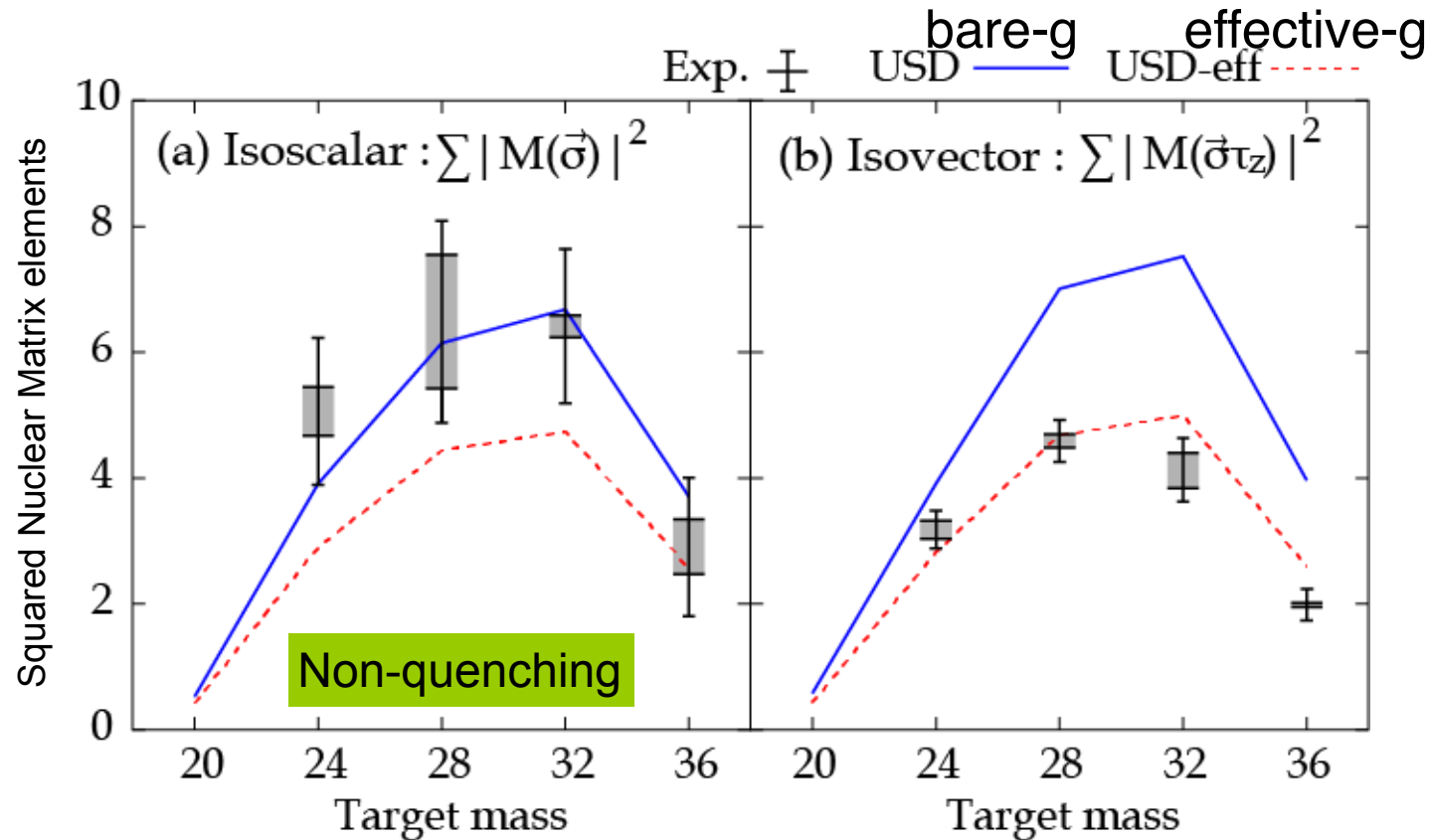


# IS / IV-spin-M1 distribution



# Spin-M1 SNME

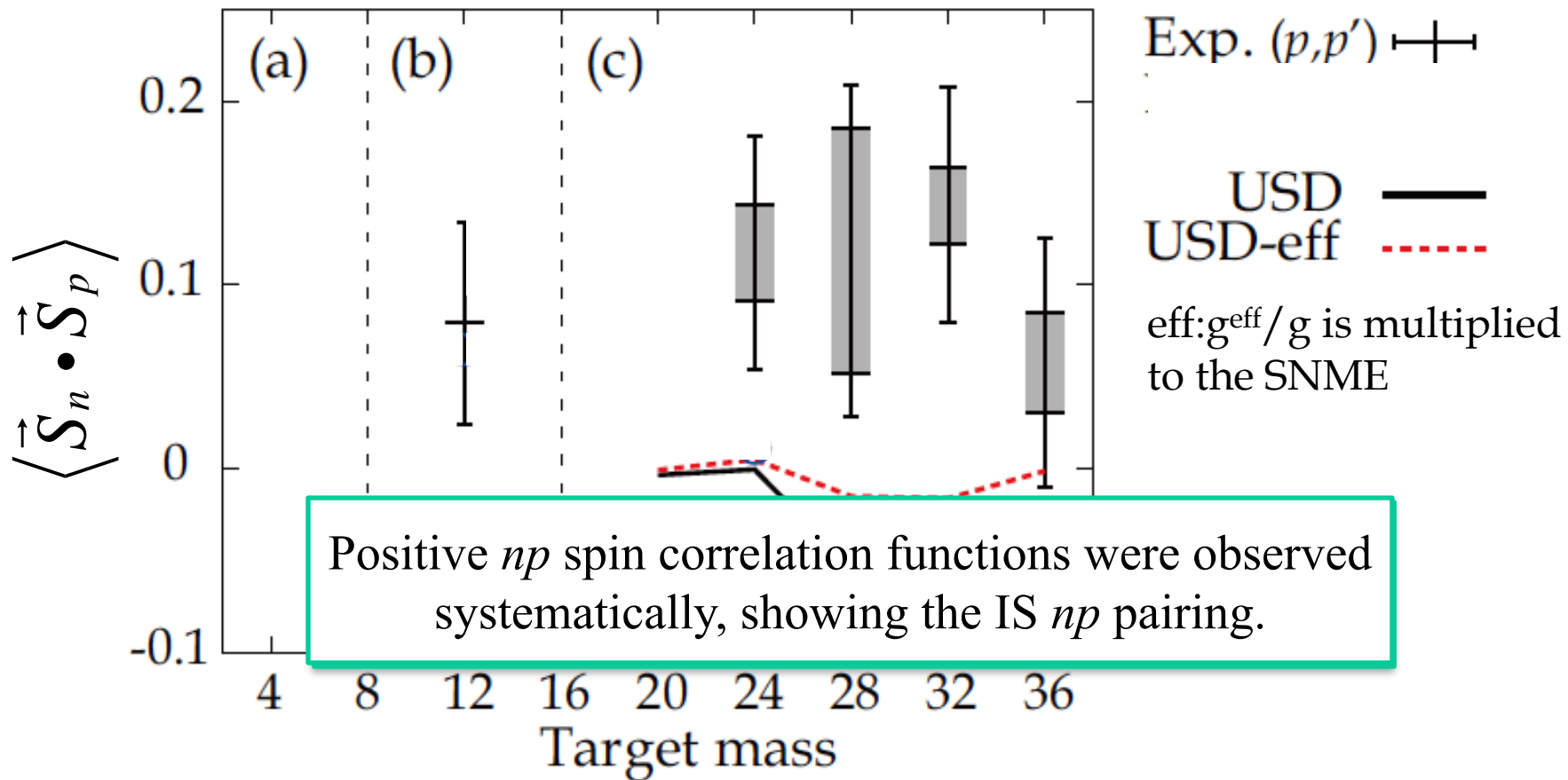
- Summed up to 16 MeV.
- Compared with shell-model predictions using the USD interaction



Isoscalar spin-M1 SNME is not quenching.

# $np$ Spin Correlation Function

Shell-Model: USD interaction



# Correlated Gaussian Calculation of the $^4\text{He}$ System with Realistic NN Interactions

by W. Horiuchi

Spin matrix elements of the  $^4\text{He}$  ground state

	$\langle \vec{S}_n^2 + \vec{S}_p^2 \rangle$	$\langle \vec{S}_n \cdot \vec{S}_p \rangle$	S=0	S=1	S=2
AV8' Stronger tensor int.	0.572	0.135	85.8%	0.4%	13.9%
G3RS Weaker tensor int.	0.465	0.109	88.5%	0.3%	11.3%
Minnesota No tensor int.	0.039	-0.020	100%	0%	0%

$$\vec{S} = \vec{S}_p + \vec{S}_n$$

Y. Suzuki, W. Horiuchi et al., FBS42, 33(2007)

H. Feldmeier, W. Horiuchi et al., PRC84, 054003(2011)



# $np$ Spin Correlation Function

Shell-Model: USD interaction

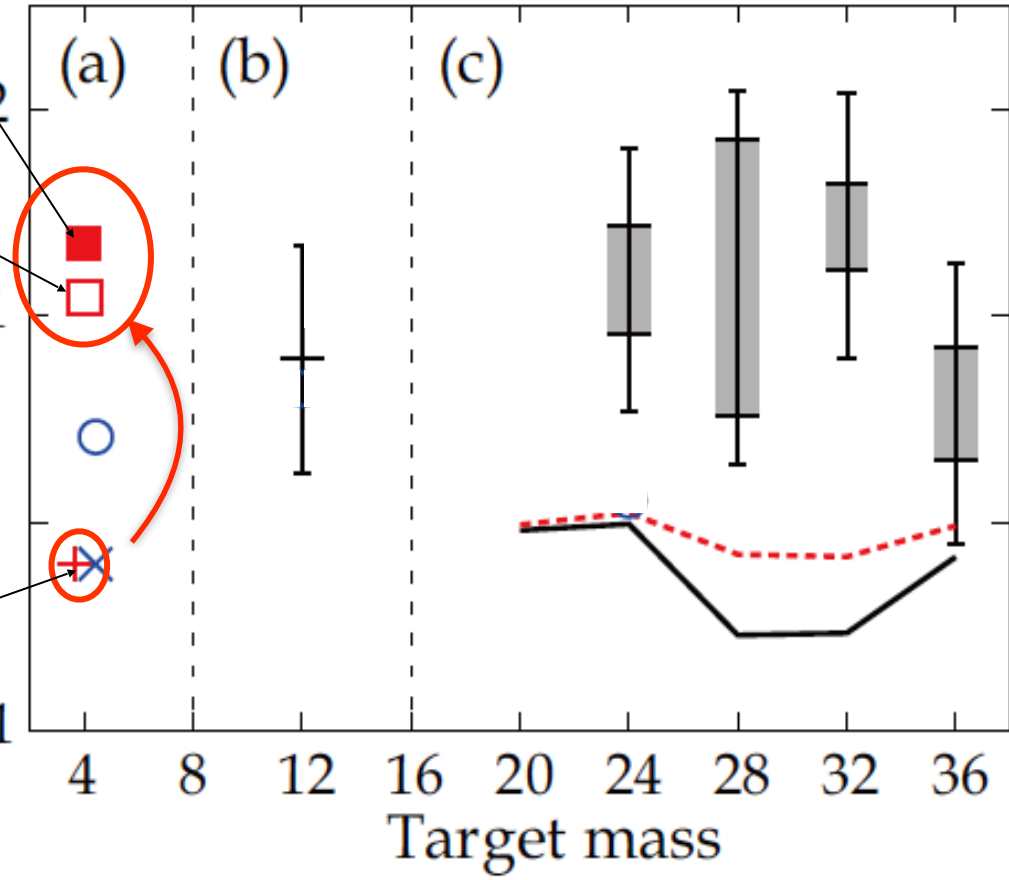
Correlated Gaussian Method: W. Horiuchi

AV8': 0.135  
(stronger tensor)

G3RS: 0.109  
(weaker tensor)

Minnesota:  
-0.020  
(no-tensor)

$$\langle \vec{S}_n \cdot \vec{S}_p \rangle$$



Exp.  $(p,p')$   $\times$

USD —

USD-eff - - -

CG-AV8'  $\blacksquare$

CG-G3RS  $\square$

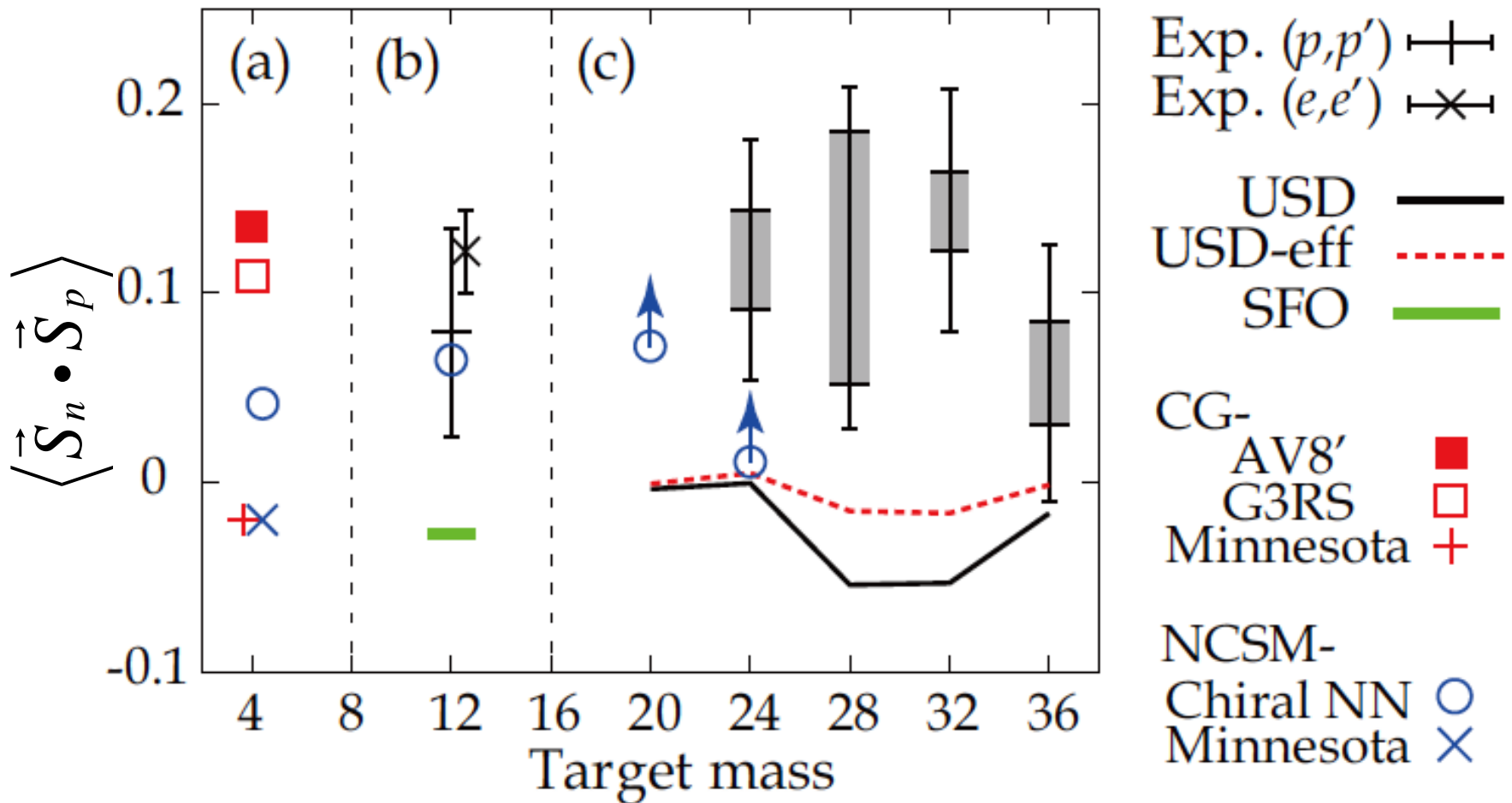
CG-Minnesota  $+$

# $np$ Spin Correlation Function

Shell-Model: USD interaction

Correlated Gaussian Method: W. Horiuchi

Non-Core Shell Model: P. Navratil

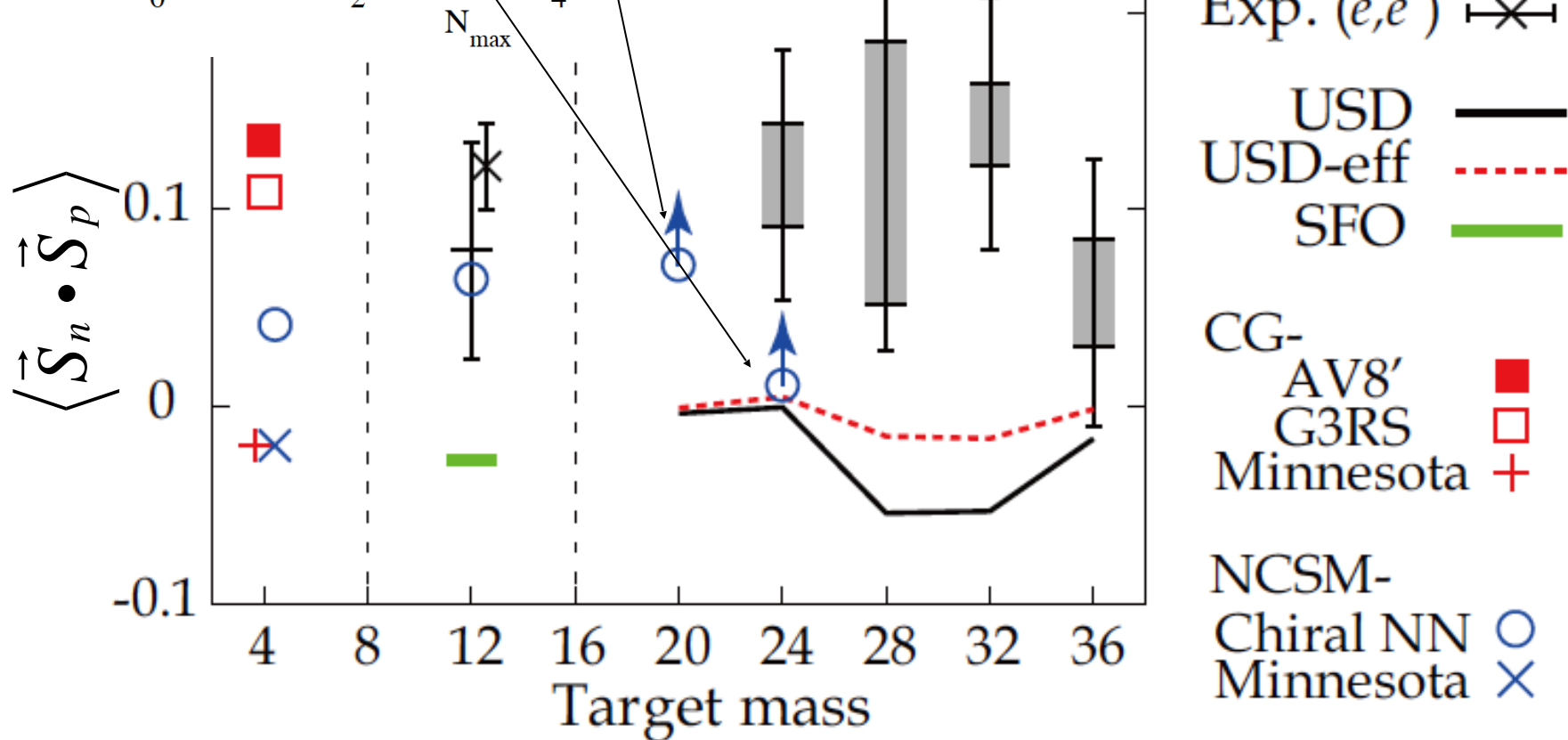
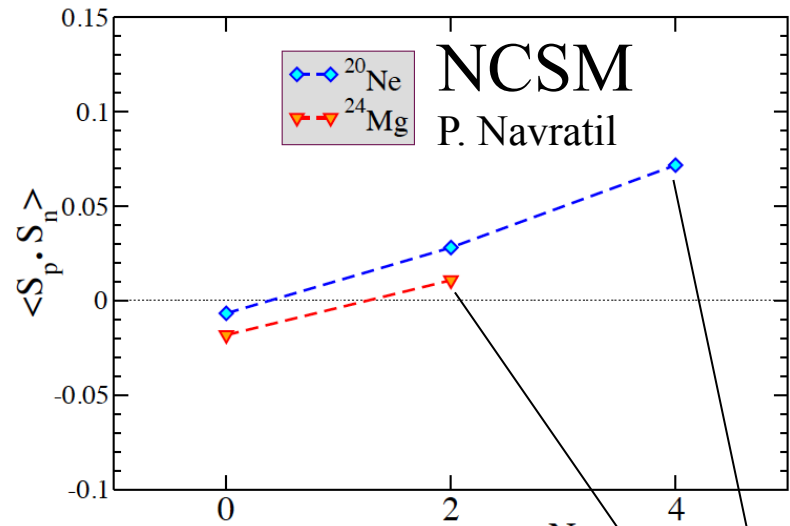


# Correlation Function

Correlated Gaussian Method: W. Horiuchi

Ion-Core Shell Model: P. Navratil

Shell-Model: USD interaction

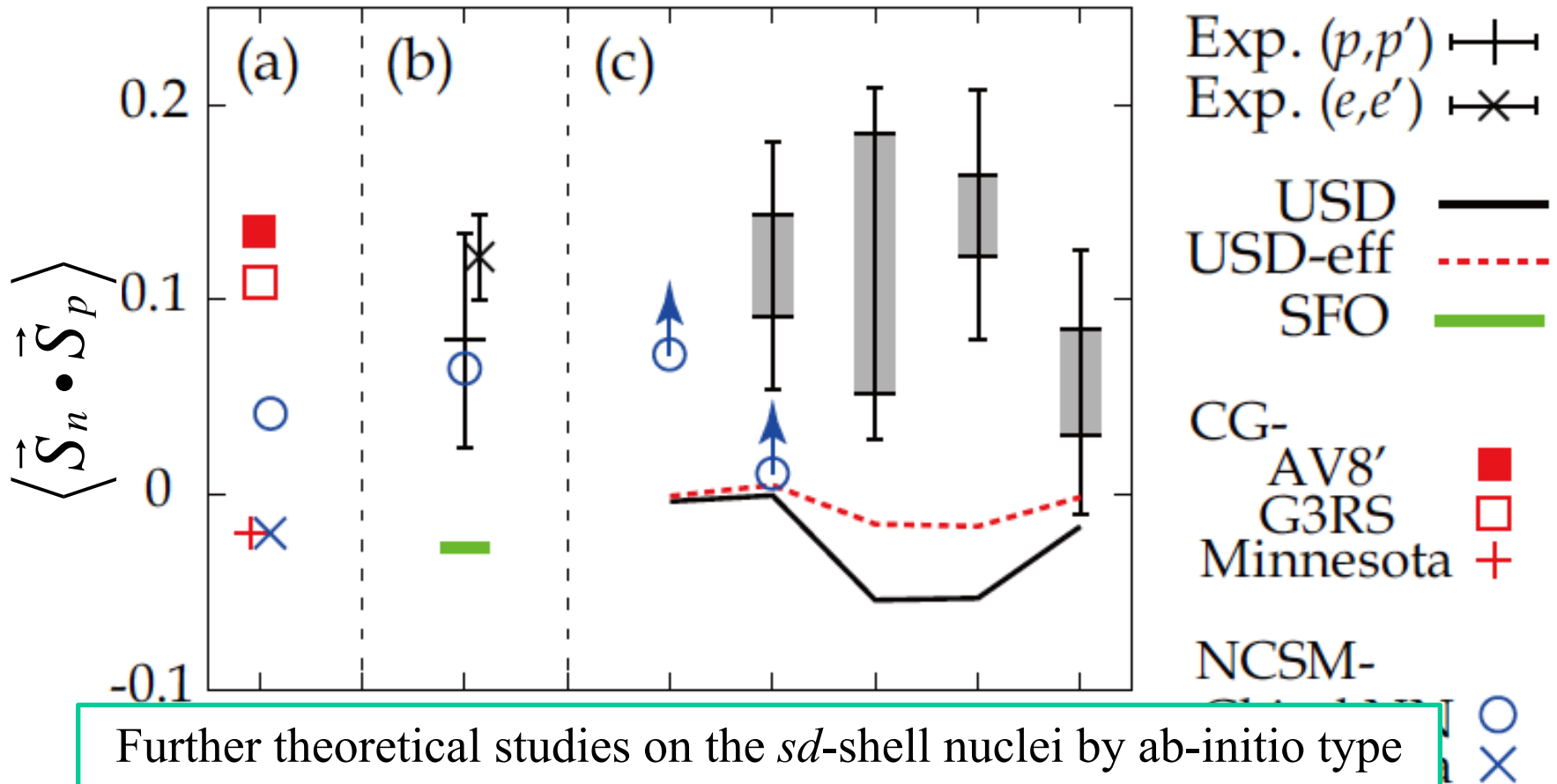


# *np* Spin Correlation Function

Shell-Model: USD interaction

Correlated Gaussian Method: W. Horiuchi

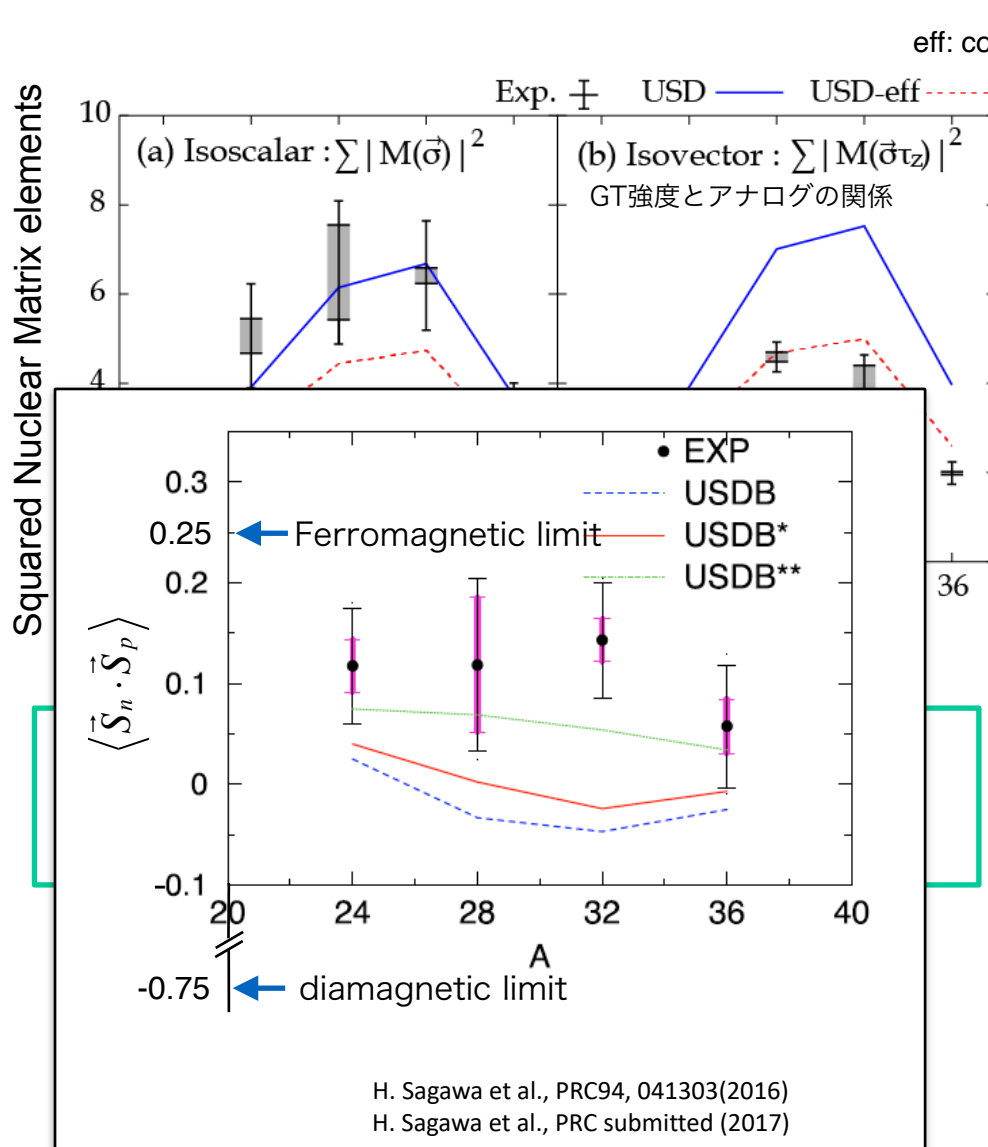
Non-Core Shell Model: P. Navratil



Further theoretical studies on the *sd*-shell nuclei by ab-initio type calculations with realistic interactions are very interesting.

# $np$ Spin Correlation Function

H. Matsubara et al., PRL115, 102501 (2015)

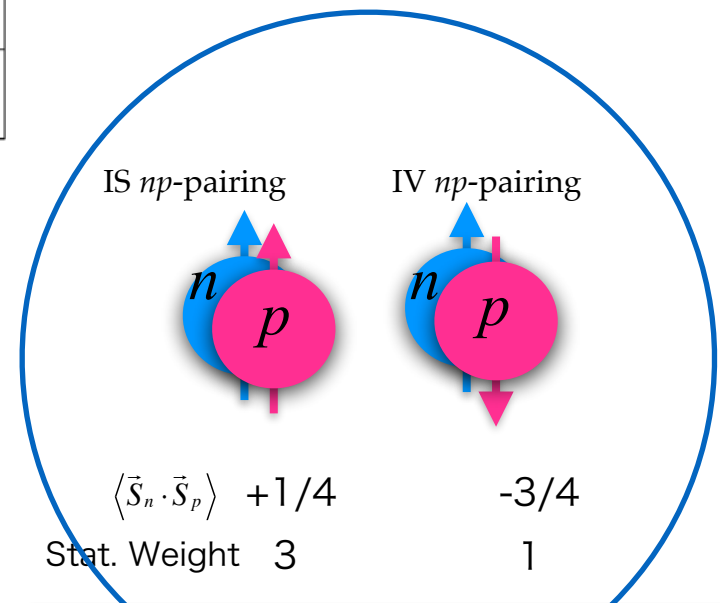


$np$  spin correlation function of a nuclear ground state

$$\langle \vec{S}_n \cdot \vec{S}_p \rangle = \frac{1}{16} \left( \sum |M(\vec{\sigma})|^2 - \sum |M(\vec{\sigma}\tau_z)|^2 \right)$$

IS - IV

> 0



IS  $np$  pairing is stronger than IV  $np$  pairing in  $N=Z$  nuclear ground states

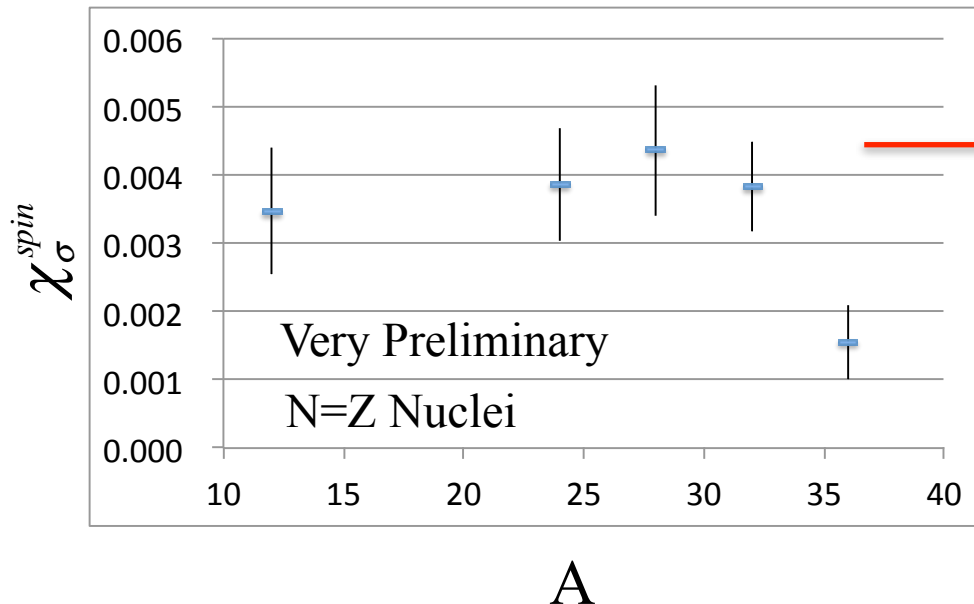
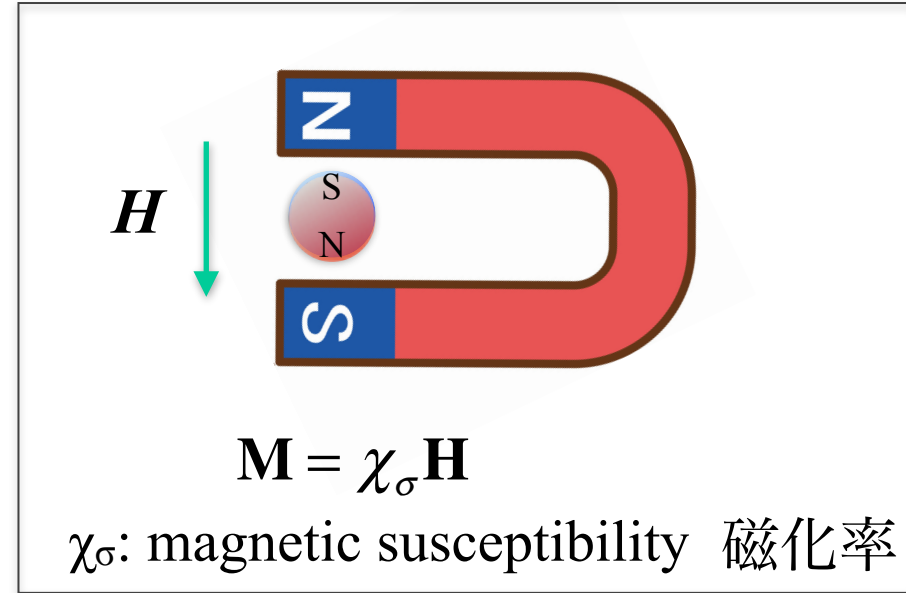
# Spin-Magnetic Susceptibility

Magnetic dipole ( $M1$ ) operator

$$O(M1) = g_\ell^{\text{IS}} \ell + \underline{g_s^{\text{IS}} \sigma} + g_\ell^{\text{IV}} \ell \cdot \tau + \underline{g_s^{\text{IV}} \sigma \cdot \tau}$$

IS(1) and IV( $\tau$ ) terms

$$\chi_\sigma^{\text{spin}} = \frac{8}{3N} \sum_f \frac{1}{\omega} \left| \langle f | \sum_i \sigma_i | 0 \rangle \right|^2$$



0.0044(7) MeV<sup>-1</sup> at  $\rho=0.16 \text{ fm}^{-3}$

Neutron Matter AFDMC model

G. Shen et al., PRC87, 025802 (2013)

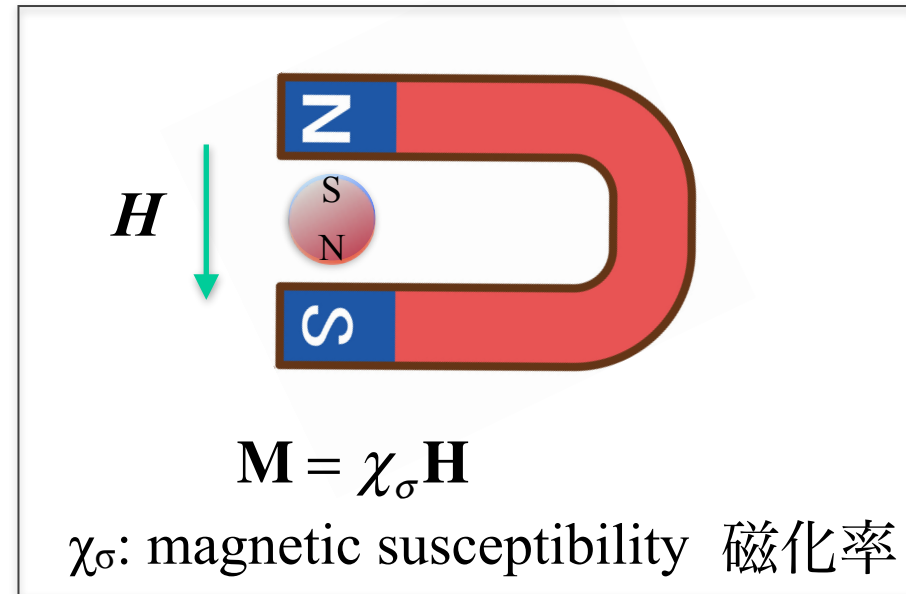
# Spin-Magnetic Susceptibility

Magnetic dipole ( $M1$ ) operator

$$O(M1) = g_\ell^{IS} \ell + \underline{g_s^{IS} \sigma} + g_\ell^{IV} \ell \cdot \tau + \underline{g_s^{IV} \sigma \cdot \tau}$$

IS(1) and IV( $\tau$ ) terms

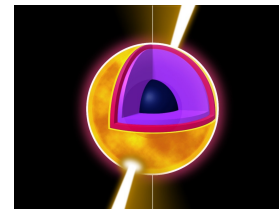
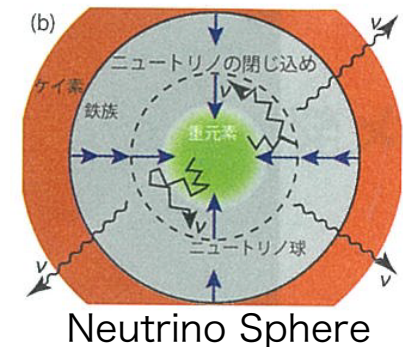
$$\chi_\sigma^{spin} = \frac{8}{3N} \sum_f \frac{1}{\omega} \left| \langle f | \sum_i \sigma_i | 0 \rangle \right|^2$$



- Spin part of magnetization of nuclear matter
- Magnetic response of nuclear matter (in e.g. magnetar)
- Neutrino trap in the core of supernova

Neutrino transparency

- Ferromagnetic state in a neutron star



Magnetar  $10^{14-16}$  Gauss